Example- Rectangular Microstrip Cavity Analysis

1) Specifications- RT/Duroid 5880 substrate (ε_r = 2.2, h = 0.1588 cm) and a resonant frequency of f_r = 9 GHz matched to a 50 Ω microstrip transmission line.

Define some constants/variables from prior example

$fr := 9 \cdot 10^9$ Hz $er := 2.2$ $h := 0.1588 \cdot 0.01$ m $R\_fd := 50$ Ω

$c := 2.9979 \cdot 10^8$ m/s $\eta0 := 376.7343$ Ω $\lambda0 := \frac{c}{fr}$ $\lambda0 \cdot 100 = 3.331$ cm

$k0 := \frac{(2 \cdot \pi)}{\lambda0}$ $k0 = 188.628$ rad/m $W := \frac{c}{2 \cdot fr} \cdot \sqrt{\frac{2}{er + 1}}$ $W \cdot 100 = 1.3167$ cm

$\frac{W}{h} = 8.292$ $er\_eff := \frac{er + 1}{2} + \frac{(er - 1)}{2} \cdot \left(1 + 12 \cdot \frac{h}{W}\right)^{-0.5}$ $er\_eff = 1.984$

$$\Delta L := 0.412 \cdot h \cdot \frac{(er\_eff + 0.3) \cdot \left(\frac{W}{h} + 0.264\right)}{(er\_eff - 0.258) \cdot \left(0.8 + \frac{W}{h}\right)} \qquad Leff := \frac{c}{2 \cdot fr \cdot \sqrt{er\_eff}} \qquad \lambda := 2 \cdot Leff$$

$Leff \cdot 100 = 1.18256$ cm $\lambda \cdot 100 = 2.36513$ cm $k0 \cdot h = 0.2995$

$L := Leff - 2 \cdot \Delta L$ $L \cdot 100 = 1.01961$ cm $\frac{L}{\lambda} = 0.431$ $\frac{W}{L} = 1.291$

$$G1 := \frac{1}{\pi \cdot \eta0} \cdot \int_0^{\pi} \left(\frac{\sin\left(\frac{k0 \cdot W \cdot \cos(\theta)}{2}\right)}{\cos(\theta)}\right)^2 \cdot (\sin(\theta))^3 \, d\theta \qquad G1 = 1.574 \times 10^{-3} \text{ S}$$

$$G12 := \frac{1}{\pi \cdot \eta0} \cdot \int_0^{\pi} \left(\frac{\sin\left(\frac{k0 \cdot W \cdot \cos(\theta)}{2}\right)}{\cos(\theta)}\right)^2 \cdot J0(k0 \cdot L \cdot \sin(\theta)) \cdot (\sin(\theta))^3 \, d\theta$$

$G12 = 5.963 \times 10^{-4}$ S

Find the directivity of this microstrip patch antenna

Method 1 (assumes k_0h <<1, a so-so assumption in this case)

$$I2 := \int_0^{\pi}\int_0^{\pi}\left(\frac{\sin\left(\frac{k0\cdot W}{2}\cdot\cos(\theta)\right)}{\cos(\theta)}\right)^2\cdot(\sin(\theta))^3\cdot\left(\cos\left(\frac{k0\cdot Leff}{2}\cdot\sin(\theta)\cdot\sin(\phi)\right)\right)^2\,d\theta\,d\phi$$

$$I2 = 3.574$$

$$Dtot1 := \left(\frac{2\cdot\pi\cdot W}{\lambda 0}\right)^2\cdot\frac{\pi}{I2} \qquad Dtot1 = 5.422 \qquad 10\cdot\log(Dtot1) = 7.342 \quad \text{dBi}$$

Method 2

$$I1 := \left[\int_0^{\pi}\left(\frac{\sin\left(\frac{k0\cdot W\cdot\cos(\theta)}{2}\right)}{\cos(\theta)}\right)^2\cdot(\sin(\theta))^3\,d\theta\right]$$

single-slot directivity $$D0 := \left(\frac{2\cdot\pi\cdot W}{\lambda 0}\right)^2\cdot\frac{1}{I1} \qquad D0 = 3.312$$

$$Dtot2 := D0\cdot\frac{2}{1+\frac{G12}{G1}} \qquad Dtot2 = 4.804 \qquad 10\cdot\log(Dtot2) = 6.816 \quad \text{dBi}$$

Method 3 (can use Figure 14.19 since ε_r is close)

$$\frac{W}{\lambda 0} = 0.395$$ Read value from Figure $$Dtot3 := 7.05 \quad \text{dBi}$$

Pretty good agreement (to within 0.5 dB) between all three values.

