EE 483/583 Antennas for Wireless Communications
Quiz #1 (Spring 2017)

Name Key A

Instructions: Open book & notes. Place answers in indicated spaces and show all work for credit.

For a lossless antenna, the radiation intensity is given by $U(\theta,\phi) = 8\cos^4(\theta)$ (W/sr) in the half-space defined by $0 \le \theta \le \frac{\pi}{2}$ & $0 \le \phi < 2\pi$, and is zero elsewhere. Find the value and direction of the maximum radiation intensity U_{max} and the elevation half-power beamwidth in degrees. Next, find the power density 160 m from the antenna in the direction of U_{max}. How much power does this antenna radiate? What is the maximum directivity (both unitless and dBi)?

For $0 \le \theta \le \pi/2$ & $0 \le \phi < 2\pi$, $U(\theta,\phi)$ has a maximum of $U_{max} = 8$ (W/sr) at $\theta_{max} = 0$ (any ϕ).

To determine HPBW, set $U(\theta,\phi) = \frac{U_{max}}{2} = 4 = 8\cos^4(\theta_{HP})$

$\Rightarrow \theta_{HP} = \cos^{-1}(0.5^{1/4}) = 32.765^\circ \Rightarrow HPBW_{EL} = 2\theta_{HP} = 65.5302^\circ$

(2-12) $W_{rad} = \frac{U}{r^2} \Rightarrow W_{rad,max}(r=160\text{m}) = \frac{U_{max}}{160^2} = \frac{8}{160^2} = 3.125\times10^{-4}\ \frac{W}{m^2}$

(2-13) $P_{rad} = \oint\oint_S U\,dS = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} 8\cos^4\theta \sin\theta\, d\theta\, d\phi$

$= 8(\phi)\Big|_{\phi=0}^{2\pi}\left(-\frac{\cos^5\theta}{5}\right)\Big|_{\theta=0}^{\pi/2} = 8(2\pi-0)\left[0+\frac{1}{5}\right]$

$P_{rad} = 10.053096491\ W$

(2-16a) $D_{max} = D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi(8)}{10.053} = 10 = 10\log_{10}10 = 10\text{ dBi}$

U_{max} = 8 (W/sr) U_{max} direction = $\theta_{max} = 0^\circ$, any ϕ

$HPBW_{EL}$ = 65.53° power density at 160 m = $3.125\times10^{-4}\ W/m^2 = 312.5\ \frac{\mu W}{m^2}$

P_{rad} = 10.0531 W maximum directivity = 10 or 10 dBi