Design an optimum (i.e., smallest possible) LPDA with a gain of 7.5 dBi and input impedance of 75 Ω to cover the over-the-air television channels 14-51 in the ultra high frequency (UHF) band. Use booms with a 5/8 inch outer diameter and the available copper/brass tubing/pipes listed in the table given with the largest elements having a 1/2 inch outer diameter.

a) Tabulate design specifications

b) Show complete design procedure (e.g., design figures, spreadsheets, …) in a fashion similar to examples given in class.

c) Make a **scale** drawing(s) of the final antenna design (i.e., show booms & transmission line) that a machinist could take and use to build the antenna. Use centimeters for all dimensions. Assume grounded boom will extend 40 cm past longest elements to allow the LPDA to be attached to an antenna mast. Allow 2 cm past the shortest elements and the longest element on the non-grounded boom for feed attachment and/or mechanical strength.

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a)

1. Select or specify design parameters

   a. Desired directivity (gain) ⇒ **7.5 dBi**

   b. Frequency range (f_{high} and f_{low})-

   Per https://en.wikipedia.org/wiki/North_American_television_frequencies, the UHF Channel 14 lower edge is at 470 MHz while Channel 51 upper edge is at 698 MHz

   ⇒ $\mathbf{f_{low} = 470\ MHz}$ **&** $\mathbf{f_{high} = 698\ MHz}$

   c. Desired input impedance R_0 (real) ⇒ $\mathbf{R_0 = 75\ \Omega}$

Parameter	Value(s)
Directivity/gain	7.5 dBi
Frequency band/range	470-698 MHz
Input impedance	75 Ω

b)

2. Use graph (Figure 11.13) on following page, which shows contours of constant directivity versus σ (relative spacing) and τ (scale factor), to select σ and τ for the desired directivity. ⇒ $\mathbf{\sigma = 0.149}$ **and** $\mathbf{\tau = 0.822}$

3. Calculate the apex half angle α using-

$$\alpha = \tan^{-1}\left(\frac{1-\tau}{4\sigma}\right) = \tan^{-1}\left(\frac{1-0.822}{4(0.149)}\right) \Rightarrow \alpha = 16.629^\circ$$

