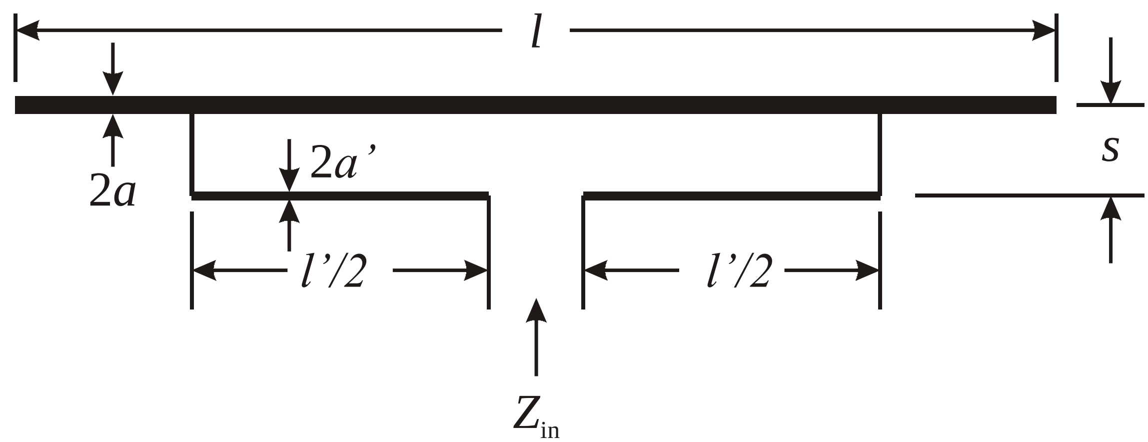
Matching Techniques For Driving Yagi-Uda Antennas: T-Match

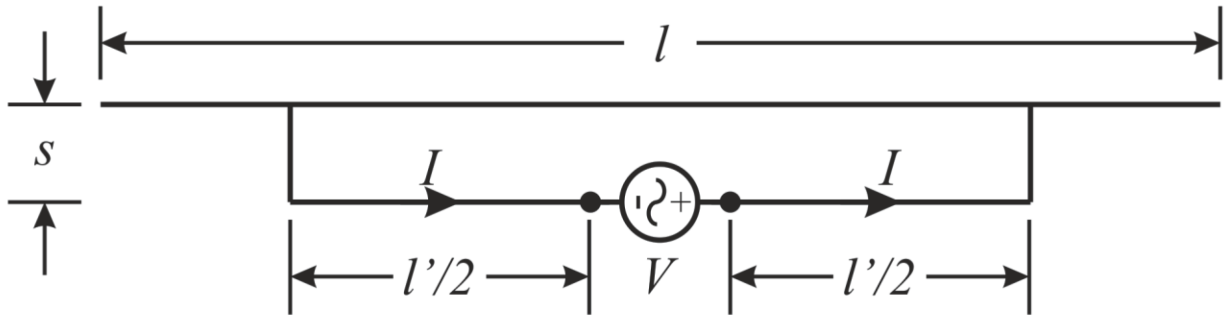
(Sections 9.5 & 9.7 of Balanis)

T-Match:

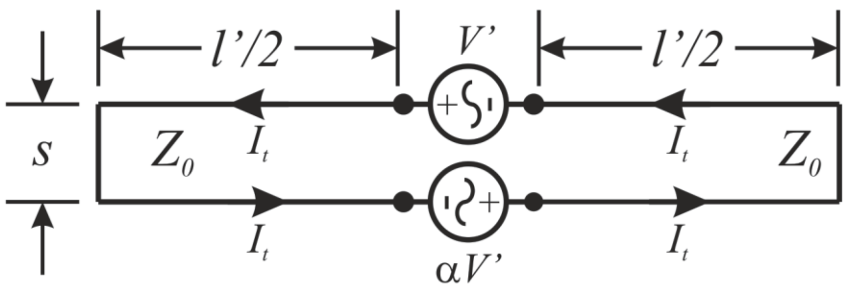


- The T-Match is a shunt-matching technique that can be used to feed the driven element of a Yagi-Uda antenna. It uses a second shorter dipole that is placed a small distance s ($s \ll \lambda$) from the driven element (parallel, and centered in the plane of the Yagi-Uda antenna).
- As it is symmetrical and balanced, it is typically used to connect twin-lead transmission lines to Yagi-Uda antennas.
- Design analysis and procedure follows that for the folded dipole.
- Due to mutual coupling with the reflector and director elements, the design of the T-Match is approximate. In practice, length adjustments will usually be required.
- The characteristic impedance of the transmission line portion of the T-Match is given by $Z_0 = \dfrac{\eta}{2\pi}\cosh^{-1}\left(\dfrac{s^2 - a^2 - a'^2}{2aa'}\right)$

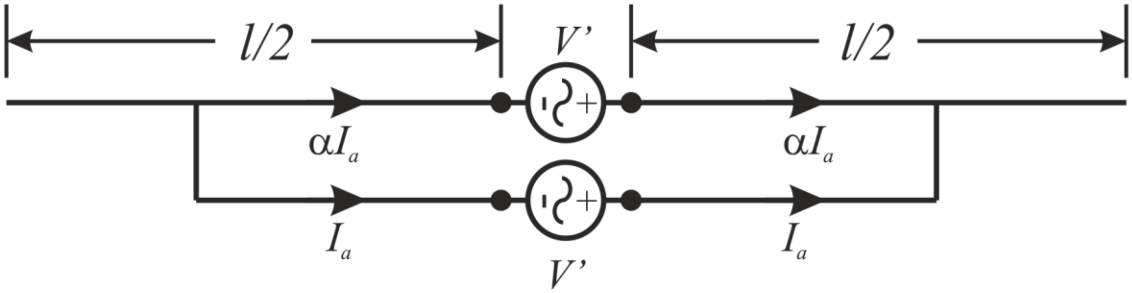
Model:

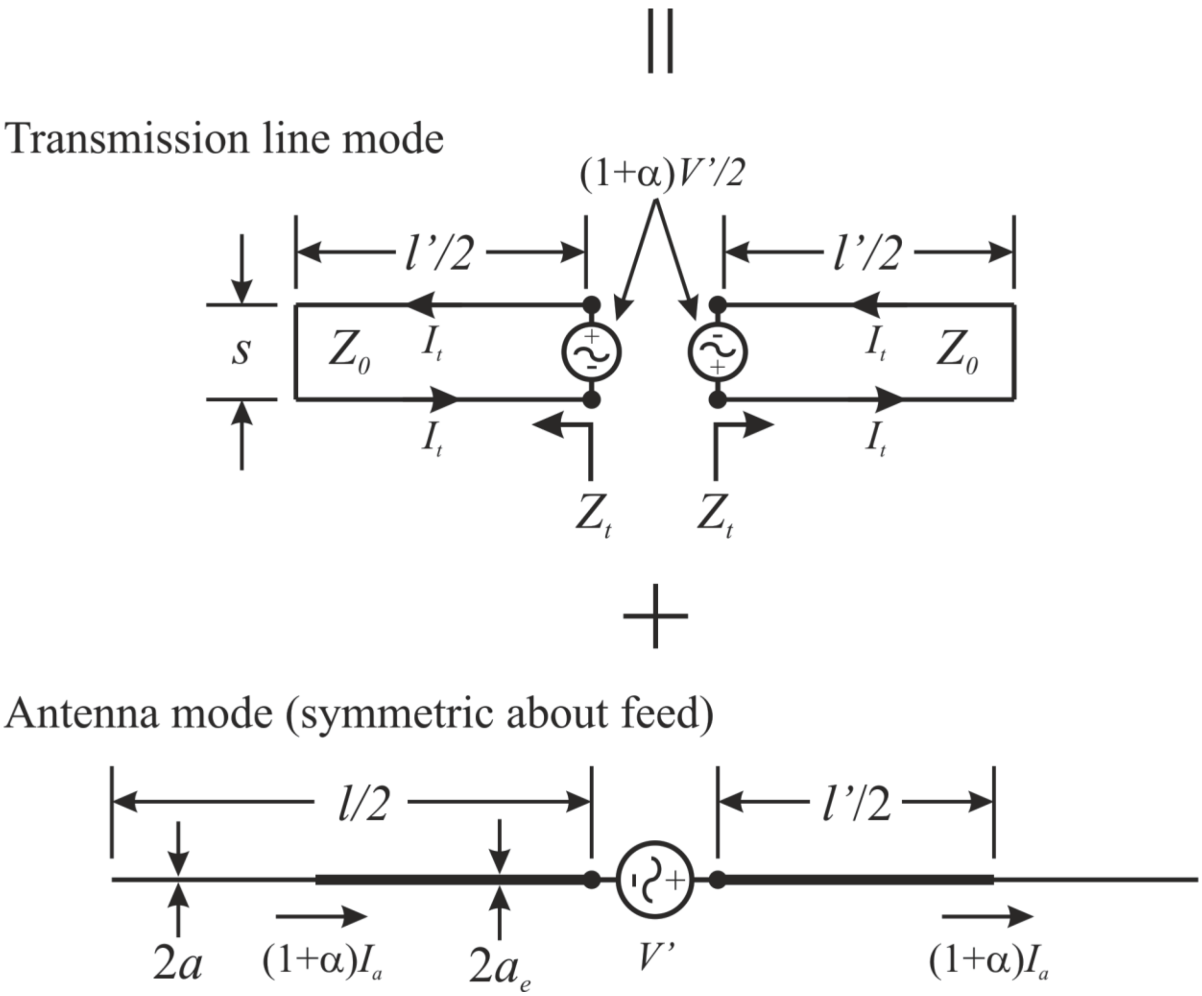


Transmission line mode



Antenna mode





where $V' = \dfrac{V}{1+\alpha}$, and we define a **current divisor factor α**

$$\alpha = \frac{\cosh^{-1}\left(\dfrac{v^2 - u^2 + 1}{2v}\right)}{\cosh^{-1}\left(\dfrac{v^2 + u^2 - 1}{2vu}\right)} \qquad \begin{array}{l} \alpha > 1 \text{ when } a > a' \\ \alpha = 1 \text{ when } a = a' \\ \alpha < 1 \text{ when } a < a' \end{array}$$

where $u = \dfrac{a}{a'}$ and $v = \dfrac{s}{a'}$. The current divisor factor α has a big impact on the magnitude of Z_{in} (i.e., when α increases $|Z_{\text{in}}|$ increases and vice versa). The current divisor factor α is inversely related to the spacing s (i.e., if s decreases α increases and vice versa).

<u>Transmission line mode impedance</u>:

Definition of transmission line input impedance

$$Z_t = \frac{\left(\frac{1+\alpha}{2}\right)V'}{I_t} = j\,Z_0 \tan\left(kl'/2\right)$$

where $k = \beta = 2\pi/\lambda$.

Note, for $0 < l' < 0.5\lambda$, we get $Z_0 \tan(kl'/2) > 0$, i.e., **inductive** reactance. This is typically the case encountered when using a T-match by itself or in a Yagi-Uda antenna. When $l' = 0.5\lambda$, $Z_t = j\,Z_0 \tan(\pi/2) \to \infty$.

