

Example- Rectangular Microstrip Cavity Analysis

- 1) Problem statement/specification- Design a rectangular microstrip antenna to operate at a frequency of 5 GHz on a RT/Duroid ($\epsilon_r = 2.2$, $h = 1.588 \text{ mm} = 1/16''$). The antenna is to be matched to a 50Ω microstrip transmission line on this substrate. Accurately sketch final design.

$$f_r := 5 \cdot 10^9 \quad \text{Hz} \quad \epsilon_r := 2.2 \quad h := 1.588 \cdot 10^{-3} \quad \text{mm}$$

$$Z_{fd} := 50 \quad \Omega \quad Y_{fd} := \frac{1}{Z_{fd}} \quad Y_{fd} = 0.02 \quad \text{S}$$

Define some constants

$$c := 2.9979 \cdot 10^8 \quad \text{m/s} \quad \eta_0 := 376.7303 \quad \Omega$$

$$\lambda_0 := \frac{c}{f_r} \quad \lambda_0 = 0.05996 \quad \text{m} \quad k_0 := \frac{2 \cdot \pi}{\lambda_0} \quad k_0 = 104.793 \quad \text{rad/m}$$

- 2) Calculate/estimate width of patch:

$$W := \frac{c}{2 \cdot f_r} \cdot \sqrt{\frac{2}{\epsilon_r + 1}} \quad \frac{W}{h} = 14.925 \quad \boxed{W \cdot 100 = 2.37} \quad \text{cm}$$

- 3) Calculate the effective permittivity

$$\epsilon_{r_eff} := \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \cdot \left(1 + 12 \cdot \frac{h}{W} \right)^{-0.5} \quad \epsilon_{r_eff} = 2.04671$$

- 4) Calculate the fringing length ΔL

$$\Delta L := 0.412 \cdot h \cdot \frac{\epsilon_{r_eff} + 0.3}{\epsilon_{r_eff} - 0.258} \cdot \frac{\frac{W}{h} + 0.264}{\frac{W}{h} + 0.8} \quad \Delta L = 8.291 \times 10^{-4} \quad \text{m}$$

$$\Delta L \cdot 1000 = 0.8291 \quad \text{mm}$$

- 5) Calculate effective patch length L_{eff} and guided wavelength λ

$$L_{eff} := \frac{c}{2 \cdot f_r \cdot \sqrt{\epsilon_{r_eff}}} \quad L_{eff} = 0.02096 \quad \text{m, or} \quad L_{eff} \cdot 100 = 2.0955 \quad \text{cm}$$

$$\lambda := 2 \cdot L_{eff} \quad \lambda = 0.04191 \quad \text{m, or} \quad \lambda \cdot 100 = 4.19101 \quad \text{cm}$$

Example- Rectangular Microstrip Cavity Analysis cont.

6) Calculate physical patch length L , L/λ , and check if $W/L > 2$?

$$L := L_{\text{eff}} - 2 \cdot \Delta L \quad L = 0.019297 \quad \text{m} \quad \boxed{L \cdot 100 = 1.9297} \quad \text{cm}$$

$$\frac{L}{\lambda} = 0.4604 \quad \frac{W}{L} = 1.228 < 2 \quad (\text{will be OK})$$

7) Calculate the slot conductances $G_1 = G_2$ and susceptances $B_1 = B_2$

$$G_{1\text{est}} := \frac{W}{120 \cdot \lambda_0} \cdot \left[1 - 24 \cdot (k_0 \cdot h)^2 \right] \quad G_{1\text{est}} = 1.105 \times 10^{-3} \quad \text{S}$$

$$B_{1\text{est}} := \frac{W}{120 \cdot \lambda_0} \cdot (1 - 0.636 \cdot \ln(k_0 \cdot h)) \quad B_{1\text{est}} = 7.051 \times 10^{-3} \quad \text{S}$$

$$G_1 := \frac{1}{\pi \cdot \eta_0} \cdot \int_0^\pi \left(\frac{\sin\left(\frac{k_0 \cdot W}{2} \cdot \cos(\theta)\right)}{\cos(\theta)} \right)^2 \cdot (\sin(\theta))^3 d\theta$$

$$G_2 := G_1 \quad G_1 = 1.5735 \times 10^{-3} \quad \text{S}$$

$$B_1 := \frac{G_1}{G_{1\text{est}}} \cdot B_{1\text{est}} \quad B_2 := B_1 \quad B_1 = 0.01004 \quad \text{S}$$

$$Y_1 := G_1 + j \cdot B_1 \quad Y_2 := Y_1 \quad Y_1 = 1.574 \times 10^{-3} + 0.01i \quad \text{S}$$

$$Z_1 := \frac{1}{Y_1} \quad Z_2 := Z_1 \quad Z_1 = 15.227 - 97.186i \quad \Omega$$