apex angle ⇒ $2\alpha = 33.257^\circ$

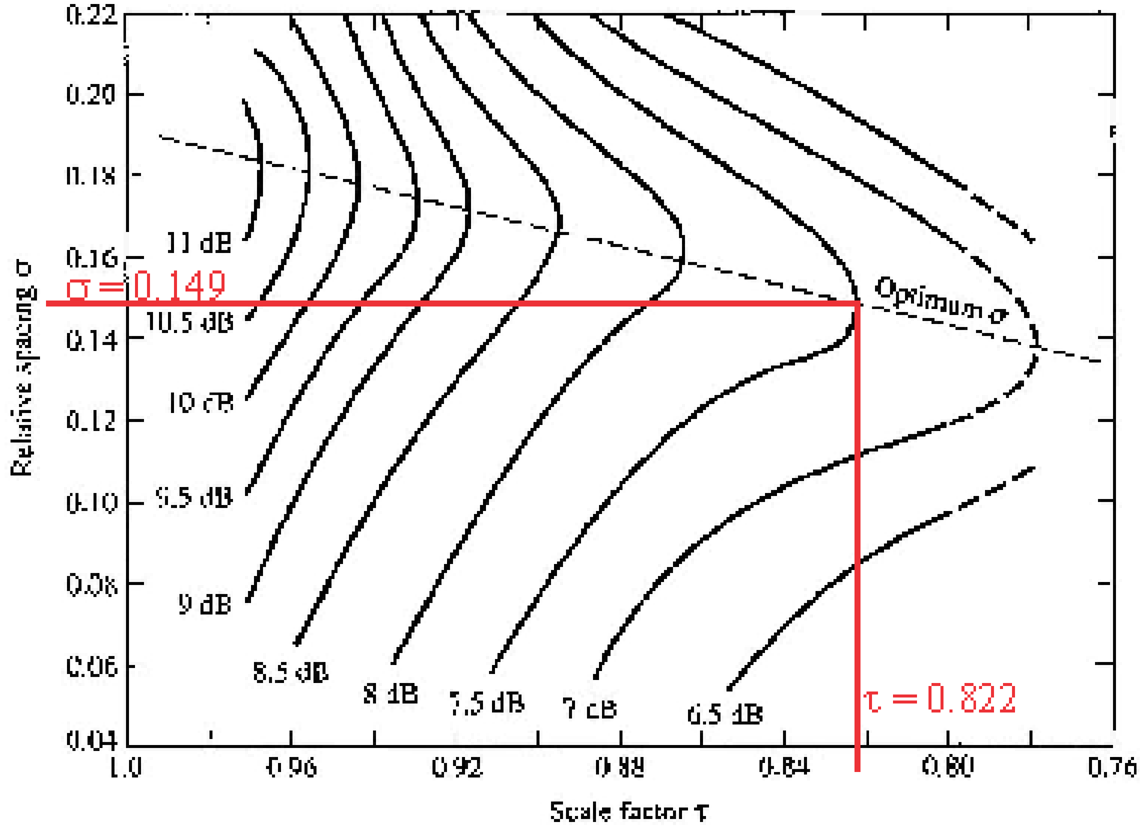
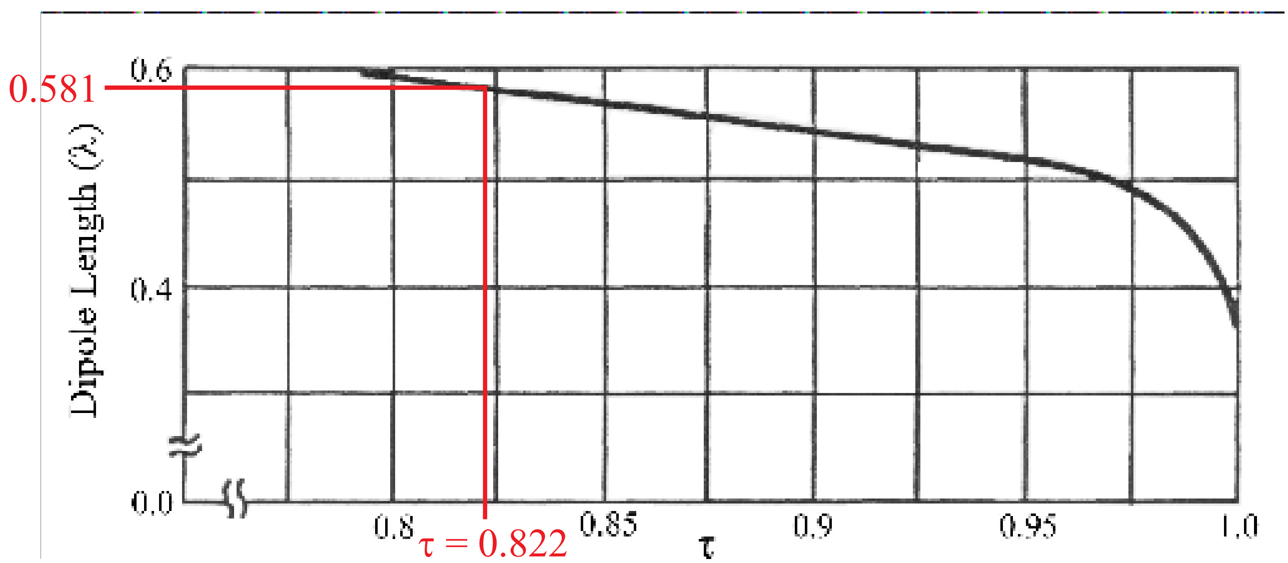


Figure 11.13 Computed contours of constant directivity versus σ and τ for log-periodic dipole arrays. [Balanis 4th Edn., p. 609]

4. Find length l_1 of the **longest** element of LPDA
   - take length in wavelengths from graph below since we are using optimum σ and τ;

