

**2.4** Find the half-power beamwidth (HPBW) and first-null beamwidth (FNBW), in radians and degrees, for the following normalized radiation intensities:

(b)  $U(\theta) = \cos^2 \theta$  ( $0 \leq \theta \leq 90^\circ, 0 \leq \phi \leq 360^\circ$ )

Find HPBW

$$U(\theta_{HP}) = 0.5 = \cos^2 \theta_{HP} \quad \text{w/in above range for } \theta$$

$$\hookrightarrow \theta_{HP} = \cos^{-1} \sqrt{0.5} = 0.7854 \text{ rad} = 45^\circ$$

$$\underline{\underline{HPBW = 2\theta_{HP} = 1.5708 \text{ rad} = \pi/2 = 90^\circ}}$$

Find FNBW

$$U(\theta_{Null}) = 0 = \cos^2 \theta_{Null} \quad \text{w/in above } \theta \text{ range}$$

$$\hookrightarrow \theta_{Null} = \cos^{-1}(0) = 1.5708 \text{ or } \pi/2 = 90^\circ$$

$$\underline{\underline{FNBW = 2\theta_{Null} = 3.14159 \text{ rad} = \pi = 180^\circ}}$$

2.4 Find the half-power beamwidth (HPBW) and first-null beamwidth (FNBW), in radians and degrees, for the following normalized radiation intensities:

(d)  $U(\theta) = \cos^2(2\theta)$  ( $0 \leq \theta \leq 90^\circ, 0 \leq \phi \leq 360^\circ$ )

Find HPBW

$$U(\theta_{HP}) = 0.5 = \cos^2(2\theta_{HP})$$

$$\hookrightarrow \cos(2\theta_{HP}) = \sqrt{0.5}$$

$$\hookrightarrow 2\theta_{HP} = \cos^{-1}(\sqrt{0.5})$$

$$\hookrightarrow \theta_{HP} = \frac{1}{2} \cos^{-1}(\sqrt{0.5}) = 0.3927 \text{ rad} = \frac{\pi}{8} = 22.5^\circ$$

w/in  $\theta$  range

$$\underline{\underline{HPBW = 2\theta_{HP} = 0.7854 \text{ rad} = \frac{\pi}{4} = 45^\circ}}$$

Find FNBW

$$U(\theta_{Null}) = 0 = \cos^2(2\theta_{Null})$$

w/in  $\theta$  range

$$\hookrightarrow \theta_{Null} = \frac{1}{2} \cos^{-1} 0 = 0.7854 \text{ rad} = \frac{\pi}{4} = 45^\circ$$

$$\underline{\underline{FNBW = 2\theta_{Null} = 1.5708 \text{ rad} = \frac{\pi}{2} = 90^\circ}}$$