Design an optimum (i.e., smallest possible) LPDA with a gain of **7.5 dBi** and input impedance of **75 Ω** to cover the over-the-air television channels 7-13 in the upper very high frequency (VHF) band [AKA: VHF High Band/Band III]. Use booms with a **7/8 inch** outer diameter and the available copper/brass tubing/pipes listed in the table given.

a) Tabulate design specifications

b) Show complete design procedure (e.g., design figures, spreadsheets, …) in a fashion similar to example given in class.

c) Make a **scale** drawing(s) of the final antenna designed (show booms & transmission line) that a machinist could take and use to build the antenna (use centimeters for all dimensions). Assume grounded boom will need to extend 60 cm past longest elements to allow the LPDA to be attached to an antenna mast. Allow 5 cm past the shortest elements and the longest element on the non-grounded boom for feed attachment and/or mechanical strength.

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a)

1. Select or specify design parameters
   a. Desired directivity (gain) ⇒ **7.5 dBi**
   b. Frequency range (f_{high} and f_{low})
   Per https://en.wikipedia.org/wiki/North_American_television_frequencies , VHF Channel 7 lower edge is at 174 MHz while Channel 13 upper edge is at 216 MHz
   ⇒ **f_{low} = 174 MHz & f_{high} = 216 MHz**
   c. Desired input impedance R_0 (real) ⇒ **R_0 = 75 Ω**

b)

2. Use graph [Balanis 4th Edn., Figure 11.13, p. 609] on following page, which shows contours of constant directivity versus σ (relative spacing) and τ (scale factor), to select σ and τ for the desired directivity.

⇒ **$\sigma = 0.1486$ and $\tau = 0.822$**

3. Calculate the apex half angle α using-

$$\alpha = \tan^{-1}\left(\frac{1-\tau}{4\sigma}\right) = \tan^{-1}\left(\frac{1-0.822}{4(0.1486)}\right) \Rightarrow \underline{\alpha = 16.67094^\circ}$$