8) Calculate $Z_{c,\text{ant}}$ and admittance $Y_{c,\text{ant}}$ of microstrip antenna

$$Z_{c\text{ant}} := \frac{\eta_0}{\sqrt{\epsilon_{r,\text{eff}}} \cdot \left(\frac{W}{h} + 1.393 + 0.667 \cdot \ln\left(\frac{W}{h} + 1.44\right) \right)}$$

$$Z_{c\text{ant}} = 14.483 \quad \Omega \quad Y_{c\text{ant}} := \frac{1}{Z_{c\text{ant}}} \quad Y_{c\text{ant}} = 0.069 \quad \text{S}$$

Example- Rectangular Microstrip Cavity Analysis cont.

9) skip

10) Calculate the mutual conductance between the slots.

$$G_{12} := \frac{1}{\pi \cdot \eta_0} \cdot \int_0^{\pi} \left(\frac{\sin\left(\frac{k_0 \cdot W}{2} \cdot \cos(\theta)\right)}{\cos(\theta)} \right)^2 \cdot J_0(k_0 \cdot L \cdot \sin(\theta)) \cdot (\sin(\theta))^3 d\theta$$

$$G_{12} = 5.163 \times 10^{-4} \quad \text{S}$$

11) Calculate expected input impedance at the edge of the rectangular patch

$$R_{in} := \frac{1}{2 \cdot (G_1 + G_{12})}$$

$$R_{in} = 239.2541 \quad \Omega$$

12) Calculate length to inset microstrip feed

$$R_{inset}(y_0) := R_{in} \cdot \left[\left(\cos\left(\frac{\pi \cdot y_0}{L}\right) \right)^2 + \frac{(G_1^2 + B_1^2)}{Y_{fd}^2} \cdot \left(\sin\left(\frac{\pi \cdot y_0}{L}\right) \right)^2 - \frac{B_1}{Y_{fd}} \cdot \sin\left(\frac{2\pi \cdot y_0}{L}\right) \right]$$

$$y_0 := 0.0042214 \quad \text{m} \quad R_{inset}(y_0) = 50 \quad \Omega$$

$$\boxed{y_0 \cdot 100 = 0.422} \quad \text{cm}$$

13) Calculate width of 50 ohm feeding microstrip:

$$\epsilon_{r_fd}(W_0) := \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \cdot \left(1 + 12 \cdot \frac{h}{W_0} \right)^{-0.5}$$

$$Z_{cfd}(W_0) := \frac{\eta_0}{\sqrt{\epsilon_{r_fd}(W_0)} \cdot \left(\frac{W_0}{h} + 1.393 + 0.667 \cdot \ln\left(\frac{W_0}{h} + 1.444\right) \right)}$$

