

Example- Rectangular Microstrip Design

- 1) Problem statement/specification- Design a rectangular microstrip antenna to operate at a frequency of 5 GHz on a RT/Duroid ($\epsilon_r = 2.2$, $h = 1.588 \text{ mm} = 1/16''$). The antenna is to be matched to a 50Ω microstrip transmission line on this substrate. Accurately sketch final design.

$$f_r := 5 \cdot 10^9 \quad \text{Hz} \quad \epsilon_r := 2.2 \quad h := 1.588 \cdot 10^{-3} \quad \text{mm}$$

$$Z_{fd} := 50 \quad \Omega \quad Y_{fd} := \frac{1}{Z_{fd}} \quad Y_{fd} = 0.02 \quad \text{S}$$

Define some constants

$$c := 2.9979 \cdot 10^8 \quad \text{m/s} \quad \eta_0 := 376.7303 \quad \Omega$$

$$\lambda_0 := \frac{c}{f_r} \quad \lambda_0 = 0.05996 \quad \text{m} \quad k_0 := \frac{2 \cdot \pi}{\lambda_0} \quad k_0 = 104.793 \quad \text{rad/m}$$

- 2) Calculate/estimate width of patch:

$$W_{est} := \frac{c}{2 \cdot f_r} \cdot \sqrt{\frac{2}{\epsilon_r + 1}} \quad W_{est} \cdot 100 = 2.37005 \quad \text{cm}$$

$$\frac{W_{est}}{h} = 14.925 \quad W := W_{est} \quad \boxed{W \cdot 100 = 2.37} \quad \text{cm}$$

Note: To operate ONLY in the TM_{010} mode, W must be less than L . If $W > L$, the dominant mode is the TM_{001} . However, if the rectangular patch is properly excited, only the TM_{010} mode will be active. Regardless, keep $W < 2L$ (where the mode that will be excited by the microstrip feed occurs).

- 3) Calculate the effective permittivity

$$\epsilon_{r_eff} := \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \cdot \left(1 + 12 \cdot \frac{h}{W} \right)^{-0.5} \quad \epsilon_{r_eff} = 2.04671$$

Example- Rectangular Microstrip Design cont.4) Calculate the fringing length ΔL

$$\Delta L := 0.412 \cdot h \cdot \frac{\epsilon_{r_eff} + 0.3}{\epsilon_{r_eff} - 0.258} \cdot \frac{\frac{W}{h} + 0.264}{\frac{W}{h} + 0.8} \quad \Delta L = 8.291 \times 10^{-4} \quad \text{m}$$

$$\Delta L \cdot 1000 = 0.8291 \quad \text{mm}$$

5) Calculate effective patch length L_{eff} and guided wavelength λ

$$L_{eff} := \frac{c}{2 \cdot fr \cdot \sqrt{\epsilon_{r_eff}}} \quad L_{eff} = 0.02096 \quad \text{m, or} \quad L_{eff} \cdot 100 = 2.0955 \quad \text{cm}$$

$$\lambda := 2 \cdot L_{eff} \quad \lambda = 0.04191 \quad \text{m, or} \quad \lambda \cdot 100 = 4.19101 \quad \text{cm}$$

6) Calculate physical patch length L , L/λ , and check if $W/L > 2$?

$$L := L_{eff} - 2 \cdot \Delta L \quad L = 0.019297 \quad \text{m} \quad \boxed{L \cdot 100 = 1.9297} \quad \text{cm}$$

$$\frac{L}{\lambda} = 0.4604 \quad \frac{W}{L} = 1.228 < 2 \quad (\text{will be OK})$$

7) Calculate the slot conductances $G_1 = G_2$ and susceptances $B_1 = B_2$

Slot conductances and susceptances based on infinite slot approximation

$$G_{1est} := \frac{W}{120 \cdot \lambda_0} \cdot \left[1 - \frac{1}{24} \cdot (k_0 \cdot h)^2 \right] \quad G_{1est} = 3.29 \times 10^{-3} \quad \text{S}$$

$$B_{1est} := \frac{W}{120 \cdot \lambda_0} \cdot (1 - 0.636 \cdot \ln(k_0 \cdot h)) \quad B_{1est} = 7.051 \times 10^{-3} \quad \text{S}$$