$$\text{apex angle} \Rightarrow \underline{2\alpha = 33.34188^\circ}$$

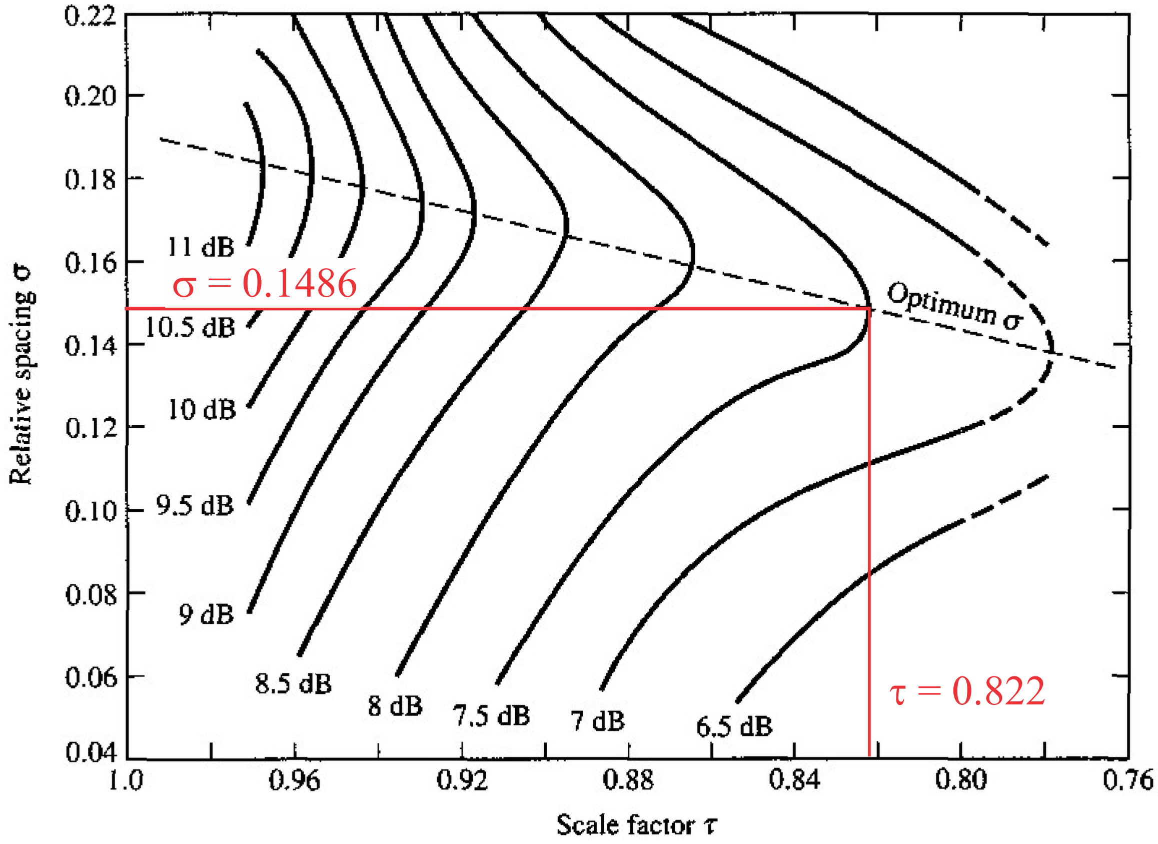
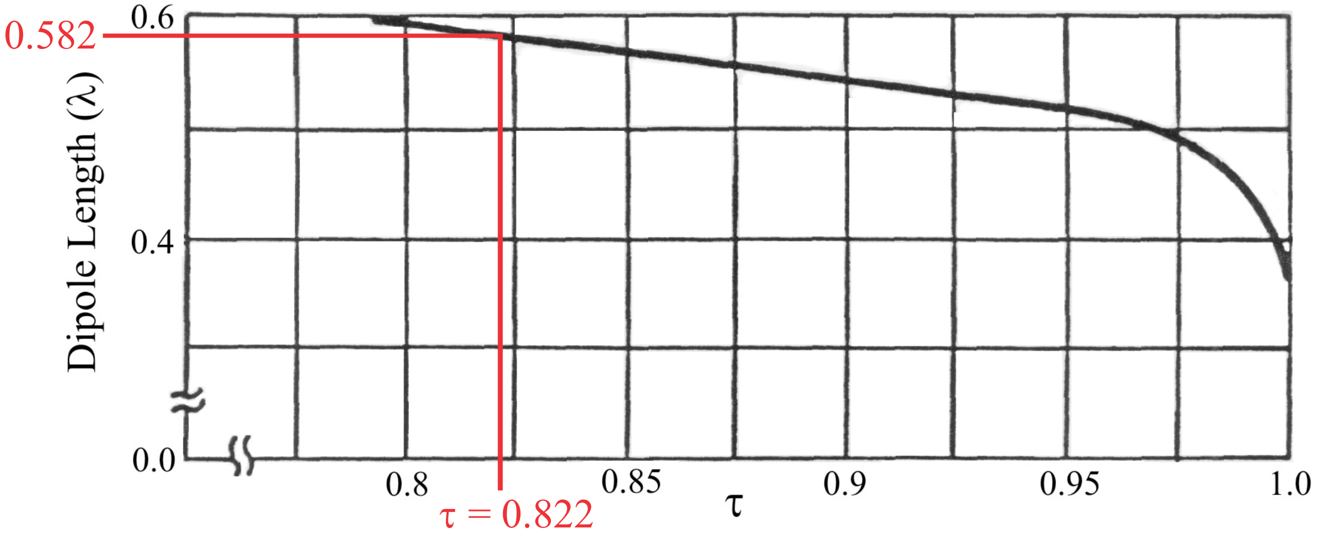


Figure 11.13 Computed contours of constant directivity versus σ and τ for log-periodic dipole arrays. [Balanis 4th Edn., p. 609]

4. Find length l_1 of the **longest** element of LPDA
   - take length in wavelengths from graph below since we are using optimum σ and τ;

