Microstrip Antennas- Rectangular Patch

(Chapter 14 in *Antenna Theory, Analysis and Design* (2nd Edition) by Balanis)

- Shown in Figures 14.1 - 14.3
- Easy to analyze using transmission line or cavity models
- Most common type of patch or microstrip antenna

Transmission line model

The rectangular microstrip antenna is represented as two slots or apertures (of width *W* and height *h*) separated by a low impedance transmission line of length *L* (see Figure 14.1).

Fringing of the fields, particularly the electric field, at the edges of the patch is an issue of concern because of the finite dimensions involved. Figures 14.1, 14.3, and 14.5 illustrate fringing.

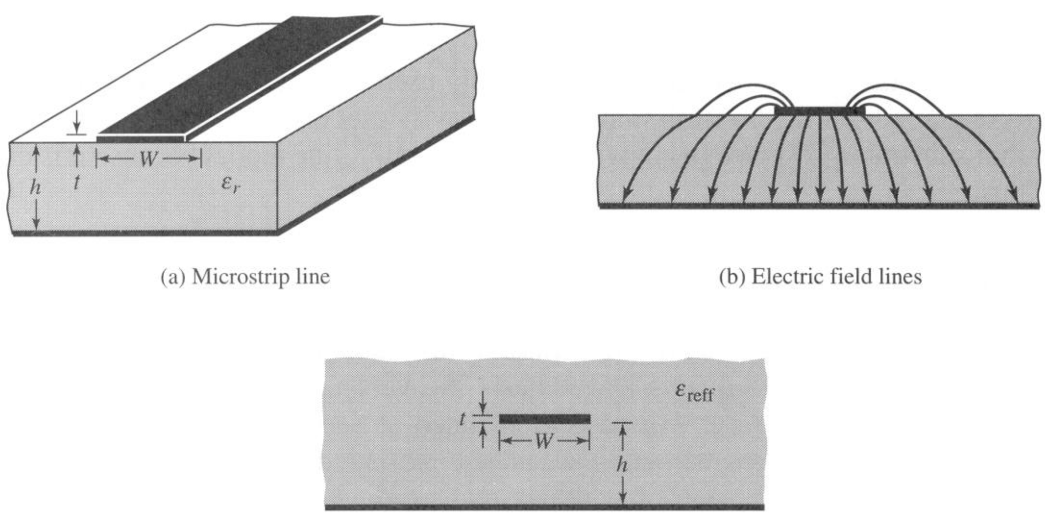


Figure 14.5 Microstrip line and its electric field lines, and effective dielectric constant geometry. (From Balanis, *Antenna Theory, Analysis and Design* (*Second Edition*))

Fringing makes the patch seem bigger (electrically) than the physical dimensions of the patch. This impacts the resonant frequency of the patch. It is dependent on the dielectric constant ε_r of the substrate as well as the physical dimensions *L*, *W* and *h*.

To account for the fringing of the electric field above the microstrip (in the air above the substrate), an effective dielectric constant $1 < \varepsilon_{r,eff} < \varepsilon_r$ is defined. As most of the field lines are confined between the patch and the ground plane (like a capacitor), $\varepsilon_{r,eff}$ tends to be closer to ε_r. The effective dielectric constant allows the microstrip to be modeled as if it were in a homogeneous dielectric medium of $\varepsilon_{r,eff}$. Figure 14.6 shows the effective dielectric constant as a function of frequency for several substrates. Note that $\varepsilon_{r,eff} \to \varepsilon_r$ as the frequency increases, i.e., the electric field concentrates in the substrate.