Example- Rectangular Microstrip Design cont.7) continued

Slot conductance based on cavity model (more accurate)

$$G1 := \frac{1}{\pi \cdot \eta_0} \cdot \int_0^{\pi} \left(\frac{\sin\left(\frac{k_0 \cdot W}{2} \cdot \cos(\theta)\right)}{\cos(\theta)} \right)^2 \cdot (\sin(\theta))^3 d\theta$$

$$G2 := G1 \qquad G1 = 1.5735 \times 10^{-3} \quad S$$

Estimate true slot susceptance

$$B1 := \frac{G1}{G1_{est}} \cdot B1_{est}$$

$$B2 := B1 \qquad B1 = 3.37206 \times 10^{-3} \quad S$$

Slot admittances and impedance

$$Y1 := G1 + j \cdot B1 \qquad Y2 := Y1 \qquad Y1 = 1.574 \times 10^{-3} + 3.372i \times 10^{-3} \quad S$$

$$Z1 := \frac{1}{Y1} \qquad Z2 := Z1 \qquad Z1 = 113.638 - 243.527i \quad \Omega$$

8) Calculate the characteristic impedance $Z_{c,ant}$ and admittance $Y_{c,ant}$ of microstrip antenna (treated as a microstrip transmission line)

Since $W/h > 1$

$$Z_{cant} := \frac{\eta_0}{\sqrt{\epsilon_{r_eff}} \cdot \left(\frac{W}{h} + 1.393 + 0.667 \cdot \ln\left(\frac{W}{h} + 1.44\right) \right)}$$

$$Z_{cant} = 14.483 \quad \Omega \qquad Y_{cant} := \frac{1}{Z_{cant}} \qquad Y_{cant} = 0.069 \quad S$$

Example- Rectangular Microstrip Design cont.

9) Use Smith chart and equations to verify that the translated slot 2 admittance Y_{2t} is approximately the complex conjugate of the slot 1 admittance Y_1 .

Smith chart approach- normalize slot 2 admittance and plot

$$y_2 := Y_2 \cdot Z_{\text{cant}} \quad y_2 = 0.023 + 0.049i \quad \text{S/S}$$

move $L/\lambda = 0.4603$ toward the generator, on circle of constant $|\Gamma|$, and read

$$y_{2t} := 0.023 - j \cdot 0.2 \quad \text{S/S} \quad \text{partial cancellation of imaginary part, real part unchanged.}$$

