2.7 The power radiated by a lossless antenna is 10 watts. The directional characteristics of the antenna are represented by the radiation intensity of

(b) $U = B_o \cos^3\theta$ (watts/unit solid angle) $(0 \le \theta \le \pi/2, 0 \le \phi \le 2\pi)$

For each, find the

(a) maximum power density (in watts/square meter) at a distance of 1,000 m (assume far-field distance). Specify the angle where this occurs.

(b) exact and approximate beam solid angle Ω_A.

(c) directivity, exact and approximate, of the antenna (dimensionless and in dB).

(d) gain, exact and approximate, of the antenna (dimensionless and in dB).

a) Per (2-12), $U = r^2 W_{rad} \Rightarrow W_{rad} = \frac{U}{r^2}$

Since $r = 1000\,m$ is fixed, the maximum in W_{rad} occurs when $U = U_{max} = B_o \cos^3 0 = B_o$ @ $\theta_{max} = 0$.

To determine the constant B_o, use the eq'n (2-13)

$$P_{rad} = \oiint_{\Omega} U\, d\Omega \Rightarrow 10\,W = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} B_o \cos^3\theta \sin\theta\, d\theta\, d\phi$$

$$10 = B_o \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi/2} \cos^3\theta \sin\theta\, d\theta = B_o (\phi)\Big|_0^{2\pi} \left(-\frac{\cos^4\theta}{4}\right)\Big|_0^{\pi/2}$$

$$10 = B_o (2\pi - 0)\Big[\underbrace{-\frac{\cos^4 \pi/2}{4}}_{0} + \frac{\cos^4 0}{4}\Big) = B_o (\pi/2)$$

$\hookrightarrow B_o = \frac{20}{\pi}$

$$W_{rad,max} = \frac{U_{max}}{r^2}\Big|_{r=1000m} = \frac{B_o}{r^2}\Big|_{r=1000m} = \frac{20/\pi}{1000^2}$$

$$\boxed{W_{rad,max} = 6.3662 \times 10^{-6}\, W/m^2 = 6.3662\, \frac{\mu W}{m^2} \quad @\ \theta_{max} = 0}$$

b) Per (2-19), $U = B_o F(\theta,\phi) \Rightarrow B_o \cos^3\theta = B_o F(\theta,\phi)$

$\longrightarrow F(\theta,\phi) = \cos^3\theta \quad (0 \le \theta \le \pi/2)$

b) cont. Per (2-24), the exact beam solid angle is

$$\Omega_A = \frac{1}{F(\theta,\phi)|_{max}} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} F(\theta,\phi) \sin\theta \, d\theta \, d\phi$$

$$= \frac{1}{(\cos^3\theta)|_{max}} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \cos^3\theta \sin\theta \, d\phi \, d\theta \quad \text{where } \cos^3\theta|_{max} = 1 \ @ \ \theta_{max} = 0$$

$$= \frac{1}{1} \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi/2} \cos^3\theta \sin\theta \, d\theta = 2\pi \left(1/4\right)$$

$$\underline{\underline{\Omega_A = \pi/2 \ sr = 1.5708 \ sr}}$$

Per (2-26a), the approximate beam solid angle is

$\Omega_A \simeq \theta_{1r} \theta_{2r}$ where θ_{1r} & θ_{2r} are HPBWs

$$U(\theta_{hp}) = \frac{B_0}{2} = B_0 \cos^3\theta_{hp} \Rightarrow \cos\theta_{hp} = (0.5)^{1/3}$$

$$\theta_{hp} = \cos^{-1}(0.5)^{1/3} = 37.467^\circ = 0.65393 \text{ rad}$$

$$\theta_{1r} = \theta_{2r} = 2\theta_{hp} = 1.307856 \text{ rad}$$

$$\underline{\underline{\text{Approx. } \Omega_A \simeq 1.307856^2 = 1.71049 \ Sr}}$$

c) Per (2-21), $D(\theta,\phi) = D(\theta) = \dfrac{4\pi F(\theta,\phi)}{\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} F(\theta,\phi) \sin\theta \, d\theta \, d\phi}$

$$\underline{\underline{D(\theta) = \frac{4\pi \cos^3\theta}{2\pi (1/4)} = 8 \cos^3\theta \quad (0 \le \theta \le \pi/2, \ 0 \le \phi \le 2\pi)}}$$

(2-23) $\underline{\underline{D_{max} = D_0 = 4\pi/\Omega_A = 4\pi/(\pi/2) = 8 = 9.0309 \text{ dBi}}}$

(2-26) $\underline{\underline{D_{max} = D_0 \simeq 4\pi/\Omega_{A,approx} = \dfrac{4\pi}{1.71049} = 7.34666 = 8.6609 \text{ dBi}}}$

d) <u>Lossless</u> antenna $\Rightarrow$ $\underline{\underline{G = D}}$ (same answers!)