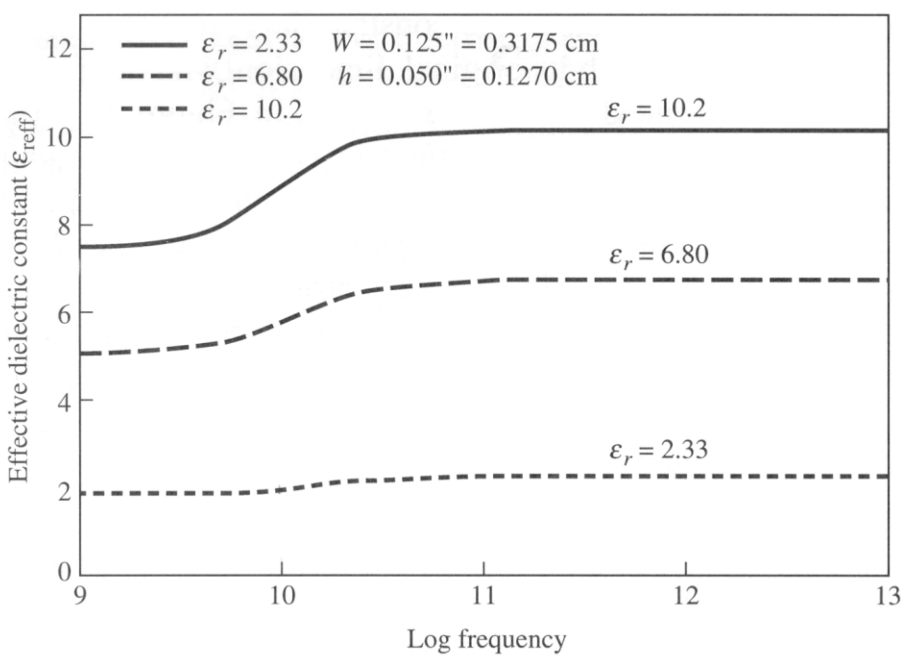


Figure 14.6 Effective dielectric constant versus frequency for typical substrates. (From Balanis, *Antenna Theory, Analysis and Design* (*Second Edition*))

The initial (low frequency) value of $\varepsilon_{r,\mathrm{eff}}$ is

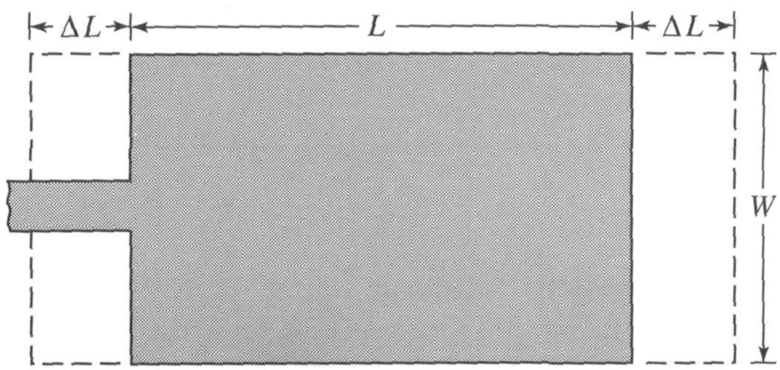
$$\varepsilon_{\mathrm{r,eff}} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\frac{h}{W}\right]^{-0.5}$$

where $W/h > 1$.

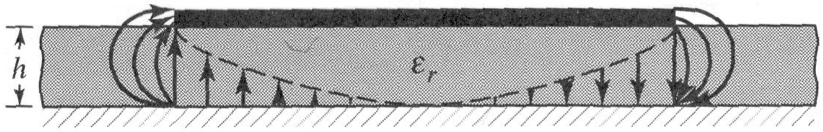
To account for fringing, the physical length of the rectangular patch is extended at both ends by a length

$$\Delta L = 0.412\,h\,\frac{\left(\varepsilon_{\mathrm{r,eff}} + 0.3\right)\left(\frac{W}{h} + 0.264\right)}{\left(\varepsilon_{\mathrm{r,eff}} - 0.258\right)\left(\frac{W}{h} + 0.8\right)}$$

to give an effective length of $L_{\mathrm{eff}} = L + 2\Delta L$ (see Figure 14.7).