Analytic approach-

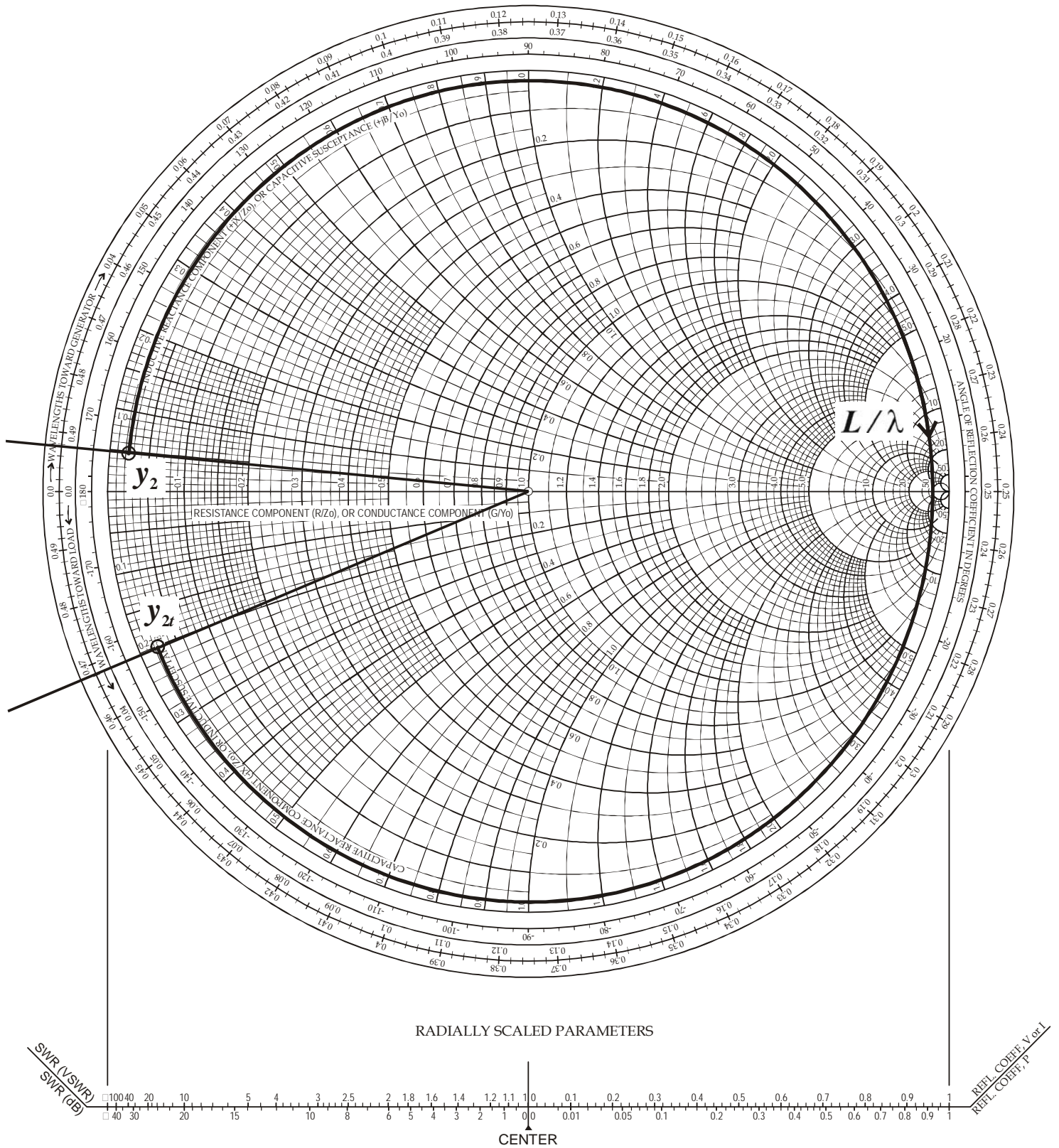
$$z_{2t} := \frac{Z_2 + j \cdot Z_{\text{cant}} \cdot \tan\left(2 \cdot \pi \cdot \frac{L}{\lambda}\right)}{Z_{\text{cant}} + j \cdot Z_2 \cdot \tan\left(2 \cdot \pi \cdot \frac{L}{\lambda}\right)} \quad \frac{1}{z_{2t}} = 0.024 - 0.202i \quad \text{About the same!}$$

From these results, we see that the approximation that $G_{2t} = G_1$ is very good. However, the approximation that $B_{2t} = -B_1$ is only roughly true. There is the issue of what distance is appropriate to use for the translation- L/λ , L_{eff}/λ , somewhere in between?

In any case, the calculation of B_1 is very approximate and using the inset feed would further change its actual value. However, we should see some cancellation/minimization of the input susceptance when Y_{2t} is added to Y_1 .

Example- Rectangular Microstrip Design cont.

