EE 483/583 Examination #1 (Spring 2017)

Name Key

Instructions: Place answers in indicated spaces, use notation as given in class for coordinates & vectors, and show all work for credit. Hand-in equation sheet with exam. Assume $c = 2.9979 \times 10^8$ m/s.

Handy integrals: $\int \cos^n(ax)\sin(ax)\,dx = -\frac{\cos^{n+1}(ax)}{(n+1)a}$, $\int \sin^n(ax)\cos(ax)\,dx = \frac{\sin^{n+1}(ax)}{(n+1)a}$

$\int \cos^2(ax)\,dx = \frac{x}{2} + \frac{\sin(2ax)}{4a}$, $\int \cos^3(ax)\,dx = \frac{\sin(ax)}{a} - \frac{\sin^3(ax)}{3a}$, $\int \cos^4(ax)\,dx = \frac{3x}{8} + \frac{\sin(2ax)}{4a} - \frac{\sin(4ax)}{32a}$

$\int \sin^2(ax)\,dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$, $\int \sin^3(ax)\,dx = \frac{-\cos(ax)}{a} + \frac{\cos^3(ax)}{3a}$, $\int \sin^4(ax)\,dx = \frac{3x}{8} - \frac{\sin(2ax)}{4a} + \frac{\sin(4ax)}{32a}$

1) A vee-dipole antenna (Gain = 5.55 dBi, Z_{ant} = 123 – j10 Ω) is used for a 400 MHz, 2.5 W, monostatic radar system driven via a 75 Ω lossless transmission line. Given that the target, a Little Tykes™ Cozy Coupe II driven by a sugared-up toddler, has an RCS of 2.5 m^2 and is located 10 m from the vee-dipole, find the received power. Also, find the maximum effective area of the antenna when losses are included. Assume that polarization losses are negligible.

(2-125) $\frac{P_r}{P_t} = e_{cdt}\, e_{cdr} (1-|\Gamma_t|^2)(1-|\Gamma_r|^2)\, \sigma \frac{D_t D_r}{4\pi}\left(\frac{\lambda}{4\pi R_1 R_2}\right)^2 |\hat{\rho}_w \cdot \hat{\rho}_r|^2$

$G_t = G_r = e_{cdt} D_t = e_{cdr} D_r = 5.55\ dB_i = 10^{5.55/10} = \underline{3.589219}$

$\Gamma_t = \Gamma_r = \frac{Z_{ant} - Z_o}{Z_{ant} + Z_o} = \frac{(123-j10)-75}{(123+j10)+75} = \underline{0.247314088 \angle -8.877^\circ}$

$|\Gamma_t|^2 = |\Gamma_r|^2 = 0.2473^2 = \underline{0.061164258}$

$\sigma = \underline{2.5 m^2}$ $\quad P_t = \underline{2.5 W}$ $\quad |\hat{\rho}_w \cdot \hat{\rho}_r|^2 = \underline{1}$

$R_1 = R_2 = \underline{10 m}$ $\quad \lambda = \frac{2.9979 \times 10^8}{400 \times 10^6} = \underline{0.749475 m}$

$P_r = (2.5W)(1-0.061164)^2 (2.5 m^2) \frac{3.589219^2}{4\pi}\left(\frac{0.749475}{4\pi\, 10^2}\right)^2 (1)$

$= \underline{2.00883356 \times 10^{-6} W}$

(2-111) $A_{em} = e_{cd}\left(\lambda^2/4\pi\right) D_o = \frac{0.749475^2}{4\pi} 10^{5.55/10} = 0.160437 m^2$

or

(2-112) $A_{em} = e_{cd}\left[1-|\Gamma|^2\right]\left(\frac{\lambda^2}{4\pi}\right) D_o |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = (1-0.06116)\frac{0.749475^2}{4\pi} 10^{\frac{5.55}{10}} (1)$

w/ $|\Gamma|$ + PLF included

Received power = $\underline{2.008834\ \mu W}$ $\qquad$ maximum effective area = $\underline{0.150624\ m^2}$

2) A 220 MHz uniform plane wave propagating in free space has an electric field given by

$$\bar{\mathcal{E}} = 4\cos(\omega t + \beta y + \pi/6)\,\hat{a}_x - 2\cos(\omega t + \beta y - \pi/3)\,\hat{a}_z \text{ (V/m)}.$$

What is the direction of wave propagation? Accurately sketch the path the tip of the electric field vector will trace out versus time (i.e., a fully-labeled polarization diagram) on the axes given below. Choose axes so that the wave propagates into the paper. Describe the polarization of this uniform plane wave. **From the polarization diagram**, find the axial ratio and the tilt angle in degrees (or N/A if not applicable) with respect to the positive vertical axis.