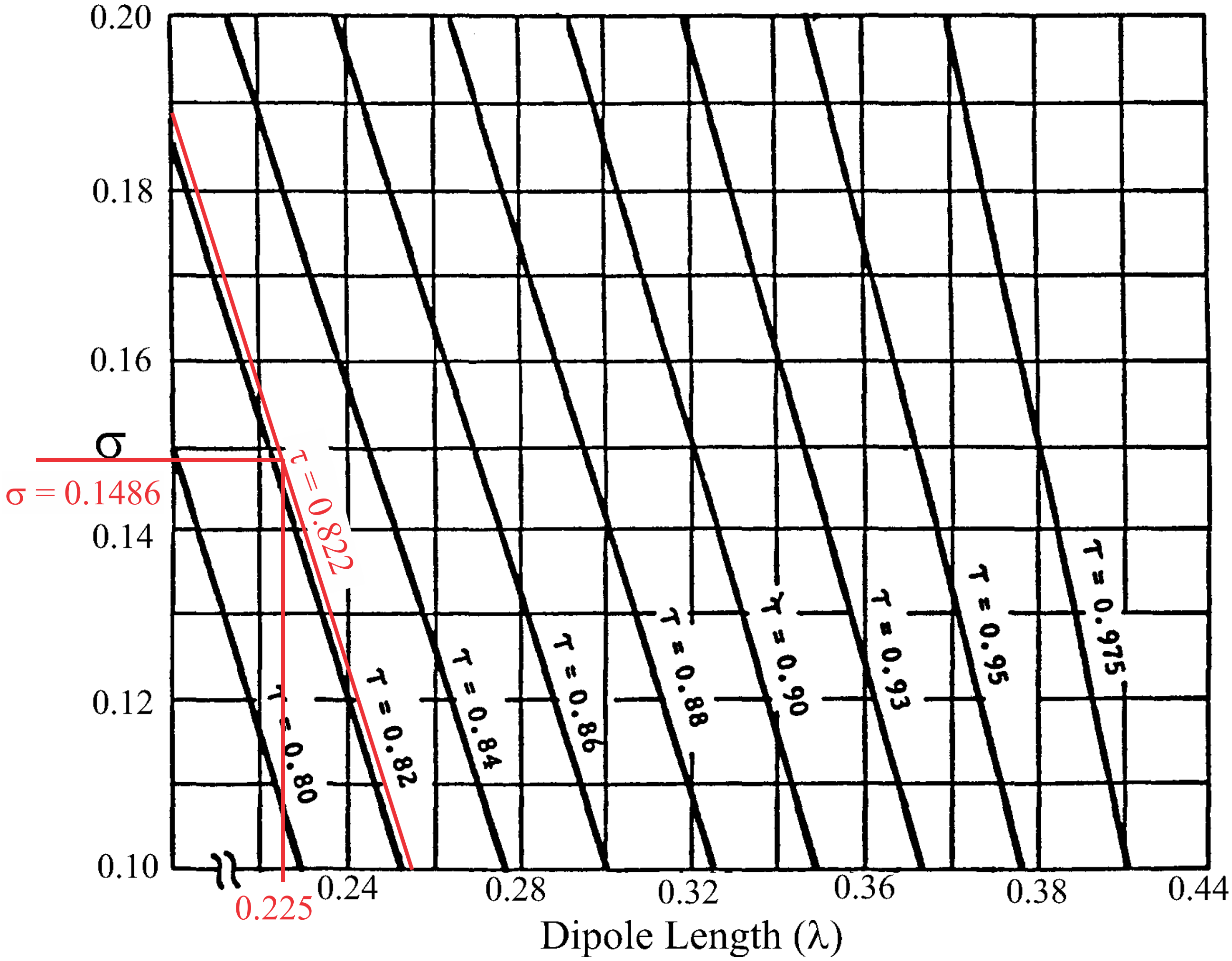
$\lambda_{max} = c/f_{low} = 2.9979\times10^8 / 174\times10^6 = 1.72293$ m ⇒ $\underline{\boldsymbol{l_1 = 0.582\,\lambda_{max} = 100.2746\ \text{cm}}}$



Measured length, normalized by λ_{max}, of longest dipole in LPDA versus optimum σ and τ.