<u>Antenna mode impedance and current</u>:

- The antenna impedance is usually found numerically using a Method of Moments (MoM) program for a dipole that has radius a_e (the equivalent radius of the two wires) over the length l' and a radius a for the portion of the dipole extending beyond the T-Match ($l' < l$). [**Note**: If T-Match used to drive a Yagi-Uda antenna, this equivalent dipole should be inserted into the Yagi-Uda antenna to determine Z_a.]
- The equivalent radius for two closely spaced (center-to-center distance s) wires of radii a and a' is determined by

$$\ln(a_e) \approx \ln(a') + \frac{1}{(1+u)^2}\left(u^2 \ln u + 2u \ln v\right)$$

Definition of antenna input impedance

$$Z_a = \frac{V'}{(1+\alpha)\,I_a}$$

<u>Total impedance and current for T-Match</u>:

The current at the terminals of the T-Match is

$$I = I_t + I_a = \frac{\left(\frac{1+\alpha}{2}\right)V'}{Z_t} + \frac{V'}{(1+\alpha)Z_a}$$

$$I = V'\left[\frac{1+\alpha}{2Z_t} + \frac{1}{(1+\alpha)Z_a}\right] = \frac{V}{1+\alpha}\left[\frac{1+\alpha}{2Z_t} + \frac{1}{(1+\alpha)Z_a}\right]$$

$$= V\left[\frac{1}{2Z_t} + \frac{1}{(1+\alpha)^2 Z_a}\right] = V\left[\frac{(1+\alpha)^2 Z_a + 2Z_t}{2(1+\alpha)^2 Z_a Z_t}\right]$$

Solving for the input admittance and impedance, yields

$$Y_{\text{in}} = \frac{I}{V} = \frac{1}{2Z_t} + \frac{1}{(1+\alpha)^2 Z_a}$$

$$= \frac{Y_t}{2} + \frac{Y_a}{(1+\alpha)^2}$$

and

$$Z_{\text{in}} = \frac{V}{I} = \frac{2(1+\alpha)^2 Z_a Z_t}{(1+\alpha)^2 Z_a + 2Z_t}$$

For the case that $l' \approx \lambda/2$ (half-wave dipole), the transmission line impedance $|Z_t| >> |Z_a|$, therefore, the input impedance becomes

$$Z_{\text{in}} \approx (1+\alpha)^2 Z_a$$

If $a = a'$, the current division factor $\alpha = 1$ and we get

$$Z_{\text{in}} \approx 4Z_a$$

as before.

Note: If Z_a has an inductive reactance (i.e., $X_a > 0$), it may not be possible to achieve a realizable match using a standard T-Match as Z_t will also have an inductive reactance. In that case, either the length l needs to be shortened to make Z_a have a capacitive reactance (i.e., $X_a < 0$) or a modified T-Match may be used.

Design Process:

- We desire to match a given Yagi-Uda antenna to a transmission line characteristic impedance $Z_{0,\text{feed}}$. Usually, a specification in terms of the VSWR is given.

1) Select a driven element length l_2 so that $l_1 < l_2 < l_3$, a', s, and l' (usually $l' < l_2/2$). These values may be changed later.

2) Calculate the characteristic impedance Z_0 of the transmission line portion of the T-Match.

3) Calculate the transmission line mode input impedance Z_t.

4) Calculate the parameters u, v, and α.

5) Calculate the equivalent radius a_e of the T-Match section.

6) Find input impedance of antenna mode Z_a.

7) Find overall input impedance Z_{in}.

8) Determine if Z_{in} meets your specification. If so, stop design process. If not, try changing l' to

$$l' = \frac{2}{k}\tan^{-1}\left[\frac{1}{2Z_0\,\mathrm{Im}\left(\dfrac{Y_a}{(1+\alpha)^2}\right)}\right]$$

to better offset the antenna mode reactance, and repeat steps 2) through 8). If necessary, l_2 , a', and s can be varied. Remember, the magnitude of the input impedance is greatly affected by α (i.e., when α increases $|Z_{\text{in}}|$ increases and vice versa). In turn, α is inversely related to s (i.e., if s decreases α increases and vice versa).