ex. Use a modified Gamma-Match to drive the 5 element, channel 43 Yagi-Uda antenna (w/out boom), that was previously designed, with a 50Ω coaxial transmission line. Matching specification: VSWR ≤ 1.1.

To save time, I'll use the final Gamma-Match design as a guide to start the design process.

1st Try

element diameters = 1/4" = 0.635cm = 2a
of Yagi-Uda ⇒ a = 0.3175cm

element spacing = S_{ij} = 0.2λ = 9.274cm

Gamma-Match diameter = 1/8" = 0.3175cm = 2a'
⇒ a' = 0.15875cm

Gamma-Match spacing = S = 2cm

Gamma-Match length = ℓ'/2 = 2" = 5.08cm

Reflector $\ell_1' = 22.7$cm

Driven $\ell_2' = 20.8$cm

Directors $\ell_3' = 19.1$cm
$\ell_4' = 19.09$cm
$\ell_5' = 19.41$cm

1st Try cont.

Using these values (see attached MathCad and NEC files), we get

$Z_o = 260.331\,\Omega$
$\alpha = 1.3675$
$a_e = 0.666\,cm$
$Z_t = j\,214.36\,\Omega$

} MathCad

$Z_a = 18.2244 + j\,1.28708\,\Omega$
$Gain = 11.5\,dBi$

} NEC (Inductive reactance, not good)

$|\Gamma| = 0.152$

$VSWR = 1.358 > 1.1$

$\left(\frac{l'}{2}\right)_{Suggested} = -0.0905\,m$ ← Not realizable

M63

1st Try

Modified Gamma-Match equations

$c := 2.9979 \cdot 10^{8}$ m/s $\quad fc := 647 \cdot 10^{6}$ Hz $\quad \lambda := \frac{c}{fc}$ $\quad \lambda = 0.46335$ m

$k := \frac{2 \cdot \pi}{\lambda}$ $\quad k = 13.56023$ rad/m $\quad Zdesired := 50$ Ω

$d := 0.635 \cdot 10^{-2}$ m $\quad a := d \cdot 0.5$ $\quad a = 0.003175$ m $\quad s := 2. \cdot 10^{-2}$ m

$dprime := 0.3175 \cdot 10^{-2}$ m $\quad aprime := dprime \cdot 0.5$ $\quad aprime = 0.0015875$ m

$$Z0 := \frac{376.734}{2 \cdot \pi} \cdot \operatorname{acosh}\left[\frac{\left(s^2 - a^2 - aprime^2\right)}{2 \cdot a \cdot aprime}\right] \qquad u := \frac{a}{aprime} \qquad v := \frac{s}{aprime}$$

$Z0 = 260.331$ Ω $\quad u = 2$ $\quad v = 12.5984$

$$\alpha := \frac{\operatorname{acosh}\left[\frac{\left(v^2 - u^2 + 1\right)}{2 \cdot v}\right]}{\operatorname{acosh}\left[\frac{\left(v^2 + u^2 - 1\right)}{2 \cdot v \cdot u}\right]} \qquad \alpha = 1.36748 \qquad ae := aprime \cdot e^{\frac{1}{(1+u)^2} \cdot \left(u^2 \cdot \ln(u) + 2 \cdot u \cdot \ln(v)\right)}$$

$ae = 0.00666$ m $\quad ae \cdot 100 = 0.66609$ cm

$lprime := 10.16 \cdot 10^{-2}$ m $\quad lprime \cdot 0.5 = 0.0508$ m

$$Zt := j \cdot Z0 \cdot \tan\left(\frac{k \cdot lprime}{2}\right) \qquad Zt = 214.36228i \ \Omega \qquad Yt := \frac{1}{Zt} \qquad Yt = -4.665i \times 10^{-3} \ S$$

Za from NEC (a MoM program)

$Za := 18.2244 + j \cdot 1.28708$ Ω $\quad Ya := \frac{1}{Za}$ $\quad Ya = 0.0546 - 0.00386i$ S

$$Yin := Yt + \frac{Ya \cdot 2}{(1+\alpha)^2} \qquad Yin = 0.019 - 6.041i \times 10^{-3} \ S \qquad \frac{1}{Zdesired} = 0.02 \ S$$