$$W_0 := 3.1037 \cdot h$$

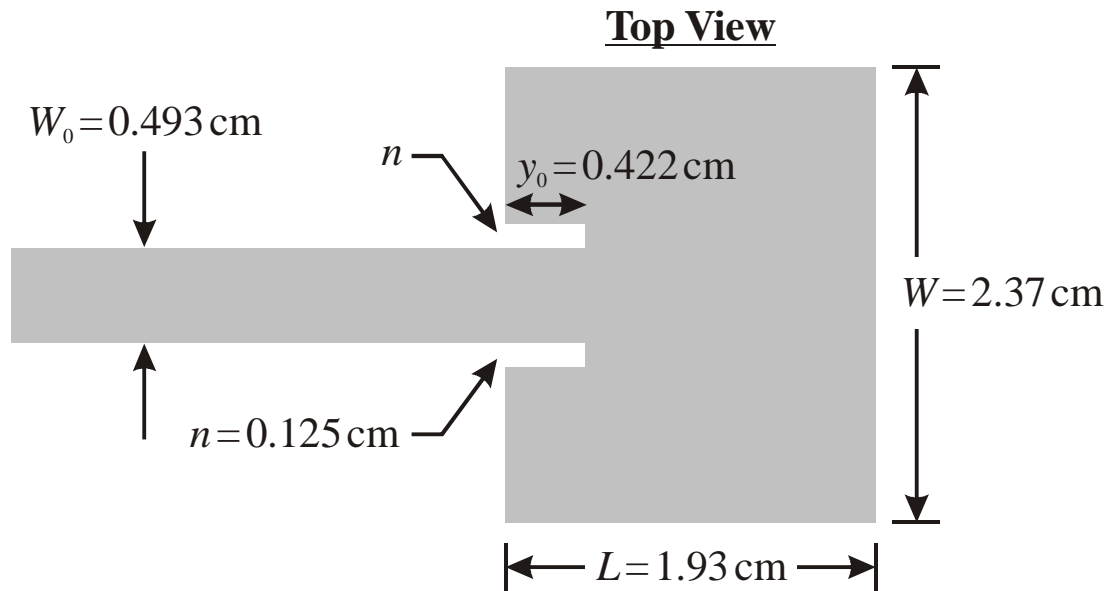
$$Z_{cfd}(W_0) = 50 \quad \Omega$$

$$\epsilon_{r_fd}(W_0) = 1.872 \quad W_0 = 0.004929 \quad \text{m} \quad \boxed{W_0 \cdot 100 = 0.4929} \quad \text{cm}$$

Example- Rectangular Microstrip Cavity Analysis cont.

14) Select a notch width n is needed. Arbitrarily select $n \sim W_0/4 = 0.125$ cm.

15) Draw resulting design

**Find the maximum directivity of this design**

Method 1 (assumes $k_0 h \ll 1$, a fair assumption)

$$k_0 \cdot h = 0.1664$$

$$I_2 := \int_0^\pi \int_0^\pi \left(\frac{\sin\left(\frac{k_0 \cdot W}{2} \cdot \cos(\theta)\right)}{\cos(\theta)} \right)^2 \cdot (\sin(\theta))^3 \cdot \left(\cos\left(\frac{k_0 \cdot L_{\text{eff}}}{2} \cdot \sin(\theta) \cdot \sin(\phi)\right) \right)^2 d\theta d\phi$$

$$I_2 = 3.625$$

$$D_{\text{tot1}} := \left(\frac{2 \cdot \pi \cdot W}{\lambda_0} \right)^2 \cdot \frac{\pi}{I_2}$$

$$D_{\text{tot1}} = 5.346$$

$$10 \cdot \log(D_{\text{tot1}}) = 7.28 \quad \text{dBi}$$

Example- Rectangular Microstrip Cavity Analysis cont.

Method 2 (assumes $k_0 h \ll 1$, a fair assumption)

$$k_0 \cdot h = 0.1664$$

$$I1 := \int_0^{\pi} \left(\frac{\sin\left(\frac{k_0 \cdot W}{2} \cdot \cos(\theta)\right)}{\cos(\theta)} \right)^2 \cdot (\sin(\theta))^3 d\theta \quad I1 = 1.862$$

First, find directivity of a single slot- $D0 := \left(\frac{2 \cdot \pi \cdot W}{\lambda_0} \right)^2 \cdot \frac{1}{I1} \quad D0 = 3.312$

Then, find overall directivity

$$Dt_{ot2} := D0 \cdot \frac{2}{1 + \frac{G12}{G1}}$$

$$\boxed{Dt_{ot2} = 4.988}$$

$$\boxed{10 \cdot \log(Dt_{ot2}) = 6.9792} \quad \text{dBi}$$

A bit less than the first method.

Method 3 Use Figure 14.19 in Antenna Theory(2nd Edn) by Balanis (valid since $\epsilon_r = 2.2 \sim 2.25$)

$$\frac{W}{\lambda_0} = 0.395$$

Plot vertical line on figure at this value (see following page).