(a) Top view



(b) Side view

Figure 14.7 Physical and effective lengths of rectangular microstrip patch. (From Balanis, *Antenna Theory, Analysis and Design* (*Second Edition*))

For the dominant TM_{010} mode (patch acts as a cavity resonator), the resonant frequency is

$$\left(f_r \right)_{010} = \frac{c}{2L\sqrt{\varepsilon_r}} \quad \text{(uncorrected)}$$

$$\left(f_{r,\mathrm{c}} \right)_{010} = \frac{c}{2L_{\mathrm{eff}}\sqrt{\varepsilon_{\mathrm{r,eff}}}} \quad \text{(corrected)}$$

$$= q\frac{c}{2L\sqrt{\varepsilon_r}}$$

where q is the fringing factor

$$q = \frac{\left(f_{r,\mathrm{c}} \right)_{010}}{\left(f_r \right)_{010}}$$

Solving for L_{eff}, we get

$$L_{\mathrm{eff}} = \frac{c}{2\left(f_{r,\mathrm{c}} \right)_{010}\sqrt{\varepsilon_{\mathrm{r,eff}}}} = \frac{\lambda}{2}$$

where λ is the guided wavelength (Note: want $(f_{r,\mathrm{c}})_{010} = f_r$).

Now, we can return to the transmission line model where we will represent the two radiating slots with parallel equivalent admittances

$$\text{Slot \#1} \quad Y_1 = G_1 + j\,B_1$$

$$\text{Slot \#2} \quad Y_2 = G_2 + j\,B_2$$

separated by a microstrip transmission line of length L with characteristic admittance $Y_c = 1/Z_c$ (see Figure 14.8). Moreover, for rectangular patches, the slots are identical

$$Y_1 = Y_2 \quad \Rightarrow \quad G_1 = G_2 \;\text{ and }\; B_1 = B_2$$

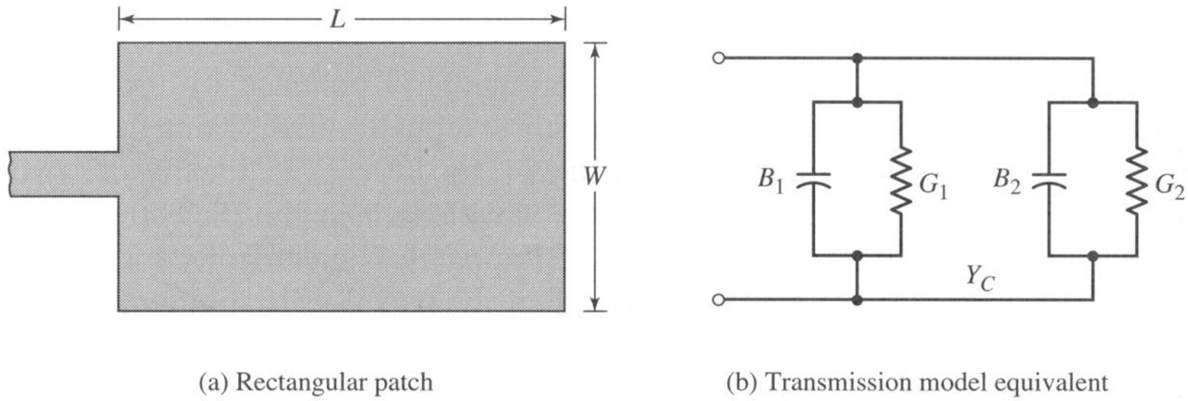


Figure 14.8 Rectangular microstrip patch and its equivalent circuit transmission model. (From Balanis, *Antenna Theory, Analysis and Design* (*Second Edition*))

How can we calculate the slot admittances? A simple derivation (based on infinitely long slots) that assumes that the electric field is uniform across the slot(s) yields

$$G_1 = \frac{W}{120\lambda_0}\left[1 - \frac{1}{24}\left(k_0 h\right)^2\right] \qquad \text{for } \frac{h}{\lambda_0} < \frac{1}{10}$$

$$B_1 = \frac{W}{120\lambda_0}\left[1 - 0.636\ln\left(k_0 h\right)\right] \quad \text{for } \frac{h}{\lambda_0} < \frac{1}{10}$$

where $k_0 = \frac{2\pi}{\lambda_0} = \frac{\omega}{c}$ is the free space wave number. This result is accurate only when $W >> \lambda_0$ and $h << \lambda_0$.