$Zin := \frac{1}{Yin}$ $\quad Zin = 46.826 + 14.519i$ Ω $\quad Zdesired = 50$ Ω

$$\Gamma := \frac{(Zin - Zdesired)}{Zin + Zdesired} \qquad \Gamma = -0.01 + 0.151i \qquad |\Gamma| = 0.152$$

$VSWR := \frac{(1 + |\Gamma|)}{1 - |\Gamma|}$ $\quad VSWR = 1.358$ $\quad lprime \cdot 0.5 = 0.0508$ m

$$lsuggested := \frac{2}{k} \cdot \operatorname{atan}\left[\frac{1}{Z0 \cdot \operatorname{Im}\left[\frac{2Ya}{(1+\alpha)^2}\right]}\right] \qquad lsuggested \cdot 0.5 = -0.0905 \ m$$

NOT realizable

MG4

```
CM Yagi-Uda Antenna for UHF channel 43
CM THIS PROGRAM ASSUMES THAT THERE IS NO BOOM.
CM
CM THIS FILE IS USED TO DETERMINE THE INPUT IMPEDANCE OF THE DRIVEN
CM ELEMENT OF A 5 ELEMENT ANTENNA. CENTER FREQUENCY IS 647 MHz
CM W/ WAVELENGTH OF 0.46335 m.
CM
CM THE DIMENSIONS ARE:
CM element diameter d=0.635cm=0.25in, radius a=d/2=0.3175cm=0.125in,
CM equivalent radius of Gamma-Match portion of driven element
CM is ae=0.00666 m which has a length of l'/2=0.0508 m  < l2/2
CM
CM l1=0.49 l=0.227 m, l3=0.4122 l=0.191 m, l4=0.412 l=0.1909m,
CM l5=0.419 l=0.1941 m, driven element l2=0.4489 l= 0.208m
CM ELEMENT SPACINGS Sij=0.2 l=0.09267m
CM SELECT SEGMENT LENGTH OF APPROX. 1.25cm=0.025 l
CE THE DRIVEN SEGMENT IS #1 on GW3 on l2.
GW 1 17 -0.1135  0.0 0.0      0.1135  0.0 0.0      0.003175
GW 2 8  -0.104   0.0 0.09267 0.0      0.0 0.09267 0.003175
GW 3 4   0.0     0.0 0.09267 0.0508   0.0 0.09267 0.00666
GW 4 4   0.0508  0.0 0.09267 0.104    0.0 0.09267 0.003175
GW 5 15 -0.0955  0.0 0.18534 0.0955   0.0 0.18534 0.003175
GW 6 15 -0.09545 0.0 0.27801 0.09545  0.0 0.27801 0.003175
GW 7 15 -0.09705 0.0 0.37068 0.09705  0.0 0.37068 0.003175
GE 0 0
FR 0 1 0 0 647 0
EX 0 3 1 0 1.0 0.0
RP 0 2 3 0000 0.0 0.0 180.0 90.0
PT -1
XQ 0
EN
```

M65

Yagi-Uda Antenna for UHF channel 43 THIS PROGRAM ASSUMES THAT THERE IS NO BOOM.

THIS FILE IS USED TO DETERMINE THE INPUT IMPEDANCE OF THE DRIVEN ELEMENT OF A 5 ELEMENT ANTENNA. CENTER FREQUENCY IS 647 MHz W/ WAVELENGTH OF 0.46335 m.

THE DIMENSIONS ARE:
element diameter d=0.635cm=0.25in, radius a=d/2=0.3175cm=0.125in,
equivalent radius of Gamma-Match portion of driven element
is ae=0.00666 m which has a length of l'/2=0.0508 m < l2

l1=0.49 l=0.227 m, l3=0.4122 l=0.191 m, l4=0.412 l=0.1909m,
l5=0.419 l=0.1941 m, driven element l2=0.4489 l= 0.208m
ELEMENT SPACINGS Sij=0.2 l=0.09267m
SELECT SEGMENT LENGTH OF APPROX. 1.25cm=0.025 l
THE DRIVEN SEGMENT IS #1 on GW3 on l2.