Estimate the HPBWs for this microstrip patch antenna

E-plane (*x*-*y* plane)

$$\Theta E := 2 \cdot acos\left[\sqrt{\frac{7.03 \cdot \lambda 0^2}{4 \cdot \pi^2 \cdot \left(3 \cdot Leff^2 + h^2\right)}}\right] \qquad \Theta Edeg := \Theta E \cdot \frac{180}{\pi} \qquad \Theta Edeg = 93.654 \quad deg$$

H-plane (*x*-*z* plane)

$$\Theta H := 2 \cdot acos\left(\sqrt{\frac{1}{2 + k0 \cdot W}}\right) \qquad \Theta Hdeg := \Theta H \cdot \frac{180}{\pi} \qquad \Theta Hdeg = 123.637 \quad deg$$

Very wide beam widths!

Find the normalized E-plane & H-plane radiation patterns

$$E(\theta, \phi) := \sin(\theta) \cdot \frac{\sin\left(\frac{k0 \cdot h}{2} \cdot \sin(\theta) \cdot \cos(\phi)\right)}{\frac{k0 \cdot h}{2} \cdot \sin(\theta) \cdot \cos(\phi)} \cdot \frac{\sin\left(\frac{k0 \cdot W}{2} \cdot \cos(\theta)\right)}{\frac{k0 \cdot W}{2} \cdot \cos(\theta)} \cdot \cos\left(\frac{k0 \cdot Leff}{2} \cdot \sin(\theta) \cdot \sin(\phi)\right)$$

$$n := 0 .. 180 \qquad phi_n := (n - 90) \cdot \frac{\pi}{180} \qquad theta_n := n \cdot \frac{\pi}{180} \qquad half\_pow_n := \frac{\sqrt{2}}{2}$$

$$Eepln_n := E\left(\frac{\pi}{2}, phi_n\right) \qquad EeplndB_n := 20 \cdot \log\left(\left|Eepln_n\right|\right) + 40 \qquad half\_dB_n := 37$$

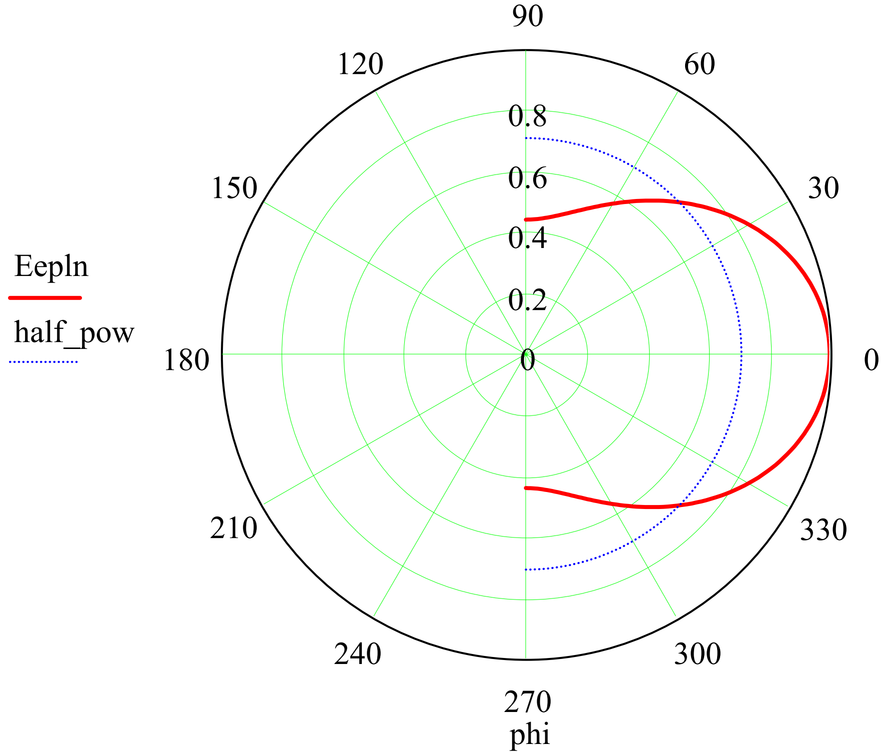
$$Eepln_{45} = 0.703 \qquad Eepln_{135} = 0.703 \qquad HPBWepln := 135 - 45 \qquad HPBWepln = 90 \quad deg$$