5. Find length l_N of the **shortest** element of the LPDA
    - take length in wavelengths from graph where $\lambda_{\min} = c\,/\,f_{\text{high}}$ is the wavelength at the highest frequency in the desired frequency range.

$$\lambda_{\min} = c/f_{\text{high}} = 2.9979\times10^8 / 216\times10^6 = 1.38792\ \text{m} \qquad \Rightarrow \quad \underline{\mathbf{l_N = 0.225\,\lambda_{\min} = 31.228\ cm}}$$



Estimated length, normalized by $\lambda_{\min}$, of shortest dipole in LPDA versus σ and τ.

6. Calculate location R_1 of **longest** element (as measured from the apex)-

$$R_1 = \frac{l_1}{2}\cot(\alpha) = \frac{100.2746}{2\tan(16.67094^\circ)} \quad \Rightarrow \quad \underline{R_1 = 167.4248\ \text{cm}}$$

7. Calculate the total bandwidth B_s, includes additional bandwidth B_{ar} due to active region, using the specified bandwidth B-

$$B = f_{\text{high}}\,/\,f_{\text{low}} = 216\ \text{MHz}/174\ \text{MHz} = \underline{\mathbf{1.24138}}$$

$$B_{\text{ar}} = 1.1 + 7.7\,(1-\tau)^2\cot(\alpha) = 1.1 + 7.7\,(1-0.822)^2\cot(16.67094^\circ) = \underline{\mathbf{1.914685}}$$

$$B_s = B_{\text{ar}}\cdot B = 1.914685\cdot 1.24138 = \underline{\mathbf{2.37685}}$$

8. Calculate the approximate number N of elements required for design

$$N = 1 + \log_{10}(B_s)/\log_{10}(1/\tau) = 1 + \log_{10}(2.37685)/\log_{10}(1/0.822) = 5.4 \Rightarrow \underline{N \approx 6}$$

9. Calculate the approximate distance L_T between the longest and shortest elements.

$$L_T = \frac{l_1}{2}(1 - 1/B_s)\cot(\alpha) = \frac{100.2746}{2}(1 - \frac{1}{2.38})\cot(16.67094^\circ) \Rightarrow \underline{L_T = 96.985\text{ cm}}$$

10. Calculate the location R_2 (from the apex) and length l_2 of the second longest element using the scale factor τ, R_1, and l_1 -

$$R_2 = R_1\,\tau = 167.4248\,(0.822) \quad \Rightarrow \quad \underline{R_2 = 137.62\text{ cm}}$$