****

- - - STRUCTURE SPECIFICATION - - -

COORDINATES MUST BE INPUT IN METERS OR BE SCALED TO METERS
BEFORE STRUCTURE INPUT IS ENDED

WIRE NO.	X1	Y1	Z1	X2	Y2	Z2	RADIUS	NO. OF SEG.	FIRST SEG.	LAST SEG.	TAG NO.
1	-0.11350	0.0	0.0	0.11350	0.0	0.0	0.00317	17	1	17	1
2	-0.10400	0.0	0.09267	0.0	0.0	0.09267	0.00317	8	18	25	2
3	0.0	0.0	0.09267	0.05080	0.0	0.09267	0.00666	4	26	29	3
4	0.05080	0.0	0.09267	0.10400	0.0	0.09267	0.00317	4	30	33	4
5	-0.09550	0.0	0.18534	0.09550	0.0	0.18534	0.00317	15	34	48	5
6	-0.09545	0.0	0.27801	0.09545	0.0	0.27801	0.00317	15	49	63	6
7	-0.09705	0.0	0.37068	0.09705	0.0	0.37068	0.00317	15	64	78	7

TOTAL SEGMENTS USED= 78 NO. SEG. IN A SYMMETRIC CELL= 78 SYMMETRY FLAG= 0

***** INPUT LINE	1	FR	0	1	0	0	6.47000E+02	0.0	0.0	0.0	0.0	0.0
***** INPUT LINE	2	EX	0	3	1	0	1.00000E+00	0.0	0.0	0.0	0.0	0.0
***** INPUT LINE	3	RP	0	2	3	0	0.0	0.0	1.80E+02	9.00E+01	0.0	0.0

FREQUENCY= 6.4700E+02 MHZ WAVELENGTH= 4.6337E-01 METERS

- - - ANTENNA INPUT PARAMETERS - - -

TAG NO.	SEG. NO.	VOLTAGE REAL	VOLTAGE IMAG.	IMPEDANCE (OHMS) REAL	IMPEDANCE (OHMS) IMAG.	ADMITTANCE (MHOS) REAL	ADMITTANCE (MHOS) IMAG.
3	26	1.0	0.0	1.82244E+01	1.28708E+00	5.45990E-02	-3.85598E-03

$Z_{in} = 18.2244 + j\,1.28708\ \Omega$

- - - RADIATION PATTERNS - - -

THETA DEGREES	PHI DEGREES	MAJOR DB
0.00	0.00	11.50571
180.00	0.00	-0.71554

***** INPUT LINE	4	PT	-1	0	0	0	0.0	0.0	0.0	0.0	0.0	0.0
***** INPUT LINE	5	XQ	0	0	0	0	0.0	0.0	0.0	0.0	0.0	0.0
***** INPUT LINE	6	EN	0	0	0	0	0.0	0.0	0.0	0.0	0.0	0.0

2nd Try ← driven element

Shorten l_2' to eliminate the positive reactance of Z_a.

letting $l_2' = 20.56\,cm$ (was 20.8cm)

we get (see attached MathCad and NEC sheets):

$Z_a = 17.378 - j\,4.09449\,\Omega$

$Z_{in} = 51.404 + j\,0.216\,\Omega$

$VSWR = 1.028 < 1.1$ (DONE!)

$Gain = 11.49\,dBi$ (OK)

Final Design Summary (see attached figure)

$Z_a = 1/4" = 0.635\,cm$

$Z_a' = 1/8" = 0.3175\,cm$

$S = 2\,cm$

$l_1'/2 = 5.08\,cm$

$l_1' = 22.7\,cm$

$l_2' = 20.56\,cm$

$l_3' = 19.1\,cm$

$l_4' = 19.09\,cm$

$l_5' = 19.41\,cm$

$Z_{in} = 51.404 + j\,0.216\,\Omega$

$Gain = 11.49\,dBi$

M67

2nd Try

Modified Gamma-Match equations

$c := 2.9979 \cdot 10^{8}$ m/s $\quad fc := 647 \cdot 10^{6}$ Hz $\quad \lambda := \frac{c}{fc}$ $\quad \lambda = 0.46335$ m

$k := \frac{2 \cdot \pi}{\lambda}$ $\quad k = 13.56023$ rad/m $\quad Zdesired := 50$ Ω

$d := 0.635 \cdot 10^{-2}$ m $\quad a := d \cdot 0.5$ $\quad a = 0.003175$ m $\quad s := 2. \cdot 10^{-2}$ m

$dprime := 0.3175 \cdot 10^{-2}$ m $\quad aprime := dprime \cdot 0.5$ $\quad aprime = 0.0015875$ m

$$Z0 := \frac{376.734}{2 \cdot \pi} \cdot \operatorname{acosh}\left[\frac{\left(s^2 - a^2 - aprime^2\right)}{2 \cdot a \cdot aprime}\right] \qquad u := \frac{a}{aprime} \qquad v := \frac{s}{aprime}$$