A more accurate formula for the conductance (based on the cavity model) is

$$G_1 = \frac{2P_{\text{rad}}}{\left|V_0\right|^2} = \frac{1}{\pi \eta_0} \int_{\theta=0}^{\pi} \left[\frac{\sin\left(\frac{k_0 W}{2} \cos\theta \right)}{\cos\theta} \right]^2 \sin^3\theta \, d\theta$$

where

$$P_{\text{rad}} = \frac{\left|V_0\right|^2}{2\pi \eta_0} \int_{\theta=0}^{\pi} \left[\frac{\sin\left(\frac{k_0 W}{2} \cos\theta \right)}{\cos\theta} \right]^2 \sin^3\theta \, d\theta$$

where η_0 is the impedance of free space. The integral can be evaluated numerically (best option), or solved to get

$$G_1 = \frac{1}{\pi\eta_0} \left[-2 + \cos\left(k_0 W\right) + k_0 W S_i\left(k_0 W\right) + \frac{\sin\left(k_0 W\right)}{k_0 W} \right]$$

where $S_i( )$ is the sine integral (see Appendix III of Balanis).

For extremely wide or narrow slots

$$G_1 = \begin{cases} \dfrac{1}{90}\left(\dfrac{W}{\lambda_0}\right)^2 & W << \lambda_0 \\ \dfrac{1}{120}\left(\dfrac{W}{\lambda_0}\right) & W >> \lambda_0 \end{cases}.$$

A plot of G_1 versus W/λ_0 is shown in Figure 14.9.

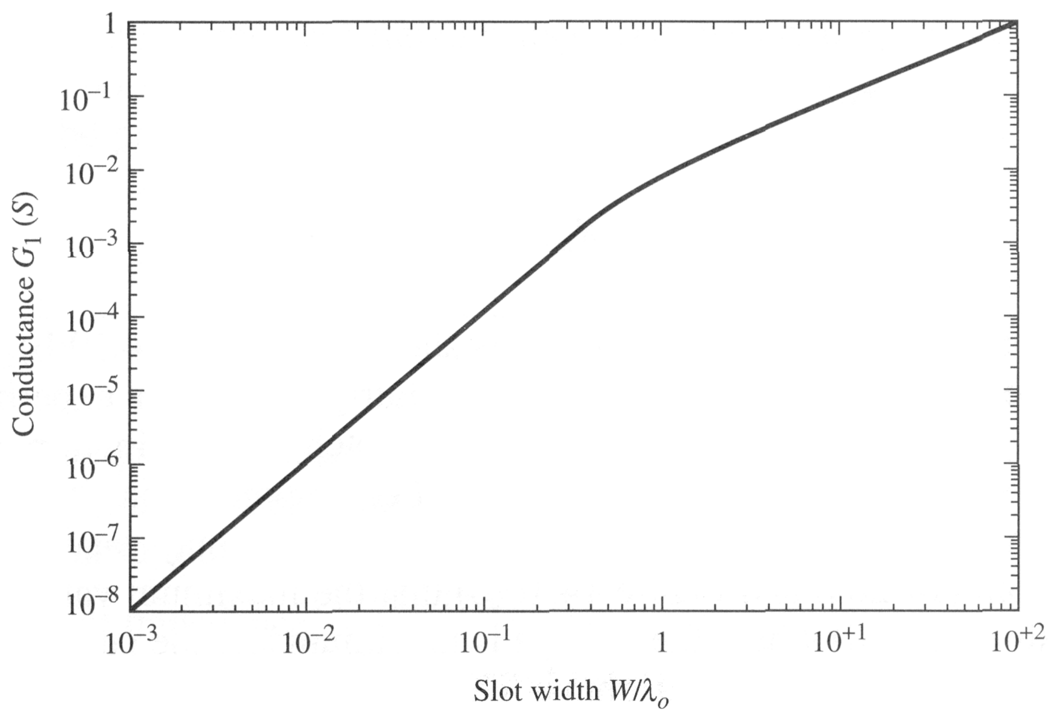


Figure 14.9 Slot conductance as a function of slot width. (From Balanis, *Antenna Theory, Analysis and Design* (*Second Edition*))