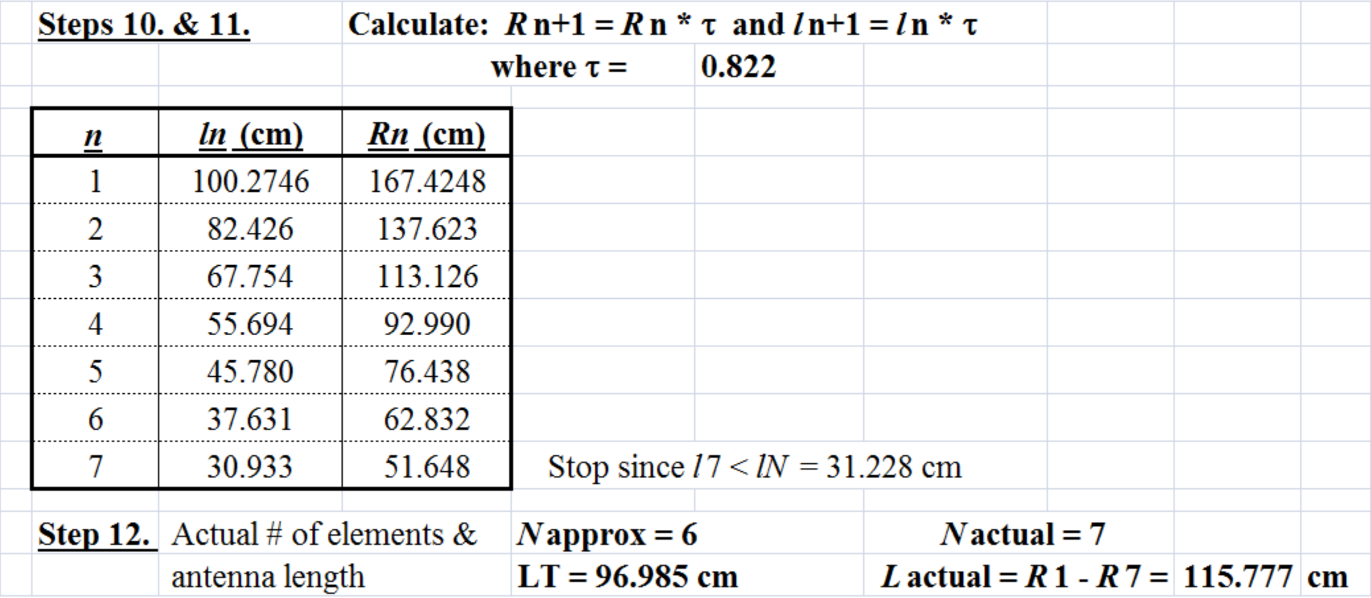
and

$$l_2 = l_1\,\tau = 100.2746\,(0.822) \quad \Rightarrow \quad \underline{l_2 = 82.43\text{ cm}}$$

11. Recursively calculate the location R_{n+1} and length l_{n+1} of the n+1th element(s) using the scale factor τ, R_n, and l_n – e.g., $R_{n+1} = R_n\,\tau = R_n\,(0.822)$ & $l_{n+1} = l_n\,\tau = l_n\,(0.822)$. Stop when l_{n+1} is less than or equal to l_N (calculated in step 5.).

12. Count actual number of elements and calculate actual length of LPDA (compare to approximate calculations in steps 8. & 9.).

- Steps 10-12 done using MS-Excel spreadsheet shown below.



Steps 10. & 11. **Calculate: $R_{n+1} = R_n * \tau$ and $l_{n+1} = l_n * \tau$**

where τ = 0.822

n	ln (cm)	Rn (cm)	
1	100.2746	167.4248	
2	82.426	137.623	
3	67.754	113.126	
4	55.694	92.990	
5	45.780	76.438	
6	37.631	62.832	
7	30.933	51.648	Stop since $l7 < lN = 31.228$ cm

Step 12.	Actual # of elements &	**Napprox = 6**	**Nactual = 7**
	antenna length	**LT = 96.985 cm**	**Lactual = R1 - R7 = 115.777 cm**

13. Select a length to diameter ratio $K = l/d$ for the elements of the LPDA. This choice is a compromise between mechanical strength for the largest and smallest elements, available tubing sizes, and the selected diameter of the boom.