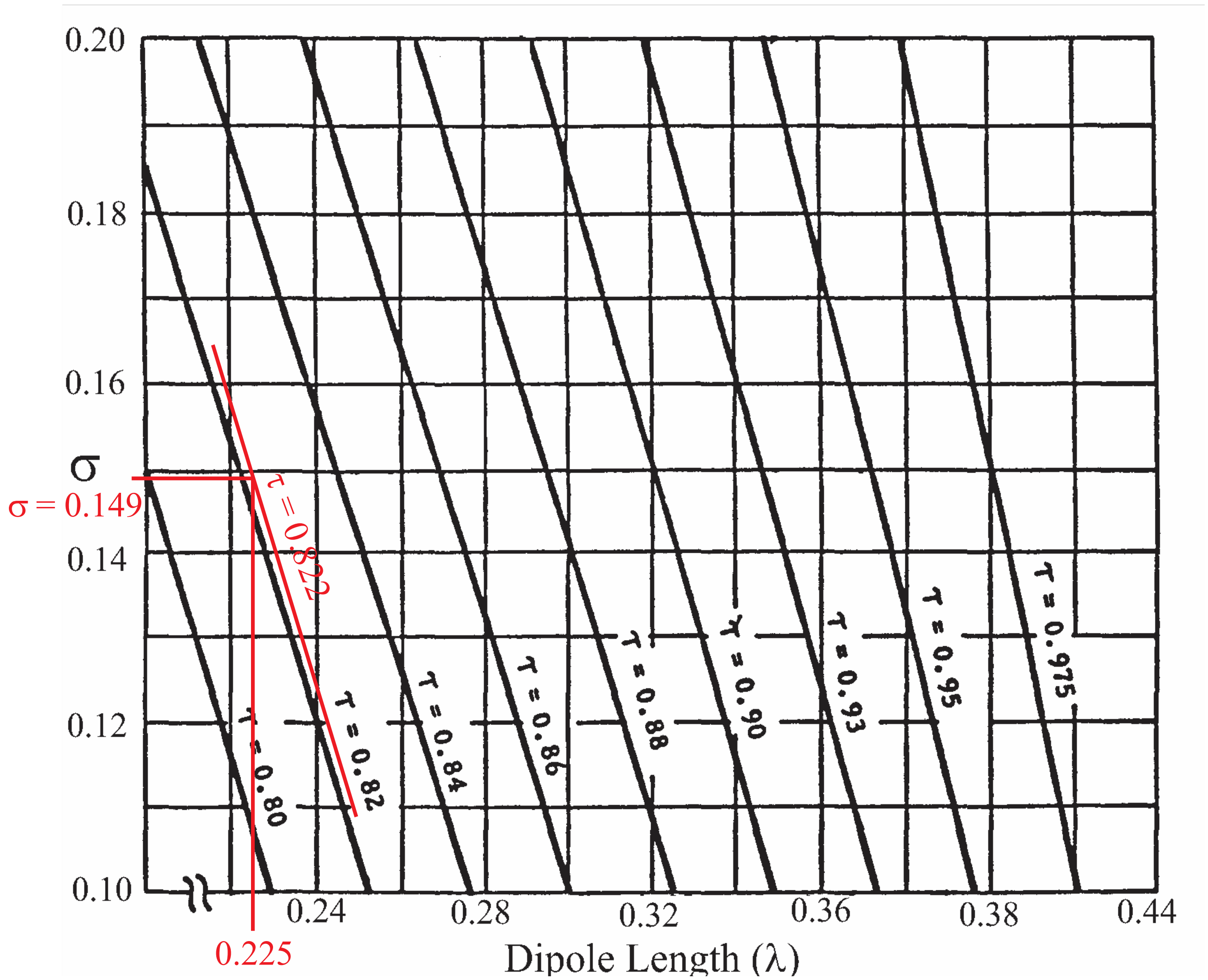
$\lambda_{max} = c/f_{low} = 2.998\times10^8 / 470\times10^6 = 63.7872$ cm ⇒ **$l_1 = 0.581\,\lambda_{max} = 37.060$ cm**



Measured length, normalized by λ_{max}, of longest dipole in LPDA versus optimum σ and τ.

5. Find length l_N of the **shortest** element of the LPDA
   - take length in wavelengths from graph where $\lambda_{min} = c / f_{high}$ is the wavelength at the highest frequency in the desired frequency range.

$\lambda_{min} = c/f_{high} = 2.998\times10^8 / 698\times10^6 = 42.9513$ m $\Rightarrow$ $\underline{\boldsymbol{l_N = 0.225\,\lambda_{min} = 9.664 \text{ cm}}}$



Estimated length, normalized by λ_{min}, of shortest dipole in LPDA versus σ and τ.