To get the input admittance Y_{in}, the admittance of slot #2 (Y_2) must be translated across the length of the rectangular patch to the location of slot #1 and added to Y_1 (they are in parallel). If this length (somewhere between L_{eff} and $L < \lambda/2$) is properly selected, the translated slot #2 admittance is

$$\tilde{Y}_2 = \tilde{G}_2 + j\tilde{B}_2 \approx G_1 - jB_1 = Y_1^*.$$

Then, the input admittance and impedance become

$$Y_{\text{in}} = Y_1 + \tilde{Y}_2 \approx 2G_1$$

and

$$Z_{\text{in}} = \frac{1}{Y_{\text{in}}} \approx R_{\text{in}} \approx \frac{1}{2G_1}.$$

Taking into account mutual coupling between the slots requires an adjustment yielding

$$Z_{\text{in}} = R_{\text{in}} = \frac{1}{2\left(G_1 \pm G_{12}\right)}$$

where G_{12} is the mutual conductance between the slots and the plus (+) sign is used for modes with asymmetric voltage distributions (e.g., the dominant TM_{010} mode) and the minus (-) sign is used for modes with symmetric voltage distributions. The mutual conductance can be calculated using

$$G_{12} = \frac{1}{\pi\eta_0}\int_{\theta=0}^{\pi}\left[\frac{\sin\left(\frac{k_0 W}{2}\cos\theta\right)}{\cos\theta}\right]^2 J_0\left(k_0 L\sin\theta\right)\sin^3\theta\, d\theta$$

where $J_0(\ )$ is a Bessel function of the first kind of order 0 (zero). Usually, $G_{12} << G_1$.

Typically, R_{in} is in the range of 150 to 300 Ω. To match a feeding microstrip transmission line to this impedance would require it to be very narrow; moreover, 50 Ω lines are a de facto standard in the RF circuit world.

Therefore, the question arises, "How can we adjust R_{in}?"

- R_{in} can be decreased by increasing W. This action is limited to $W/L < 2$ because the aperture efficiency of the slots drops for $W/L > 2$.

- An alternative is to use an **inset or recessed microstrip feed** (see Figure 14.10). This works because the voltage is a maximum and the current a minimum at the edges of the patch, leading to large impedance values. As we go into the patch, the voltage drops and the current increases, leading to smaller impedances until we reach the midpoint of the patch.

The input resistance of the inset feed is given by

$$R_{\text{in}}(y_0) \simeq \frac{1}{2(G_1 \pm G_{12})}\left[\cos^2\left(\frac{\pi}{L}y_0\right) + \frac{G_1^2 + B_1^2}{Y_{c,\text{feed}}^2}\sin^2\left(\frac{\pi}{L}y_0\right) - \frac{B_1}{Y_{c,\text{feed}}}\sin\left(\frac{2\pi}{L}y_0\right)\right]$$

where $Y_{c,\text{feed}} = 1/Z_{c,\text{feed}}$ and $Z_{c,\text{feed}}$ is the characteristic impedance of the feeding microstrip transmission line (width W_0). Note: the inset distance y_0 must be in the range $0 < y_0 < L/2$.

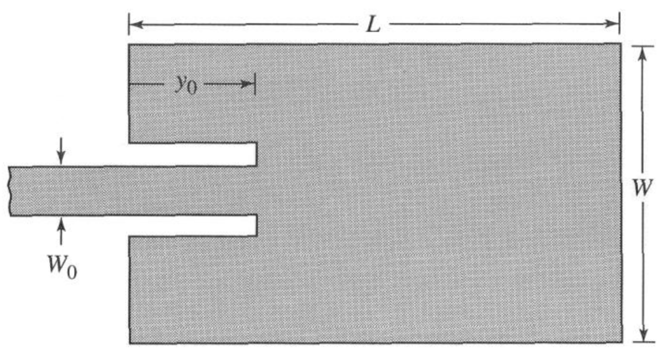
The characteristic impedance Z_c for any microstrip transmission line of width W (i.e., the feed or patch) can be calculated using