$Z0 = 260.331$ Ω $\quad u = 2$ $\quad v = 12.5984$

$$\alpha := \frac{\operatorname{acosh}\left[\frac{\left(v^2 - u^2 + 1\right)}{2 \cdot v}\right]}{\operatorname{acosh}\left[\frac{\left(v^2 + u^2 - 1\right)}{2 \cdot v \cdot u}\right]} \qquad \alpha = 1.36748 \qquad ae := aprime \cdot e^{\frac{1}{(1+u)^2} \cdot \left(u^2 \cdot \ln(u) + 2 \cdot u \cdot \ln(v)\right)}$$

$ae = 0.00666$ m $\quad ae \cdot 100 = 0.66609$ cm

$lprime := 10.16 \cdot 10^{-2}$ m $\quad lprime \cdot 0.5 = 0.0508$ m

$$Zt := j \cdot Z0 \cdot \tan\left(\frac{k \cdot lprime}{2}\right) \qquad Zt = 214.36228i \ \Omega \qquad Yt := \frac{1}{Zt} \qquad Yt = -4.665i \times 10^{-3} \ S$$

Za from NEC (a MoM program)

$Za := 17.378 + j \cdot -4.09449$ Ω $\quad Ya := \frac{1}{Za}$ $\quad Ya = 0.05452 + 0.01285i$ S

$$Yin := Yt + \frac{Ya \cdot 2}{(1+\alpha)^2} \qquad Yin = 0.019 - 8.154i \times 10^{-5} \ S \qquad \frac{1}{Zdesired} = 0.02 \ S$$

$Zin := \frac{1}{Yin}$ $\quad Zin = 51.404 + 0.215i$ Ω $\quad Zdesired = 50$ Ω

$$\Gamma := \frac{(Zin - Zdesired)}{Zin + Zdesired} \qquad \Gamma = 0.014 + 2.095i \times 10^{-3} \qquad |\Gamma| = 0.014$$

← DONE!

$VSWR := \frac{(1 + |\Gamma|)}{1 - |\Gamma|}$ $\quad VSWR = 1.028$ < 1.1 $\quad lprime \cdot 0.5 = 0.0508$ m

$$lsuggested := \frac{2}{k} \cdot \operatorname{atan}\left[\frac{1}{Z0 \cdot \operatorname{Im}\left[\frac{2Ya}{(1+\alpha)^2}\right]}\right] \qquad lsuggested \cdot 0.5 = 0.0514 \ m$$

↑ could tweak l/2 slightly to do better

M68

```
CM Yagi-Uda Antenna for UHF channel 43
CM THIS PROGRAM ASSUMES THAT THERE IS NO BOOM.
CM
CM THIS FILE IS USED TO DETERMINE THE INPUT IMPEDANCE OF THE DRIVEN
CM ELEMENT OF A 5 ELEMENT ANTENNA. CENTER FREQUENCY IS 647 MHz
CM W/ WAVELENGTH OF 0.46335 m.
CM
CM THE DIMENSIONS ARE:
CM element diameter d=0.635cm=0.25in, radius a=d/2=0.3175cm=0.125in,
CM equivalent radius of Gamma-Match portion of driven element
CM is ae=0.00666 m which has a length of l'/2=0.0508 m  < l2/2
CM
CM l1=0.49 l=0.227 m, l3=0.4122 l=0.191 m, l4=0.412 l=0.1909m,
CM l5=0.419 l=0.1941 m, driven element l2=0.4437 l= 0.2056 m
CM ELEMENT SPACINGS Sij=0.2 l=0.09267m
CM SELECT SEGMENT LENGTH OF APPROX. 1.25cm=0.025 l
CE THE DRIVEN SEGMENT IS #1 on GW3 on l2.
GW 1 17 -0.1135  0.0 0.0     0.1135  0.0 0.0     0.003175
GW 2 8  -0.1028  0.0 0.09267 0.0     0.0 0.09267 0.003175
GW 3 4   0.0     0.0 0.09267 0.0508  0.0 0.09267 0.00666
GW 4 4   0.0508  0.0 0.09267 0.1028  0.0 0.09267 0.003175
GW 5 15 -0.0955  0.0 0.18534 0.0955  0.0 0.18534 0.003175
GW 6 15 -0.09545 0.0 0.27801 0.09545 0.0 0.27801 0.003175
GW 7 15 -0.09705 0.0 0.37068 0.09705 0.0 0.37068 0.003175
GE 0 0
FR 0 1 0 0 647 0
EX 0 3 1 0 1.0 0.0
RP 0 2 3 0000 0.0 0.0 180.0 90.0
PT -1
XQ 0
EN
```