$\Rightarrow$ From $+\beta y$ term, wave propagates in $-y$-direction

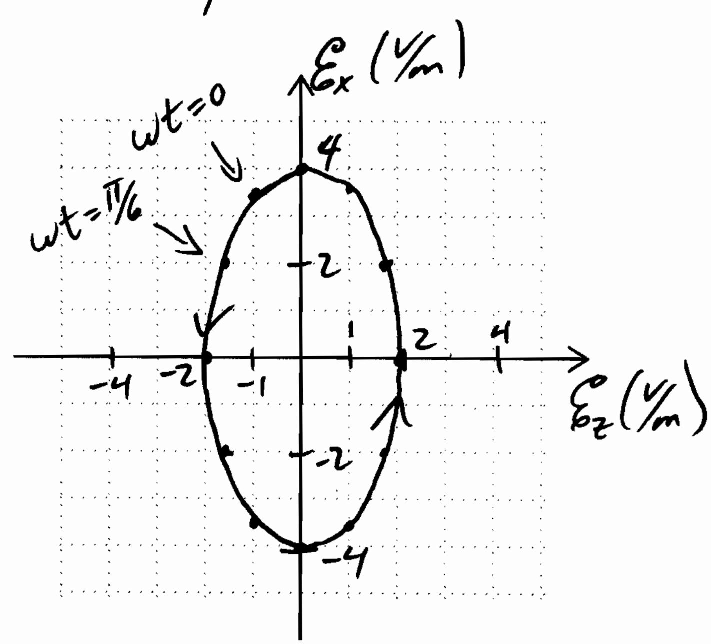
Choose $y=0$ plane, $\bar{\mathcal{E}}(y=0) = 4\cos(\omega t + \underbrace{\pi/6}_{30^\circ})\hat{a}_x - 2\cos(\omega t - \underbrace{\pi/3}_{60^\circ})\hat{a}_z$

ωt	$\bar{\mathcal{E}}$ (V/m)
0	$3.464\hat{a}_x - 1\hat{a}_z$
$\pi/6$	$2\hat{a}_x - 1.732\hat{a}_z$
$\pi/3$	$0 - 2\hat{a}_z$
$\pi/2$	$-2\hat{a}_x - 1.732\hat{a}_z$
$2\pi/3$	$-3.464\hat{a}_x - 1\hat{a}_z$
$5\pi/6$	$-4\hat{a}_x + 0\hat{a}_z$
π	$-3.464\hat{a}_x + 1\hat{a}_z$

$AR = \dfrac{\text{Major}}{\text{minor}} = \dfrac{8}{4} = \underline{\underline{2}}$

$\underline{\underline{\tau = 0^\circ}}$

$-y$-direction into page



Direction of propagation is $-y$-direction

Polarization is LH elliptical

Axial ratio = 2 Tilt angle = 0°

3) Given that the vector electric potential for an antenna is $\bar{F} = \hat{a}_\phi F_0 \sin^2\theta \dfrac{e^{-jkr}}{r}\left[1 - \dfrac{jk}{r}\right]$ C/m, find the electric and magnetic fields in the **far-field** in spherical coordinates. Assume $\bar{A} = 0$.

For far-field, neglect all $\frac{1}{r^n}$ terms where $n \geq 2$

$$\bar{F}_{FF} = \hat{a}_\phi F_0 \sin^2\theta \frac{e^{-jkr}}{r} \quad \text{(dropped } \tfrac{-jk}{r} \text{ term)}$$

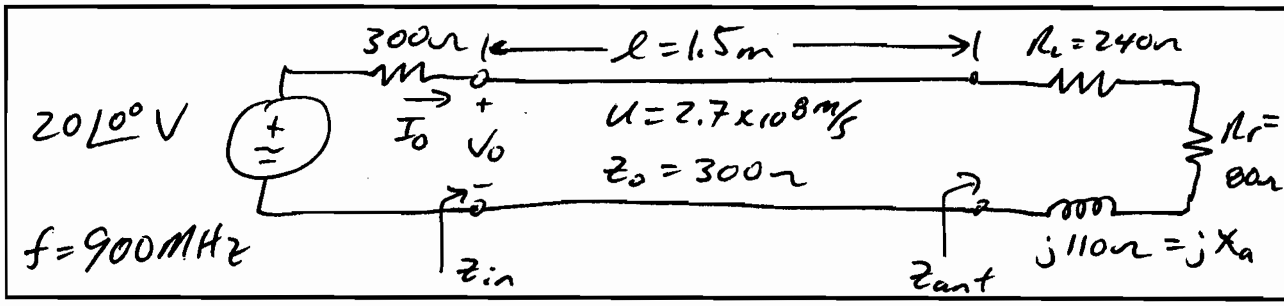
Per (3-59a) + (3-59b):