6. Calculate location R_1 of **longest** element (as measured from the apex)-

$$R_1 = \frac{l_1}{2}\cot(\alpha) = \frac{37.0604}{2\tan(16.629°)} \Rightarrow \underline{R_1 = 62.045 \text{ cm}}$$

7. Calculate the total bandwidth B_s, includes additional bandwidth B_{ar} due to active region, using the specified bandwidth B-

$B = f_{high} / f_{low} = 698$ MHz/470 MHz $\Rightarrow$ $\underline{\boldsymbol{B = 1.48511}}$

$B_{ar} = 1.1 + 7.7\,(1\text{-}\tau)^2 \cot(\alpha) = 1.1 + 7.7\,(1\text{-}0.822)^2 \cot(16.629°) = \underline{\mathbf{1.91688}}$

$B_s = B_{ar} \cdot B = 1.91688 \cdot 1.48511 = \underline{\mathbf{2.84677}}$

8. Calculate the approximate number N of elements required for design

$$N = 1 + \log_{10}(B_s)/\log_{10}(1/\tau) = 1 + \log_{10}(2.84677)/\log_{10}(1/0.822) = 6.33 \quad \Rightarrow \quad \underline{N \approx 6}$$

9. Calculate the approximate distance L_T between the longest and shortest elements.

$$L_T = \frac{l_1}{2}(1 - 1/B_s)\cot(\alpha) = \frac{37.06}{2}\left(1 - \frac{1}{2.84677}\right)\cot(16.629°) \quad \Rightarrow \quad \underline{L_T = 40.25 \text{ cm}}$$

10. Calculate the location R_2 (from the apex) and length l_2 of the second longest element using the scale factor τ, R_1, and l_1 -

$$R_2 = R_1\,\tau = 62.045\,(0.822) \quad \Rightarrow \quad \underline{R_2 = 51.001 \text{ cm}}$$

and

$$l_2 = l_1\,\tau = 37.06\,(0.822) \quad \Rightarrow \quad \underline{l_2 = 30.464 \text{ cm}}$$

11. Recursively calculate the location R_{n+1} and length l_{n+1} of the n+1[th] element(s) using the scale factor τ, R_n, and l_n – e.g., $R_{n+1} = R_n\,\tau = R_n\,(0.822)$ & $l_{n+1} = l_n\,\tau = l_n\,(0.822)$. Stop when l_{n+1} is less than or equal to l_N (calculated in step 5.).

12. Count actual number of elements and calculate actual length of LPDA (compare to approximate calculations in steps 8. & 9.).

- Steps 10-12 done using MS-Excel spreadsheet shown below.

Steps 10. & 11.		**Calculate: $R_{n+1} = R_n * \tau$ and $l_{n+1} = l_n * \tau$**					
		where τ =	**0.822**				
n	**l_n (cm)**	**R_n (cm)**					
1	37.06038	62.04491					
2	30.464	51.001					
3	25.041	41.923					
4	20.584	34.461					
5	16.920	28.327					
6	13.908	23.284					
7	11.432	19.140					
8	9.398	15.733	Stop since $l8 < lN = 9.664$ cm				
Step 12.	Actual # of elements &	**N_{approx} = 6**		**N_{actual} = 8**			
	antenna length	**L_T = 40.25 cm**		**$L_{actual} = R_1 - R_8 =$**		**46.312**	**cm**

13. Select a length to diameter ratio $K = l/d$ for the elements of the LPDA. This choice is a compromise between mechanical strength for the largest and smallest elements, available tubing sizes, and the selected diameter of the boom.

Choose boom diameter **D = 5/8" = 1.5875 cm**

If $d_1 = 1/2" = 1.27$ cm, then $K_1 = l_1/d_1 = 37.0604/1.27 = 29.18$

If $d_8 = 1/8" = 0.3175$ cm, then $K_8 = l_8/d_8 = 9.398/0.3175 = 29.6$

Select K to be ~average of the two values above $\Rightarrow$ **$K = 29.39$**.