2nd Try

MG9

Yagi-Uda Antenna for UHF channel 43 THIS PROGRAM ASSUMES THAT THERE IS NO BOOM.

THIS FILE IS USED TO DETERMINE THE INPUT IMPEDANCE OF THE DRIVEN ELEMENT OF A 5 ELEMENT ANTENNA. CENTER FREQUENCY IS 647 MHz W/ WAVELENGTH OF 0.46335 m.

THE DIMENSIONS ARE:
element diameter d=0.635cm=0.25in, radius a=d/2=0.3175cm=0.125in, equivalent radius of Gamma-Match portion of driven element is ae=0.00666 m which has a length of l'/2=0.0508 m < 12

l1=0.49 l=0.227 m, l3=0.4122 l=0.191 m, l4=0.412 l=0.1909m, l5=0.419 l=0.1941 m, driven element l2=0.4489 l= 0.208m
ELEMENT SPACINGS Sij=0.2 l=0.09267m
SELECT SEGMENT LENGTH OF APPROX. 1.25cm=0.025 l
THE DRIVEN SEGMENT IS #1 on GW3 on 12.

2nd Try

****

- - - STRUCTURE SPECIFICATION - - -

COORDINATES MUST BE INPUT IN METERS OR BE SCALED TO METERS
BEFORE STRUCTURE INPUT IS ENDED

WIRE NO.	X1	Y1	Z1	X2	Y2	Z2	RADIUS	NO. OF SEG.	FIRST SEG.	LAST SEG.	TAG NO.
1	-0.11350	0.0	0.0	0.11350	0.0	0.0	0.00317	17	1	17	1
2	-0.10280	0.0	0.09267	0.0	0.0	0.09267	0.00317	8	18	25	2
3	0.0	0.0	0.09267	0.05080	0.0	0.09267	0.00666	4	26	29	3
4	0.05080	0.0	0.09267	0.10280	0.0	0.09267	0.00317	4	30	33	4
5	-0.09550	0.0	0.18534	0.09550	0.0	0.18534	0.00317	15	34	48	5
6	-0.09545	0.0	0.27801	0.09545	0.0	0.27801	0.00317	15	49	63	6
7	-0.09705	0.0	0.37068	0.09705	0.0	0.37068	0.00317	15	64	78	7

TOTAL SEGMENTS USED= 78 NO. SEG. IN A SYMMETRIC CELL= 78 SYMMETRY FLAG= 0

***** INPUT LINE 1 FR 0 1 0 0 6.47000E+02 0.0 0.0 0.0 0.0 0.0
***** INPUT LINE 2 EX 0 3 1 0 1.00000E+00 0.0 0.0 0.0 0.0 0.0
***** INPUT LINE 3 RP 0 2 3 0 0.0 0.0 1.800E+02 9.0E+01 0.0 0.0

FREQUENCY= 6.4700E+02 MHZ WAVELENGTH= 4.6337E-01 METERS

- - - ANTENNA INPUT PARAMETERS - - -

TAG NO.	SEG. NO.	VOLTAGE REAL	VOLTAGE IMAG.	IMPEDANCE (OHMS) REAL	IMPEDANCE (OHMS) IMAG.	ADMITTANCE (MHOS) REAL	ADMITTANCE (MHOS) IMAG.
3	26	1.0	0.0	1.73780E+01	-4.09449E+00	5.45177E-02	1.28451E-02