Choose boom diameter ***D* = 7/8" = 2.2225 cm**

If d_1 = 5/8" = 1.5875 cm, then $K_1 = l_1/d_1 = 100.2746/1.5875 = 63.165$

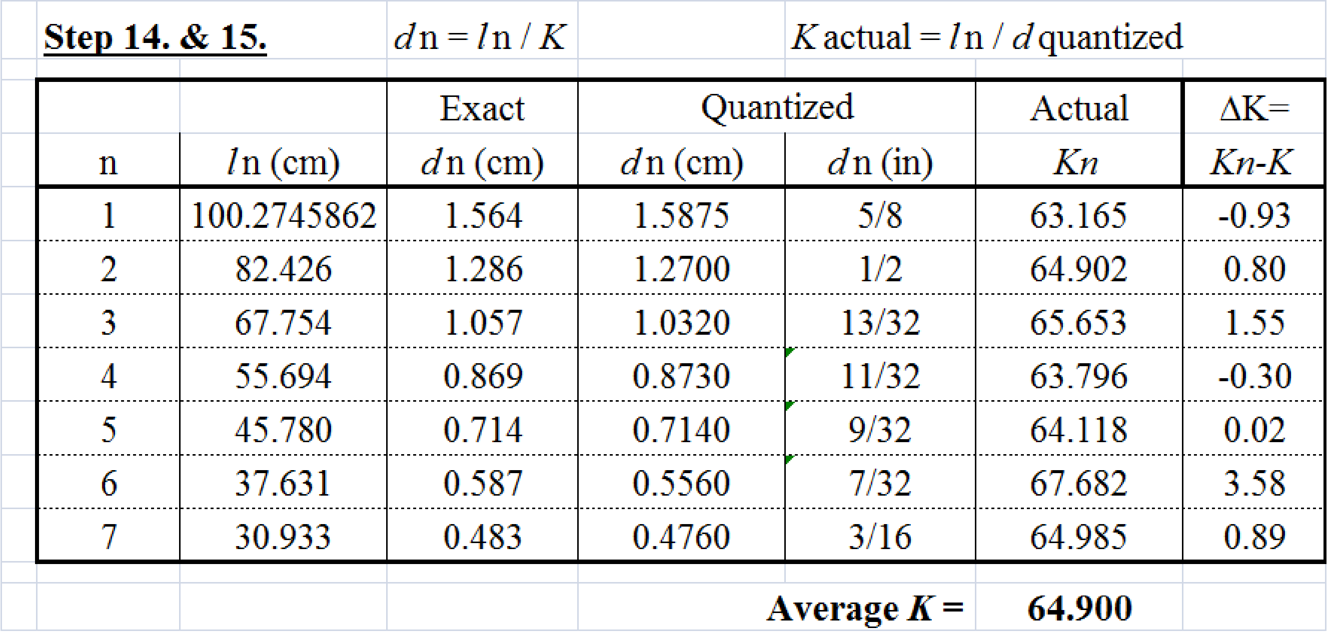
If d_7 = 3/16" = 0.47625 cm, then $K_7 = l_7/d_{12} = 30.933/0.47625 = 64.951$

Select K to be ~average of the two values above ⇒ ***K* = 64.1**.

14. Calculate the diameter $d_n = l_n / K$ for each element. Then, select the closest available tube/pipe/rod diameter to the calculated value.

15. Calculate the actual length to diameter ratio K_n for each element and the average length to diameter ratio K_{ave} after quantization. Check for unusually large deviations from desired K (may want to go back to step 13. and select another value of K).

- Steps 14-15 done using MS-Excel spreadsheet shown below.



Step 14. & 15. $d_n = l_n / K$ $K_{actual} = l_n / d_{quantized}$

		Exact	Quantized		Actual	ΔK=
n	l_n (cm)	d_n (cm)	d_n (cm)	d_n (in)	K_n	K_n-K
1	100.2745862	1.564	1.5875	5/8	63.165	-0.93
2	82.426	1.286	1.2700	1/2	64.902	0.80
3	67.754	1.057	1.0320	13/32	65.653	1.55
4	55.694	0.869	0.8730	11/32	63.796	-0.30
5	45.780	0.714	0.7140	9/32	64.118	0.02
6	37.631	0.587	0.5560	7/32	67.682	3.58
7	30.933	0.483	0.4760	3/16	64.985	0.89
				Average *K* =	**64.900**	

16. Calculate the approximate average characteristic impedance of the active region elements-

$$Z_a = 60\ln(2XK_{ave}/\pi) = 60\ln[2(0.53633)\,64.900/\pi] \quad \Rightarrow \quad \underline{Z_a = 185.8956\ \Omega}$$

where $X = 8\tau\sigma/(1+\tau) = 8\,(0.822)\,0.1486/(1+0.822) = 0.53633$.

17. Find the characteristic impedance of the unloaded transmission line Z_0 for the desired input impedance R_0-

$$\begin{aligned} Z_0 &= \frac{R_0^{\ 2}}{4Z_aX} + R_0\sqrt{\left(\frac{R_0}{4Z_aX}\right)^2 + 1} \\ &= \frac{75^2}{4(185.8956)0.53633} + 75\sqrt{\left(\frac{75}{4(185.8956)0.53633}\right)^2 + 1} \\ &= 14.10462 + 76.3147 \\ &= \underline{90.4194\Omega} \end{aligned}$$

18. Calculate the **center-to-center** spacing S of the booms using the unloaded, cylindrical, twin-lead transmission line formula-

$$S = D\cosh(Z_0/120) = 2.2225\cosh(90.4194/120) \quad \Rightarrow \quad \underline{\mathbf{S = 2.8838\ cm}}$$