$$Z_c = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_{\text{r,eff}}}}\ln\left[\dfrac{8h}{W} + \dfrac{W}{4h}\right] & \dfrac{W}{h} \le 1 \\[2ex] \dfrac{\eta_0}{\sqrt{\varepsilon_{\text{r,eff}}}\left[\dfrac{W}{h} + 1.393 + 0.667\ln\left(\dfrac{W}{h} + 1.444\right)\right]} & \dfrac{W}{h} > 1 \end{cases}.$$

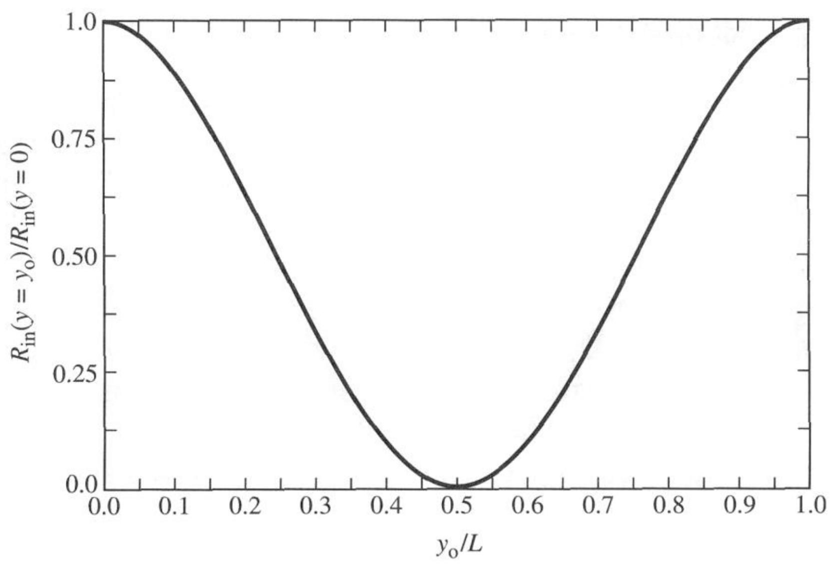
If $G_1/Y_{c,\text{feed}} << 1$ and $B_1/Y_{c,\text{feed}} << 1$ (the typical case since the width of the feeding microstrip is much less than that of the rectangular patch), the input resistance of the inset feed becomes

$$R_{\text{in}}(y_0) \simeq \frac{1}{2(G_1 \pm G_{12})} \cos^2\left(\frac{\pi}{L} y_0\right) = R_{\text{in}}(0) \cos^2\left(\frac{\pi}{L} y_0\right)$$

which is plotted in Figure 14.10 (normalized by $R_{\text{in}}(0)$).



(a) Recessed microstrip-line feed



(b) Normalized input resistance

Figure 14.10 Recessed microstrip-line feed and variation of normalized input resistance. (From Balanis, *Antenna Theory, Analysis and Design* (*Second Edition)*)

If we set $R_{\text{in}}(y_0)$ equal to the characteristic impedance of the feeding transmission line $Z_{c,\text{feed}}$ and note that $R_{\text{in}}(0) \to R_{\text{in}}$ in this case, we can solve for the inset length

$$y_0 = \frac{L}{\pi}\cos^{-1}\left(\sqrt{\frac{R_{\text{in}}(y_0)}{R_{\text{in}}(0)}}\right) = \frac{L}{\pi}\cos^{-1}\left(\sqrt{\frac{Z_{c,\text{feed}}}{R_{\text{in}}}}\right)$$

The notch width n on either side of the inset feed introduces some capacitance. This can impact the resonant frequency slightly ($\approx$1%) and can change the input impedance. Also, the feeding microstrip transmission line will perturb the radiation from slot #1 (i.e., Y_1 changes) which argues for minimizing n. As a starting point, select the notch width n to fall in the range $0.2W_0 < n < 0.5W_0$. To get truly accurate results, a full-wave numerical model of the antenna should be run after using the design based on the transmission line model to get accurate lengths and widths.