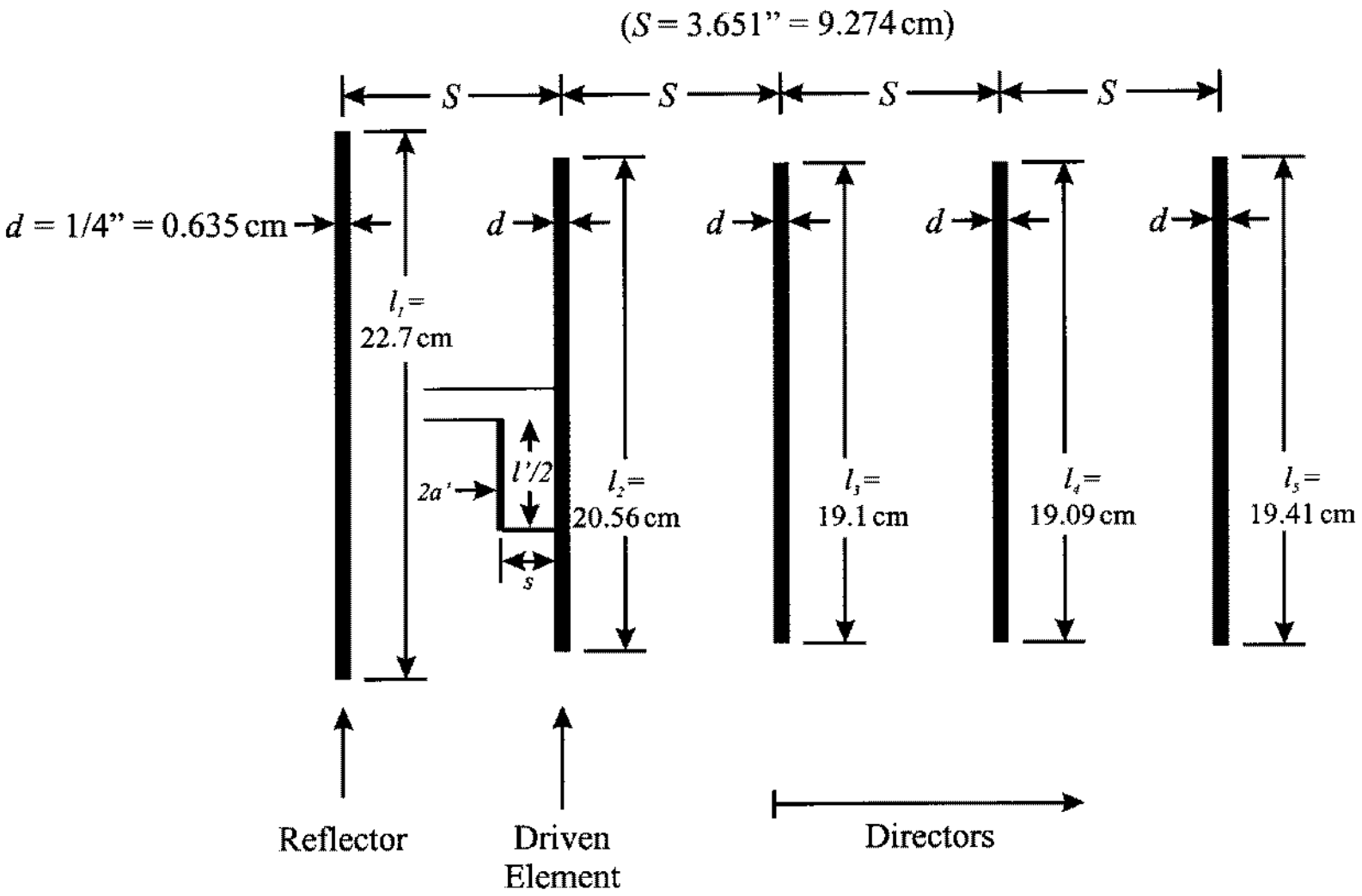
$Z_a = 17.378 - j\,4.09449\ \Omega$

- - - RADIATION PATTERNS - - -

THETA DEGREES	PHI DEGREES	Gain DB
0.00	0.00	11.49238
180.00	0.00	-0.70477

***** INPUT LINE 4 PT -1 0 0 0 0.0 0.0 0.0 0.0 0.0 0.0
***** INPUT LINE 5 XQ 0 0 0 0 0.0 0.0 0.0 0.0 0.0 0.0
***** INPUT LINE 6 EN 0 0 0 0 0.0 0.0 0.0 0.0 0.0 0.0

5 element, channel 43 Yagi-Uda antenna w/ modified Gamma-Match



Modified Gamma-Match Dimensions: $2a'$ = 1/8" = 0.3175cm, s = 2.0 cm, and l'/2 = 5.08 cm.

MG10