$$\left.\begin{aligned} H_r &\simeq 0 \\ H_\theta &\simeq -j\omega \cancel{F_\theta}^{0} = 0 \\ H_\phi &\simeq -j\omega F_\phi = -j\omega F_0 \sin^2\theta \frac{e^{-jkr}}{r} \end{aligned}\right\} \Rightarrow \bar{H}_{FF} = \hat{a}_\phi H_\phi$$

$$\left.\begin{aligned} E_r &\simeq 0 \\ E_\theta &\simeq -j\omega\eta F_\phi = -j\omega\eta F_0 \sin^2\theta \frac{e^{-jkr}}{r} \\ E_\phi &\simeq +j\omega\eta \cancel{F_\theta}^{0} = 0 \end{aligned}\right\} \Rightarrow \bar{E}_{FF} = \hat{a}_\theta E_\theta$$

$\bar{E}_{FF} =$ $-\hat{a}_\theta j\omega\eta F_0 \sin^2\theta \dfrac{e^{-jkr}}{r} \left(\dfrac{V}{m}\right)$ $\bar{H}_{FF} =$ $-\hat{a}_\phi j\omega F_0 \sin^2\theta \dfrac{e^{-jkr}}{r} \left(A/m\right)$

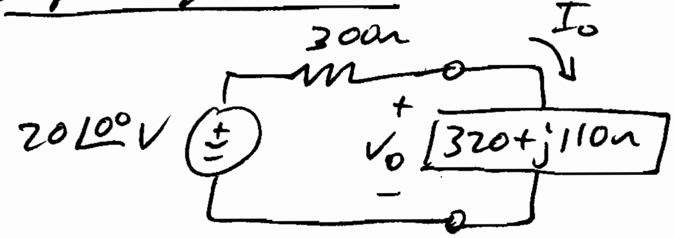
4). An antenna is connected to a generator operating at 900 MHz by a **lossless** 300 Ω transmission line ($u = 2.7 \times 10^8$ m/s) of length 1.5 m (5λ). The generator has an open circuit voltage of $20\underline{/0^\circ}$ V and equivalent impedance of 300 Ω. If the antenna has a loss resistance of 240 Ω, radiation resistance of 80 Ω, and reactance of 110 Ω, draw & label the equivalent circuit in the box provided. Then, find the antenna impedance Z_{ant} and impedance looking into the transmission line Z_{in} in rectangular format. Next, calculate the time-average power delivered to the antenna P_{ant}, dissipated in the antenna P_{ohmic}, and radiated by the antenna P_{rad}.



$$Z_{ant} = R_r + R_L + jX_a = 80 + 240 + j110 = 320 + j110\,\Omega$$

For $\ell = 5\lambda$ on a <u>lossless</u> T.L., $Z_{in} = Z_{ant}$

<u>Input Equiv. Ckt.</u>



$$I_o = \frac{20\angle 0^\circ}{300 + (320 + j110\,\Omega)} = 0.031762041\angle{-10.06069^\circ}\ A$$

$$V_o = 20\angle 0^\circ \frac{320 + j110}{300 + 320 + j110} = 10.7475913\angle 8.909718^\circ\ V$$

For a lossless T.L.,

$$P_{ant} = P_{in} = \tfrac{1}{2}\,\mathrm{Re}\{V_o I_o^*\} = \tfrac{1}{2}\,\mathrm{Re}\{(10.7476\angle 8.91^\circ)(0.03176\angle{+10.0607^\circ})\}$$
$$= \underline{0.16141\ W}$$

$$P_{ohmic} = P_{ant}\frac{R_L}{R_L + R_r} = 0.16141\left(\frac{240}{240+80}\right) = \underline{0.121059\ W}$$

$$P_{rad} = P_{ant}\frac{R_r}{R_L + R_r} = 0.16141\left(\frac{80}{240+80}\right) = \underline{0.040353\ W}$$

Z_{ant} = 320 + j110 Ω Z_{in} = 320 + j110 Ω

P_{ant} = 0.161412 W P_{ohmic} = 0.12106 W P_{rad} = 0.04035 W