A similar process applies if a coaxial probe feed is used, i.e., the input resistance can be decreased by moving the coaxial probe in from the edge of the patch.

Design Procedure

1) Specify ε_r and h of substrate, the desired resonant frequency f_r, and the impedance $Z_{c,\text{feed}}$ of the feeding transmission line.

2) Calculate width of patch using

$$W = \frac{c}{2 f_r}\sqrt{\frac{2}{\varepsilon_r + 1}}$$

(selected to give good radiation efficiency). Strictly, the patch width should be less than the length (i.e., $W < L$) to ensure operation only in the TM_{010} mode. Practically, $W > L$ can be used if the patch is excited/driven so as not to excite other modes (e.g., TM_{001} mode is dominant when $W > L$). W can be changed so long as $W/L < 2$, avoid aperture efficiency decrease.

3) Calculate $\varepsilon_{\text{r,eff}} = \dfrac{\varepsilon_r + 1}{2} + \dfrac{\varepsilon_r - 1}{2}\left[1 + 12\dfrac{h}{W}\right]^{-0.5}$

4) Calculate $\Delta L = 0.412\, h \dfrac{\left(\varepsilon_{\text{r,eff}} + 0.3\right)\left(\dfrac{W}{h} + 0.264\right)}{\left(\varepsilon_{\text{r,eff}} - 0.258\right)\left(\dfrac{W}{h} + 0.8\right)}$

5) Calculate the effective length and guided wavelength λ.

$$L_{\text{eff}} = \frac{c}{2 f_r \sqrt{\varepsilon_{\text{r,eff}}}}$$

$$\lambda = 2 L_{\text{eff}}$$

6) Calculate the patch length L.

$$L = L_{\text{eff}} - 2\Delta L = \frac{\lambda}{2} - 2\Delta L,$$

L/λ, and calculate & check that $W/L < 2$.

7) Calculate $G_1 \approx G_2$ and $B_1 \approx B_2$.

$$G_{1,\text{est}} = \frac{W}{120\lambda_0}\left[1 - \frac{1}{24}\left(k_0 h\right)^2\right] \qquad \text{for} \quad \frac{h}{\lambda_0} < \frac{1}{10}$$

$$B_{1,\text{est}} = \frac{W}{120\lambda_0}\left[1 - 0.636\ln\left(k_0 h\right)\right] \quad \text{for} \quad \frac{h}{\lambda_0} < \frac{1}{10}$$

$$G_1 = \frac{1}{\pi\eta_0}\int_{\theta=0}^{\pi}\left[\frac{\sin\left(\dfrac{k_0 W}{2}\cos\theta\right)}{\cos\theta}\right]^2 \sin^3\theta \, d\theta$$

$$B_1 = \left(\frac{G_1}{G_{1,\text{est}}}\right) B_{1,\text{est}}$$

where $k_0 = \dfrac{2\pi}{\lambda_0} = \dfrac{\omega}{c}$ is the free space wave number.

8) Calculate the characteristic impedance $Z_{c,\text{ant}}$ and admittance $Y_{c,\text{ant}}$ for the rectangular microstrip antenna.

$$Z_{c,\text{ant}} = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_{\text{r,eff}}}}\ln\left[\dfrac{8h}{W} + \dfrac{W}{4h}\right] & \dfrac{W}{h} \le 1 \\[2ex] \dfrac{\eta_0}{\sqrt{\varepsilon_{\text{r,eff}}}\left[\dfrac{W}{h} + 1.393 + 0.667\ln\left(\dfrac{W}{h} + 1.444\right)\right]} & \dfrac{W}{h} > 1 \end{cases}$$

and

$$Y_{c,\text{ant}} = \frac{1}{Z_{c,\text{ant}}}$$

9) (Optional) Use Smith chart or direct calculation to verify

$$\tilde{Y}_2 = \tilde{G}_2 + j\tilde{B}_2 \approx G_1 - jB_1 = Y_1^*.$$

10) Calculate the mutual conductance between the slots

$$G_{12} = \frac{1}{\pi\eta_0}\int_{\theta=0}^{\pi}\left[\frac{\sin\left(\frac{k_0 W}{2}\cos\theta\right)}{\cos\theta}\right]^2 J_0\left(k_0 L\sin\theta\right)\sin^3\theta\, d\theta$$