Read directivity (in dBi) as-

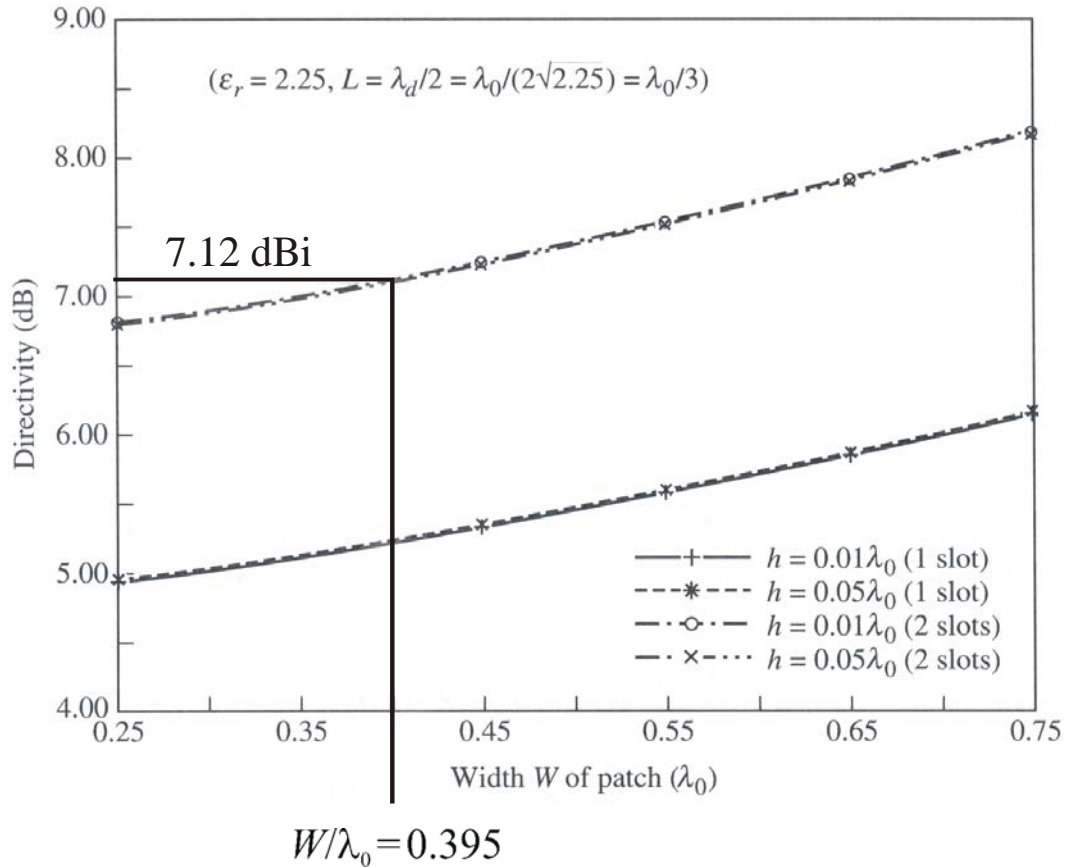
$$\boxed{Dt_{ot3} := 7.12} \quad \text{dBi}$$

About halfway in between previous results.

All three methods agree to within 0.3 dB.

Example- Rectangular Microstrip Cavity Analysis cont.

Method 3 cont.



Estimate the HPBW's for this design

$$\Theta_E := 2 \cdot \arccos \left[\sqrt{\frac{7.03 \cdot \lambda_0^2}{4(3 \cdot L_{\text{eff}}^2 + h^2) \cdot \pi^2}} \right]$$

$$\Theta_E \cdot \frac{180}{\pi} = 91.717 \quad \text{deg}$$

$$\Theta_H := 2 \cdot \arccos \left(\sqrt{\frac{1}{2 + k_0 \cdot W}} \right)$$

$$\Theta_H \cdot \frac{180}{\pi} = 123.637 \quad \text{deg}$$

Example- Rectangular Microstrip Cavity Analysis cont.**Define function for normalized far-zone electric field and spherical angles θ & ϕ**

$$X(\theta, \phi) := \frac{k_0 \cdot h}{2} \cdot \sin(\theta) \cdot \cos(\phi) \qquad Z(\theta) := \frac{k_0 \cdot W}{2} \cdot \cos(\theta)$$