Simple Smith Chart



Example- Rectangular Microstrip Design cont.10) Calculate the mutual conductance between the slots.

$$G_{12} := \frac{1}{\pi \cdot \eta_0} \cdot \int_0^{\pi} \left(\frac{\sin\left(\frac{k_0 \cdot W}{2} \cdot \cos(\theta)\right)}{\cos(\theta)} \right)^2 \cdot J_0(k_0 \cdot L \cdot \sin(\theta)) \cdot (\sin(\theta))^3 d\theta$$

$$G_{12} = 5.163 \times 10^{-4} \quad \text{S}$$

11) Calculate expected input impedance (assumed to be real) at the edge of the rectangular patch

$$R_{in} := \frac{1}{2 \cdot (G_1 + G_{12})}$$

$$R_{in} = 239.2541 \quad \Omega$$

Not a good match to 50 Ω .12) Calculate length to inset microstrip feed

Check to see if conditions met for approximation- $\frac{G_1}{Y_{fd}} = 0.079 \ll 1 \quad \text{OK}$

$$\frac{B_1}{Y_{fd}} = 0.169 \ll 1? \quad \text{So-so}$$

Estimate inset length as start point for iterative solution- $y_{0est} := \frac{L}{\pi} \cdot \text{acos}\left(\sqrt{\frac{Z_{fd}}{R_{in}}}\right)$

$$y_{0est} = 0.0067 \quad \text{m. Is it} < L/2? \quad 0.5 \cdot L = 9.648 \times 10^{-3} \quad \text{m. OK}$$

$$\frac{L}{4} = 0.0048 \quad \text{m is another possible starting value}$$

Example- Rectangular Microstrip Design cont.

12) continued

$$\text{Rinset}(y_0) := \text{Rin} \cdot \left[\left(\cos\left(\frac{\pi \cdot y_0}{L}\right) \right)^2 + \frac{(G_1^2 + B_1^2)}{Y_{fd}^2} \cdot \left(\sin\left(\frac{\pi \cdot y_0}{L}\right) \right)^2 - \frac{B_1}{Y_{fd}} \cdot \sin\left(\frac{2\pi \cdot y_0}{L}\right) \right]$$

$\text{Rinset}(y_{0est}) = 23.749 \quad \Omega$ Too low, try a smaller/shorter inset.

Iteratively select y_0 - $y_{01} := 0.0048 \quad \text{m} \quad \text{Rinset}(y_{01}) = 84.341 \quad \Omega$

$y_{02} := 0.006 \quad \text{m} \quad \text{Rinset}(y_{02}) = 43.209 \quad \Omega$

$y_{03} := 0.0055 \quad \text{m} \quad \text{Rinset}(y_{03}) = 59.195 \quad \Omega$

$y_0 := 0.00578013 \quad \text{m} \quad \text{Rinset}(y_0) = 50 \quad \Omega$

$$\boxed{y_0 \cdot 100 = 0.578} \quad \text{cm}$$

13) Calculate width of 50 ohm feeding microstrip:

$$\epsilon_{r_fd}(W_0) := \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \cdot \left(1 + 12 \cdot \frac{h}{W_0} \right)^{-0.5}$$

$$Z_{cfd}(W_0) := \frac{\eta_0}{\sqrt{\epsilon_{r_fd}(W_0)} \cdot \left(\frac{W_0}{h} + 1.393 + 0.667 \cdot \ln\left(\frac{W_0}{h} + 1.444\right) \right)}$$

Iteratively select W_0 (in terms of substrate thickness h)-

$W_0 := 3.1037 \cdot h \quad Z_{cfd}(W_0) = 50 \quad \Omega$

$\epsilon_{r_fd}(W_0) = 1.872 \quad W_0 = 0.004929 \quad \text{m} \quad \boxed{W_0 \cdot 100 = 0.4929} \quad \text{cm}$

$\frac{W_0}{W} = 0.208$ Microstrip feed only blocks about 20% of slot 1, should be OK.

Example- Rectangular Microstrip Design cont.

14) Select a notch width n is needed. Arbitrarily select $n \sim W_0/4 = 0.125$ cm.

15) Draw resulting design

