Matching Techniques For Driving Yagi-Uda Antennas: T-Match
(Sections 9.5 & 9.7 of Balanis)

T-Match:

- The T-Match is a shunt-matching technique that can be used to feed the driven element of a Yagi-Uda antenna. It uses a second shorter dipole that is placed a small distance $s$ ($s << \lambda$) from the driven element (parallel, and centered in the plane of the Yagi-Uda antenna).

- As it is symmetrical and balanced, it is typically used to connect twin-lead transmission lines to Yagi-Uda antennas.

- Design analysis and procedure follows that for the folded dipole.

- Due to mutual coupling with the reflector and director elements, the design of the T-Match is approximate. In practice, length adjustments will usually be required.

- The characteristic impedance of the transmission line portion of the T-Match is given by $Z_0 = \frac{\eta}{2\pi} \cosh^{-1}\left(\frac{s^2 - a^2 - a'^2}{2aa'}\right)$
Model:

Transmission line mode

Antenna mode
Transmission line mode

<table>
<thead>
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<th>Transmission line mode</th>
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<tr>
<td>(1+(\alpha))V''/2</td>
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<td>l'/2</td>
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<tr>
<td>s</td>
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<tr>
<td>Z₀</td>
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<td>Zₜ₁</td>
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<td>Zₜ₂</td>
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<td>l'/2</td>
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Antenna mode (symmetric about feed)

\[
\alpha = \begin{cases} 
\cosh^{-1}\left(\frac{v^2 - u^2 + 1}{2v}\right) & \text{\(\alpha > 1\) when } a > a' \\
\cosh^{-1}\left(\frac{v^2 + u^2 - 1}{2vu}\right) & \text{\(\alpha < 1\) when } a < a' \\
\cosh^{-1}\left(\frac{v^2}{2v}\right) & \text{\(\alpha = 1\) when } a = a'
\end{cases}
\]

where \(V' = \frac{V}{1+\alpha}\), and we define a current divisor factor \(\alpha\)

where \(u = \frac{a}{a'}\) and \(v = \frac{s}{a'}\). The current divisor factor \(\alpha\) has a big impact on the magnitude of \(Z_{\text{in}}\) (i.e., when \(\alpha\) increases \(|Z_{\text{in}}|\) increases and vice versa). The current divisor factor \(\alpha\) is inversely related to the spacing \(s\) (i.e., if \(s\) decreases \(\alpha\) increases and vice versa).
Transmission line mode impedance:

Definition of transmission line input impedance

\[ Z_t = \frac{\left(\frac{1+\alpha}{2}\right) V'}{I_t} = j Z_0 \tan(kl'/2) \]

where \( k = \beta = 2\pi/\lambda \).

Note, for \( 0 < l' < 0.5\lambda \), we get \( Z_0 \tan(kl'/2) > 0 \), i.e., inductive reactance. This is typically the case encountered when using a T-match by itself or in a Yagi-Uda antenna. When \( l' = 0.5\lambda \), \( Z_t = j Z_0 \tan(\pi/2) \to \infty \).

Antenna mode impedance and current:

- The antenna impedance is usually found numerically using a Method of Moments (MoM) program for a dipole that has radius \( a_e \) (the equivalent radius of the two wires) over the length \( l' \) and a radius \( a \) for the portion of the dipole extending beyond the T-Match (\( l' < l \)). [Note: If T-Match used to drive a Yagi-Uda antenna, this equivalent dipole should be inserted into the Yagi-Uda antenna to determine \( Z_a \).]

- The equivalent radius for two closely spaced (center-to-center distance \( s \)) wires of radii \( a \) and \( a' \) is determined by

\[ \ln(a_e) \approx \ln(a') + \frac{1}{(1+u)^2} \left( u^2 \ln u + 2u \ln v \right) \]

Definition of antenna input impedance

\[ Z_a = \frac{V'}{(1+\alpha)I_a} \]

Total impedance and current for T-Match:

The current at the terminals of the T-Match is
\[ I = I_t + I_a = \left( \frac{1+\alpha}{2} \right) V' + \frac{V'}{(1+\alpha)Z_a} \]

\[ I = V' \left[ \frac{1+\alpha + 1}{2Z_t} + \frac{1}{(1+\alpha)Z_a} \right] = V \left[ \frac{1+\alpha + 1}{2Z_t} + \frac{1}{(1+\alpha)Z_a} \right] \]

\[ = V \left[ \frac{1}{2Z_t} + \frac{1}{(1+\alpha)^2 Z_a} \right] = V \left[ \frac{(1+\alpha)^2 Z_a + 2Z_t}{2(1+\alpha)^2 Z_a Z_t} \right] \]

Solving for the input admittance and impedance, yields

\[ Y_{in} = \frac{I}{V} = \frac{1}{2Z_t} + \frac{1}{(1+\alpha)^2 Z_a} \]

\[ = \frac{Y_t}{2} + \frac{Y_a}{(1+\alpha)^2} \]

and

\[ Z_{in} = \frac{V}{I} = \frac{2(1+\alpha)^2 Z_a Z_t}{(1+\alpha)^2 Z_a + 2Z_t} \]

For the case that \( l' \approx \lambda/2 \) (half-wave dipole), the transmission line impedance \( |Z_t| >> |Z_a| \), therefore, the input impedance becomes

\[ Z_{in} \approx (1+\alpha)^2 Z_a \]

If \( a = a' \), the current division factor \( \alpha = 1 \) and we get

\[ Z_{in} \approx 4Z_a \]

as before.

Note: If \( Z_a \) has an inductive reactance (i.e., \( X_a > 0 \)), it may not be possible to achieve a realizable match using a standard T-Match as \( Z_t \) will also have an inductive reactance. In that case, either the length \( l \) needs to be shortened to make \( Z_a \) have a capacitive reactance (i.e., \( X_a < 0 \)) or a modified T-Match may be used.
Design Process:

- We desire to match a given Yagi-Uda antenna to a transmission line characteristic impedance $Z_{0,\text{feed}}$. Usually, a specification in terms of the VSWR is given.

1) Select a driven element length $l_2$ so that $l_1 < l_2 < l_3$, $a'$, $s$, and $l'$ (usually $l' < l_2/2$). These values may be changed later.

2) Calculate the characteristic impedance $Z_0$ of the transmission line portion of the T-Match.

3) Calculate the transmission line mode input impedance $Z_t$.

4) Calculate the parameters $u$, $v$, and $\alpha$.

5) Calculate the equivalent radius $a_e$ of the T-Match section.

6) Find input impedance of antenna mode $Z_a$.

7) Find overall input impedance $Z_{\text{in}}$.

8) Determine if $Z_{\text{in}}$ meets your specification. If so, stop design process. If not, try changing $l'$ to

$$l' = \frac{2}{k} \tan^{-1} \left[ \frac{1}{2Z_0 \text{Im} \left( \frac{Y_a}{(1+\alpha)^2} \right)} \right]$$

to better offset the antenna mode reactance, and repeat steps 2) through 8). If necessary, $l_2$, $a'$, and $s$ can be varied. Remember, the magnitude of the input impedance is greatly affected by $\alpha$ (i.e., when $\alpha$ increases $|Z_{\text{in}}|$ increases and vice versa). In turn, $\alpha$ is inversely related to $s$ (i.e., if $s$ decreases $\alpha$ increases and vice versa).