$$E(\theta, \phi) := \sin(\theta) \cdot \frac{\sin(X(\theta, \phi))}{X(\theta, \phi)} \cdot \frac{\sin(Z(\theta))}{Z(\theta)} \cdot \cos\left(\frac{k_0 \cdot L_{\text{eff}}}{2} \cdot \sin(\theta) \cdot \sin(\phi)\right)$$

$$n := 0..180 \qquad \text{phi}_n := (n - 90) \cdot \frac{\pi}{180} \qquad \text{theta}_n := n \cdot \frac{\pi}{180}$$

Calculate normalized E-plane (x-y plane for $x > 0$) far-zone electric field & HPBW

$$E_{\text{Eplane}_n} := \left| E\left(\frac{\pi}{2}, \text{phi}_n\right) \right|$$

$$E_{\text{Eplane_dB}_n} := \text{if}(E_{\text{Eplane}_n} < 0.01, 0, 20 \cdot \log(E_{\text{Eplane}_n}) + 40)$$

$$\phi_1 := 45.63 \cdot \frac{\pi}{180} \qquad \left| E\left(\frac{\pi}{2}, \phi_1\right) \right| = 0.7071$$

$$\phi_2 := -45.63 \cdot \frac{\pi}{180} \qquad \left| E\left(\frac{\pi}{2}, \phi_2\right) \right| = 0.7071$$

**Symmetric about $\phi = 0$ at
 $\theta = 90$ deg**

$$\text{HPBW}_{\text{Eplane}} := (\phi_1 - \phi_2) \cdot \frac{180}{\pi} \qquad \text{HPBW}_{\text{Eplane}} = 91.26 \quad \text{degrees}$$

$$\Theta_E \cdot \frac{180}{\pi} = 91.72 \quad \text{degrees}$$

Very close agreement

Example- Rectangular Microstrip Cavity Analysis cont.**Calculate normalized H-plane (x - z plane for $x > 0$) far-zone electric field & HPBW**

$$E_{\text{Hplane}_n} := |E(\theta_n, 0)|$$

$$E_{\text{Hplane_dB}_n} := \text{if}(E_{\text{Hplane}_n} < 0.01, 0, 20 \cdot \log(E_{\text{Hplane}_n}) + 40)$$

$$\theta_1 := 51.55 \cdot \frac{\pi}{180} \quad |E(\theta_1, 0)| = 0.7071$$

**Symmetric about $\theta = 90$ deg
at $\phi = 0$**

$$\theta_2 := 128.45 \cdot \frac{\pi}{180} \quad |E(\theta_2, 0)| = 0.7071$$

$$\text{HPBW}_{\text{Hplane}} := (\theta_2 - \theta_1) \cdot \frac{180}{\pi}$$

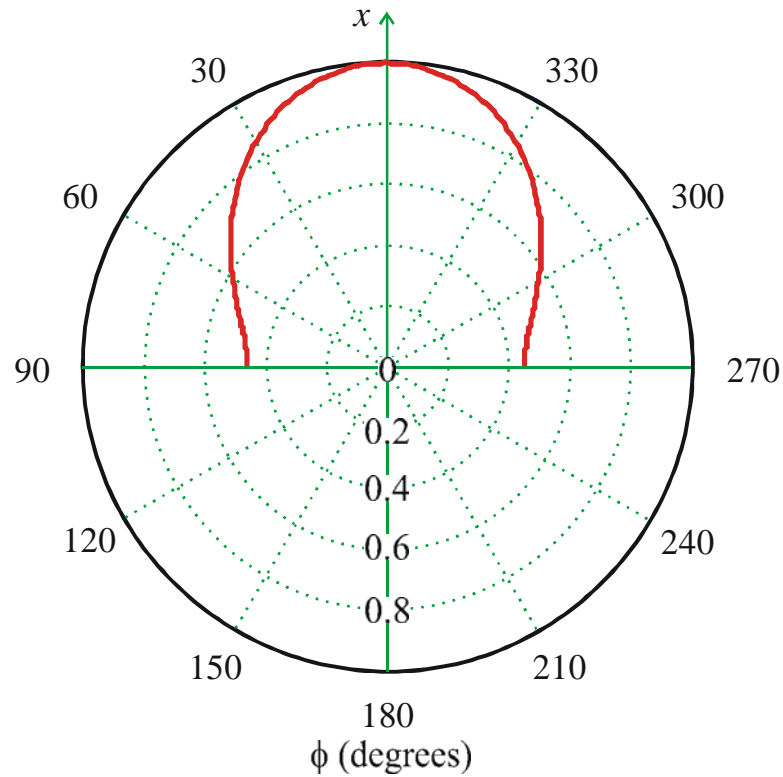
$$\text{HPBW}_{\text{Hplane}} = 76.9 \quad \text{degrees}$$