$$Ehpln_n := E\left(theta_n, 0\right) \qquad EhplndB_n := if\left(Ehpln_n < 0.01, 0, 20\log\left(\left|Ehpln_n\right|\right) + 40\right)$$

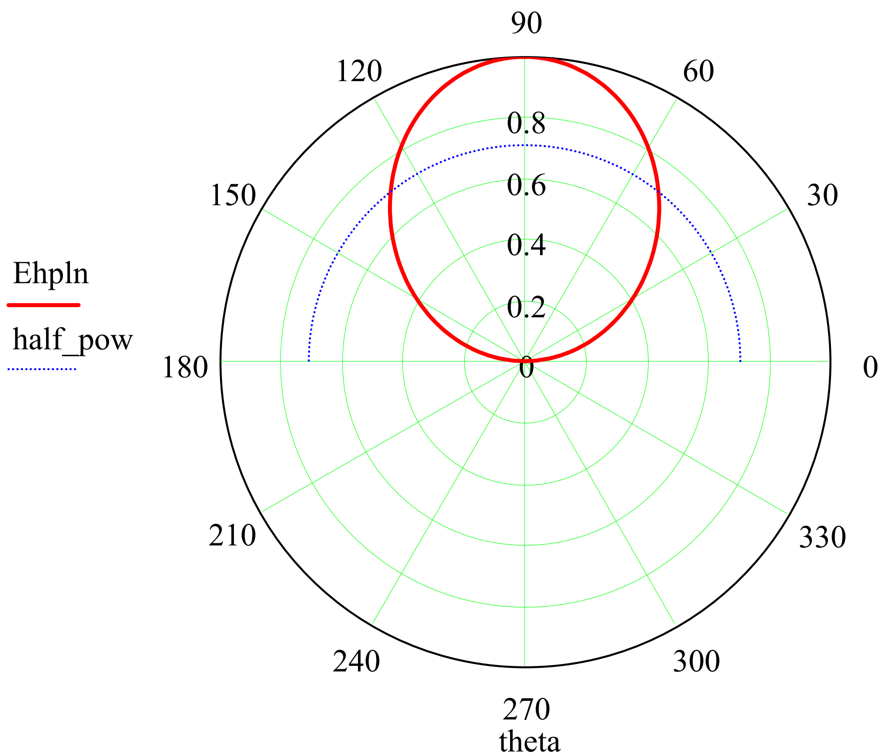
$$Ehpln_{52} = 0.712 \qquad Ehpln_{128} = 0.712 \qquad HPBWhpln := 128 - 52 \qquad HPBWhpln = 76 \quad deg$$