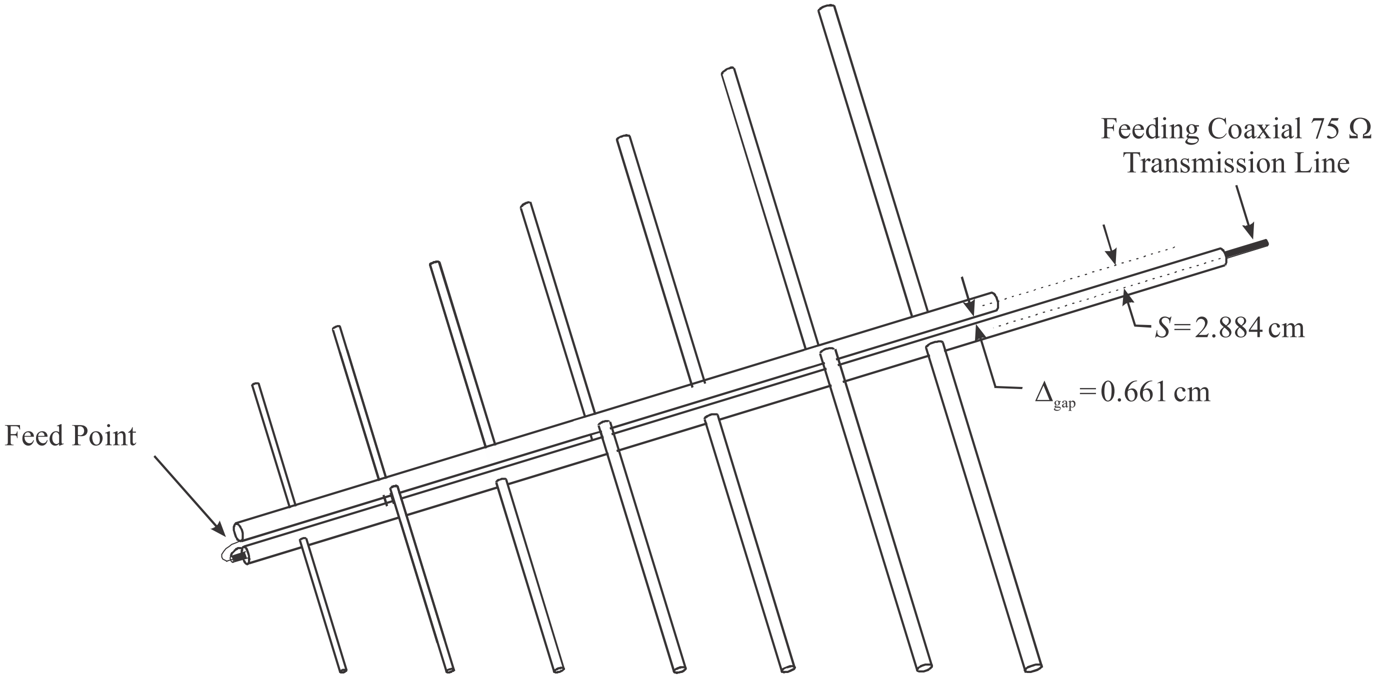
where D is the diameter of the booms (assumed to be identical). The air gap Δ_{gap} between the inner surfaces of the booms is-

$$\Delta_{\text{gap}} = S - D = 2.8838 - 2.2225 \quad \Rightarrow \quad \underline{\mathbf{\Delta_{gap} = 0.661\ cm = 6.61\ mm}}.$$

c) Make a **scale** drawing(s) of the final antenna designed (show booms & transmission line) that a machinist could take and use to build the antenna (use centimeters for all dimensions). Assume grounded boom will need to extend 60 cm past longest elements to allow the LPDA to be attached to an antenna mast. Allow 5 cm past the shortest elements and the longest element on the non-grounded boom for feed attachment and/or mechanical strength.