$$\Theta_{\text{H}} \cdot \frac{180}{\pi} = 123.64 \quad \text{degrees}$$

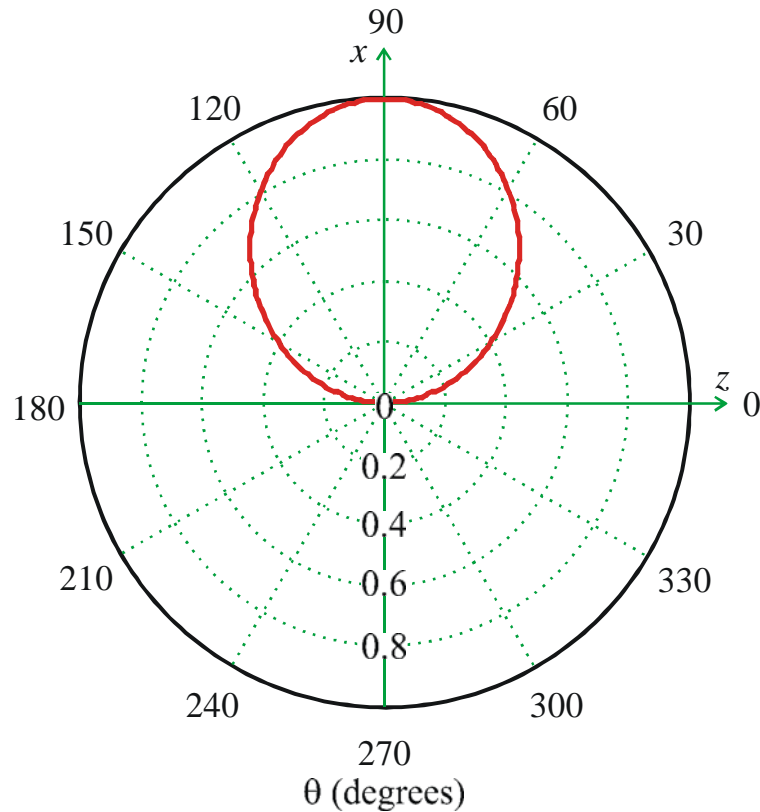
Not very good agreement

Example- Rectangular Microstrip Cavity Analysis cont.

Normalized $|E|$ in E-plane (x - y plane with $x > 0$)