14. Calculate the diameter $d_n = l_n / K$ for each element. Then, select the closest available tube/pipe/rod diameter to the calculated value.

15. Calculate the actual length to diameter ratio K_n for each element and the average length to diameter ratio K_{ave} after quantization. Check for unusually large deviations from desired K (may want to go back to step 13. and select another value of K).

- Steps 14-15 done using MS-Excel spreadsheet shown below.

Step 13.	Select K =	29.39				
Step 14. & 15.		$d_n = l_n / K$		$K_{actual} = l_n / d_{quantized}$		
		Exact	**Quantized**		**Actual**	**ΔK =**
n	**l_n (cm)**	**d_n (cm)**	**d_n (cm)**	**d_n (in)**	**K_n**	**$K_n - K$**
1	37.06038	1.261	1.2700	1/2	29.181	-0.209
2	30.464	1.037	1.0320	13/32	29.519	0.129
3	25.041	0.852	0.8730	11/32	28.684	-0.706
4	20.584	0.700	0.7140	9/32	28.829	-0.561
5	16.920	0.576	0.5560	7/32	30.431	1.041
6	13.908	0.473	0.4760	3/16	29.219	-0.171
7	11.432	0.389	0.3970	5/32	28.797	-0.593
8	9.398	0.320	0.3175	1/8	29.598	0.208
			Average K =		**29.282**	

16. Calculate the approximate average characteristic impedance of the active region elements-

$$Z_a = 60\ln(2\,X\,K_{ave}/\pi) = 60\ln[2(0.5378)\,29.282/\pi] \quad \Rightarrow \quad \underline{\mathbf{Z_a = 138.304\ \Omega}}$$

where $X = 8\,\tau\,\sigma/(1+\tau) = 8\,(0.822)\,0.149/(1+0.822) = 0.5378$.

17. Find the characteristic impedance of the unloaded transmission line Z_0 for the desired input impedance R_0-

$$Z_0 = \frac{R_0^{\,2}}{4Z_aX} + R_0\sqrt{\left(\frac{R_0}{4Z_aX}\right)^2 + 1}$$

$$= \frac{75^2}{4(138.304)0.5378} + 75\sqrt{\left(\frac{75}{4(185.8956)0.5378}\right)^2 + 1}$$

$$= \underline{96.254\Omega}$$

18. Calculate the **center-to-center** spacing S of the booms using the unloaded, cylindrical, twin-lead transmission line formula-

$$S = D\cosh(Z_0/120) = 1.5875\cosh(96.254/120) \quad \Rightarrow \quad \underline{\mathbf{S = 2.1262\ cm}}$$

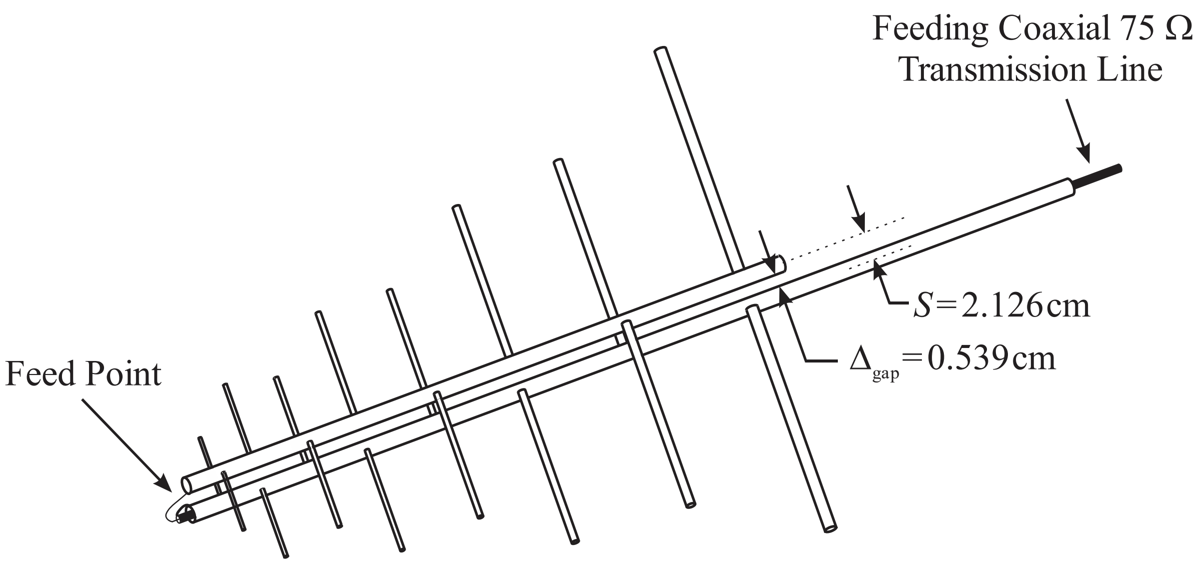
where D is the diameter of the booms (assumed to be identical). The air gap Δ_{gap} between the inner surfaces of the booms is-