11) Calculate R_{in} (used plus (+) sign in original equation).

$$Z_{\text{in}} = R_{\text{in}} = \frac{1}{2\left(G_1 + G_{12}\right)}$$

12) If an inset microstrip feed is required (i.e., $R_{\text{in}} \neq Z_{c,\text{feed}}$), calculate length y_0 of the inset needed to match the rectangular patch to the feeding transmission line. When $G_1/Y_{c,\text{feed}} << 1$ and $B_1/Y_{c,\text{feed}} << 1$,

$$y_0 = \frac{L}{\pi}\cos^{-1}\left(\sqrt{\frac{Z_{c,\text{feed}}}{R_{\text{in}}}}\right).$$

This answer can be checked using

$$R_{\text{in}}(y_0) \simeq \frac{1}{2\left(G_1 \pm G_{12}\right)}\left[\cos^2\left(\frac{\pi y_0}{L}\right) + \frac{G_1^{\,2} + B_1^{\,2}}{Y_{c,\text{feed}}^{\,2}}\sin^2\left(\frac{\pi y_0}{L}\right) - \frac{B_1}{Y_{c,\text{feed}}}\sin\left(\frac{2\pi y_0}{L}\right)\right]$$

If $R_{\text{in}}(y_0)$ is not equal to $Z_{c,\text{feed}}$, the length y_0 should be iteratively adjusted until $R_{\text{in}}(y_0) = Z_{c,\text{feed}}$.

13) Determine the width W_0 of the feeding microstrip transmission line.

Method 1:

If available, use information/tools from the manufacturer of the substrate/PCB. For example, the Rogers Corporation has an on-line JAVA calculator for determining the width W_0 of a microstrip transmission line based on desired Z_0 and the particular substrate at

http://www.rogerscorporation.com/mwu/mwi_java/Mwij_vp.html

Method 2:

Iteratively (i.e., guess a starting value of $W_0 < W$) determine the width W_0 of the feeding transmission line using

$$\varepsilon_{\mathrm{r,eff}} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\frac{h}{W_0}\right]^{-0.5}$$

and

$$Z_{c,\mathrm{feed}} = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_{\mathrm{r,eff}}}} \ln\left[\dfrac{8h}{W_0} + \dfrac{W_0}{4h}\right] & \dfrac{W_0}{h} \le 1 \\ \dfrac{\eta_0}{\sqrt{\varepsilon_{\mathrm{r,eff}}}\left[\dfrac{W_0}{h} + 1.393 + 0.667\ln\left(\dfrac{W_0}{h} + 1.444\right)\right]} & \dfrac{W_0}{h} > 1 \end{cases}.$$

Note that the effective permittivity must be recalculated for the feeding transmission line because it has different dimensions than the patch antenna (i.e., it is much narrower).

Method 3:

Use the design equation

$$\frac{W_0}{h} = \begin{cases} \dfrac{8e^A}{e^{2A}-2} & \dfrac{W_0}{h} \le 2 \\ \dfrac{2}{\pi}\left\{B - 1 - \ln(2B-1) + \dfrac{\varepsilon_r - 1}{2\varepsilon_r}\left[\ln(B-1) + 0.39 - \dfrac{0.61}{\varepsilon_r}\right]\right\} & \dfrac{W_0}{h} > 2 \end{cases}.$$

where

$$A = \frac{Z_{c,\text{feed}}}{60}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right)$$

and

$$B = \frac{\eta_0 \pi}{2Z_{c,\text{feed}}\sqrt{\varepsilon_r}}.$$

14) Select the notch width n in the range $0.2W_0 < n < 0.5W_0$.

15) Draw resulting design.

Top View