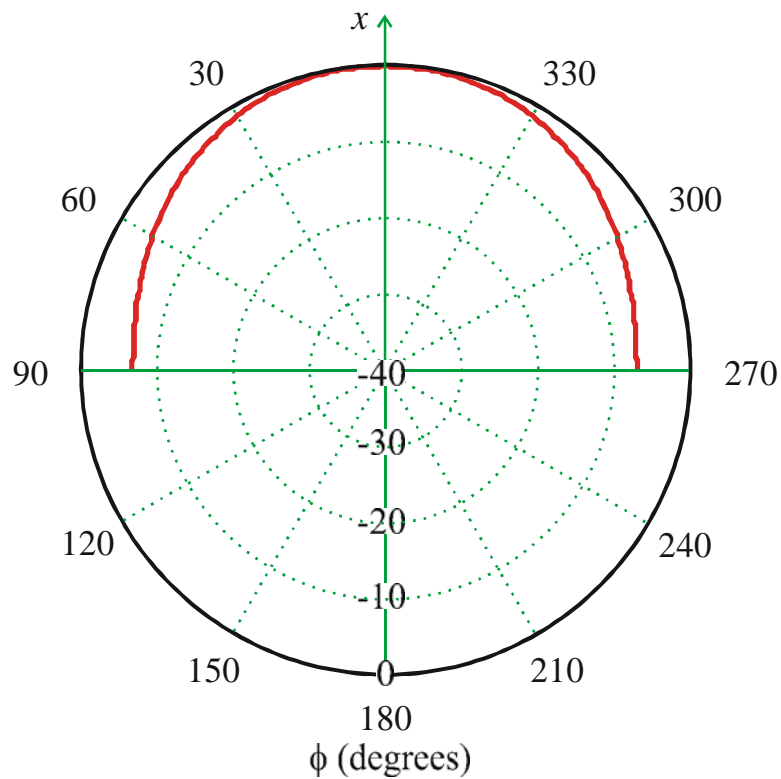


Normalized $|E|$ in H-plane (x - z plane with $x > 0$)

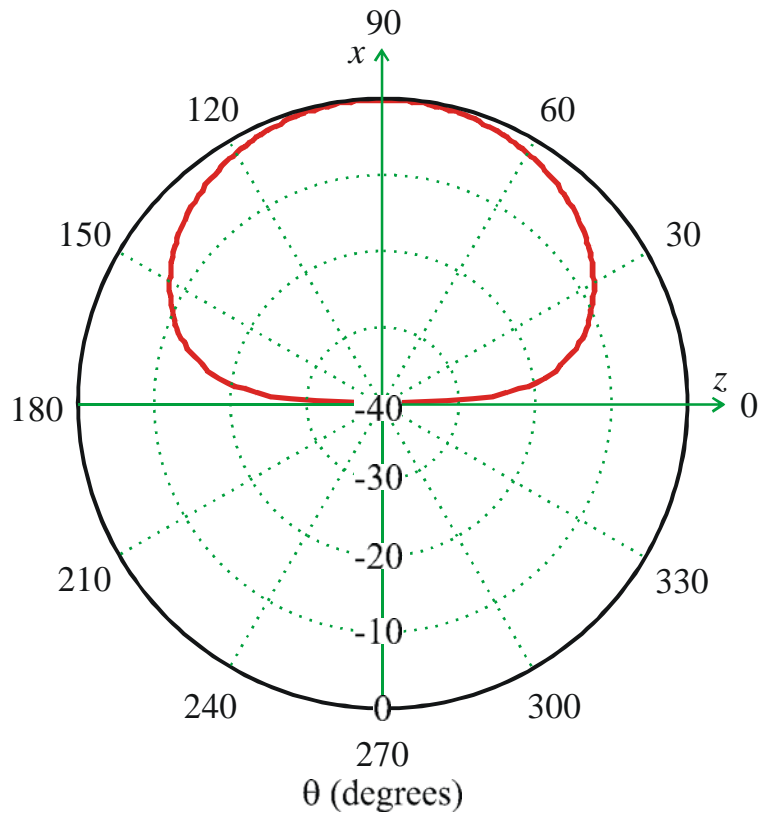


Example- Rectangular Microstrip Cavity Analysis cont.