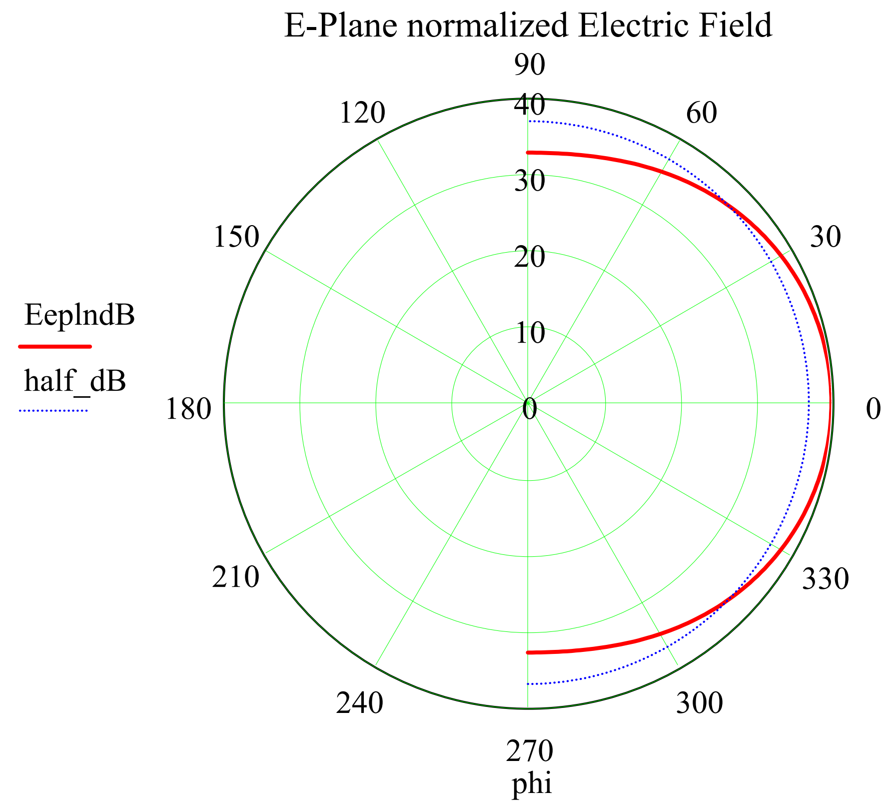
The E-Plane HPBW agrees well with that estimated. However, the H-Plane HPBW is not very close to that estimated.

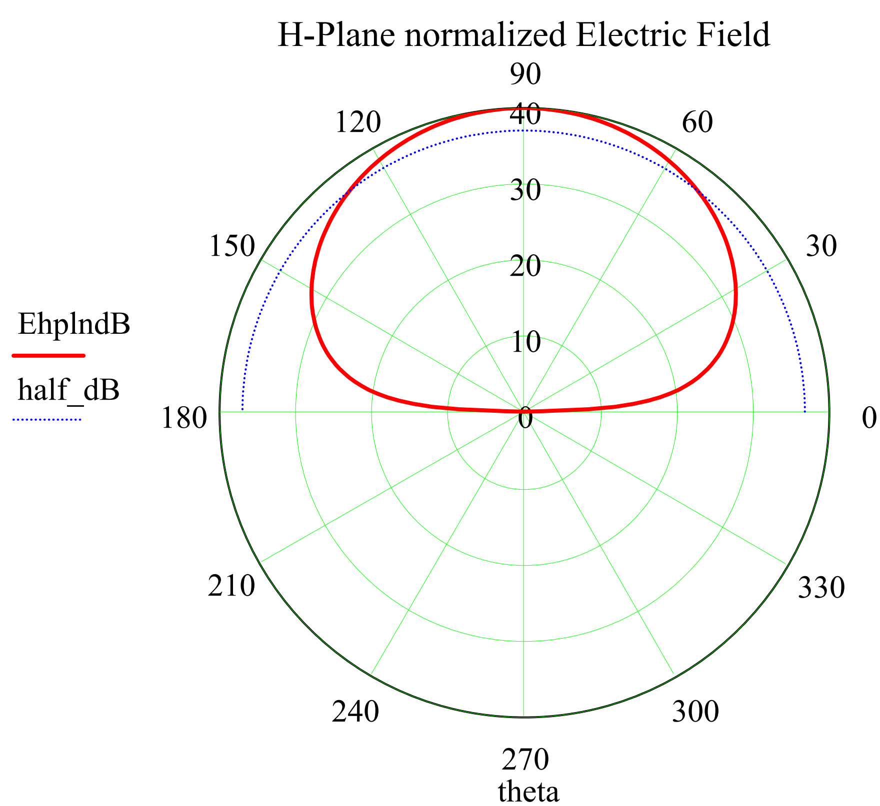
E-Plane normalized Electric Field



H-Plane normalized Electric Field




E-Plane normalized Electric Field
90
60
30
0
330
300
270
240
210
180
150
120
40
30
20
10
0
phi
EeplndB
half_dB


H-Plane normalized Electric Field
90
60
30
0
330
300
270
240
210
180
150
120
40
30
20
10
0
theta
EhplndB
half_dB