$$\Delta_{gap} = S - D = 2.1262 - 1.5875 \quad \Rightarrow \quad \underline{\mathbf{\Delta_{gap} = 0.539\ cm = 5.39\ mm}}.$$

c) Make a **scale** drawing(s) of the final antenna design (i.e., show booms & transmission line) that a machinist could take and use to build the antenna. Use centimeters for all dimensions. Assume grounded boom will need to extend 40 cm past longest elements to allow the LPDA to be attached to an antenna mast. Allow 2 cm past the shortest elements and the longest element on the non-grounded boom for feed attachment and/or mechanical strength.

see following pages

Perspective View of 8 Element LPDA for UHF Channels 14-51 w/ 7.5 dBi gain

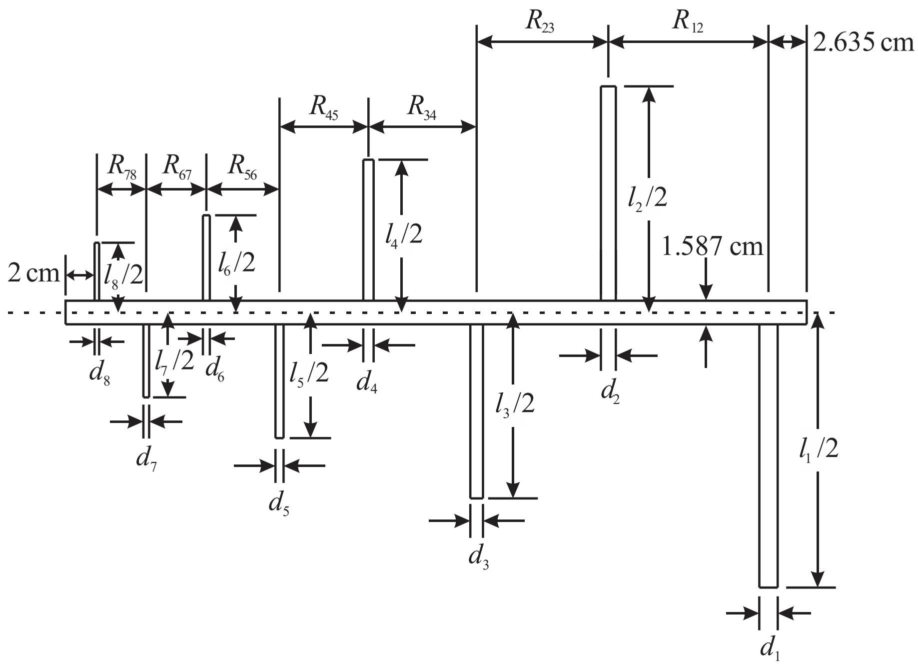


Not to scale. Other dimensions shown on following views of bottom & top booms

Table of Dimensions for following top and bottom booms

n	l_n **(cm)**	d_n **(cm)**	R_n **(cm)**	element-element spacing	(cm)
1	37.06038	1.2700	62.04491		
2	30.464	1.0320	51.001	R_{12}	11.044
3	25.041	0.8730	41.923	R_{23}	9.078
4	20.584	0.7140	34.461	R_{34}	7.462
5	16.920	0.5560	28.327	R_{45}	6.134
6	13.908	0.4760	23.284	R_{56}	5.042
7	11.432	0.3970	19.140	R_{67}	4.145
8	9.398	0.3175	15.733	R_{78}	3.407

Top View of Top Boom



Top View of Bottom Boom