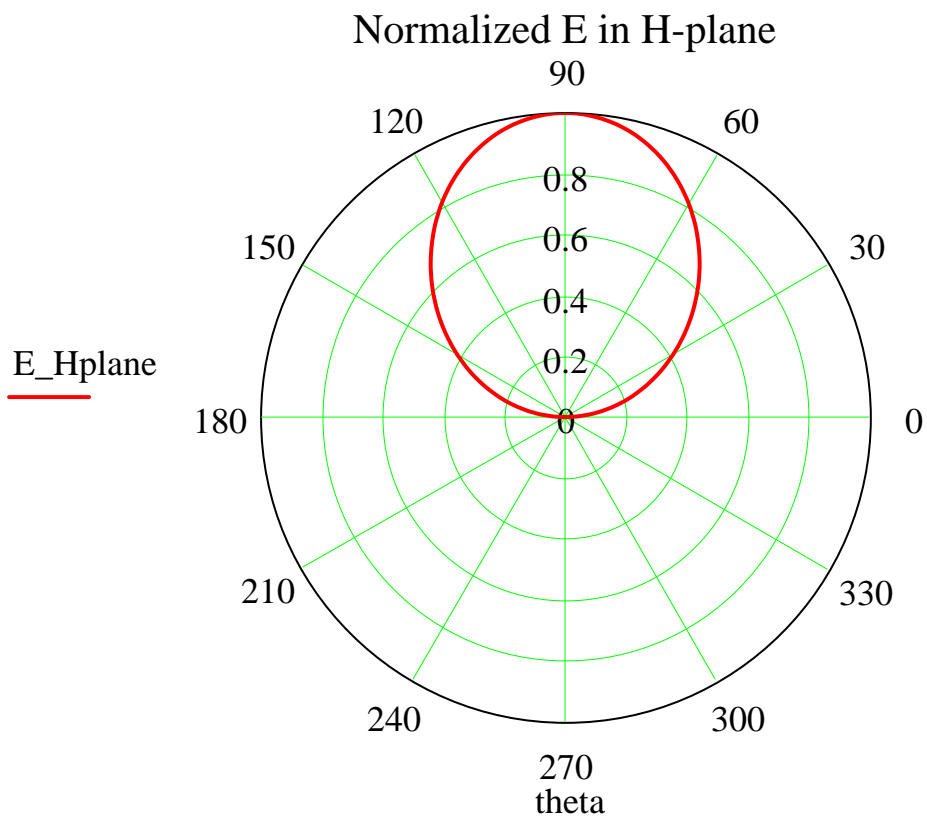
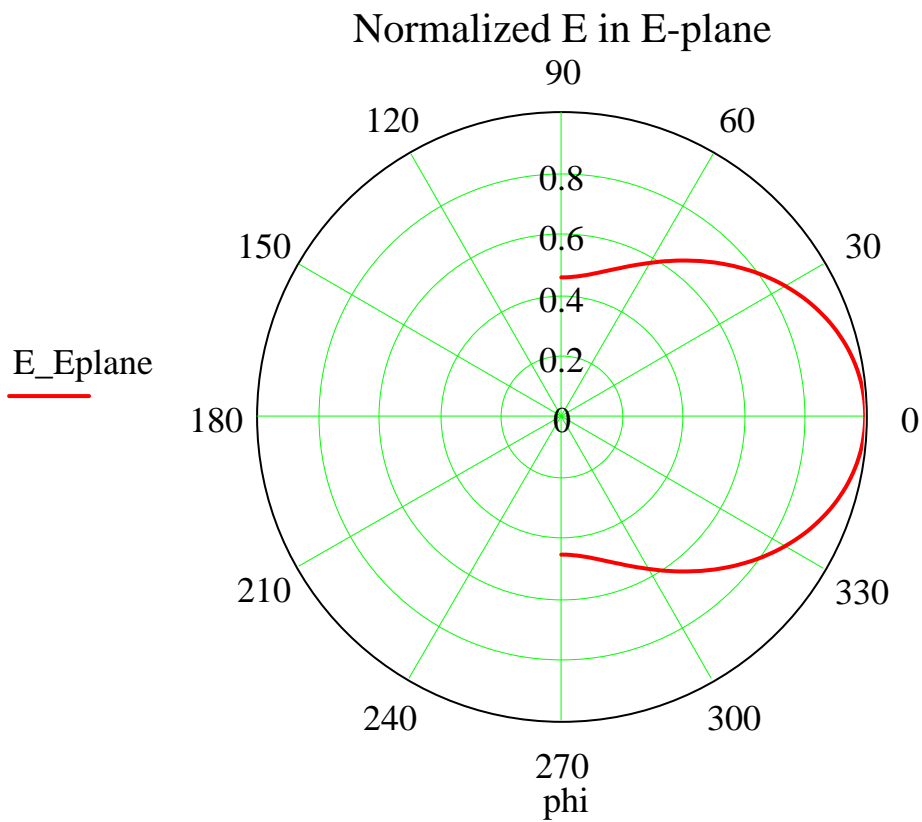
Normalized $|E|$ (dB) in E-plane (x - y plane with $x > 0$)



Normalized $|E|$ (dB) in H-plane (x - z plane with $x > 0$)



Example- Rectangular Microstrip Cavity Analysis cont.



Example- Rectangular Microstrip Cavity Analysis cont.