see following pages

Perspective View of 7 Element LPDA for VHF Ch 7-13

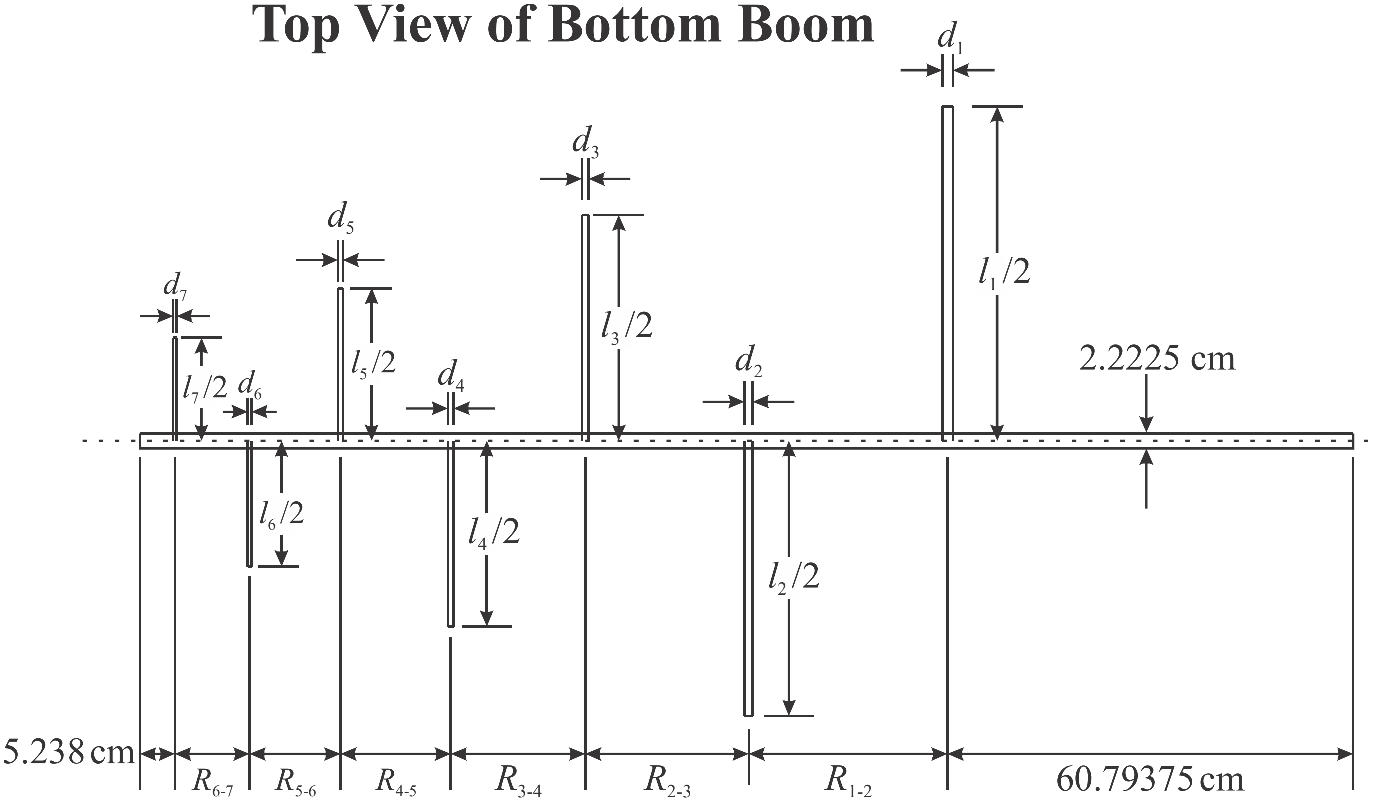


Not to scale. Dimensions shown on following views of bottom and top booms

Table of Dimensions for following Top and Bottom booms

n	ln (cm)	dn (cm)	Rn (cm)	element-element spacing	
1	100.275	1.5875	167.425		(cm)
2	82.426	1.2700	137.623	R1 - R2	29.802
3	67.754	1.0320	113.126	R2 - R3	24.497
4	55.694	0.8730	92.990	R3 - R4	20.136
5	45.780	0.7140	76.438	R4 - R5	16.552
6	37.631	0.5560	62.832	R5 - R6	13.606
7	30.933	0.4760	51.648	R6 - R7	11.184

Top View of Bottom Boom



Top View of Top Boom