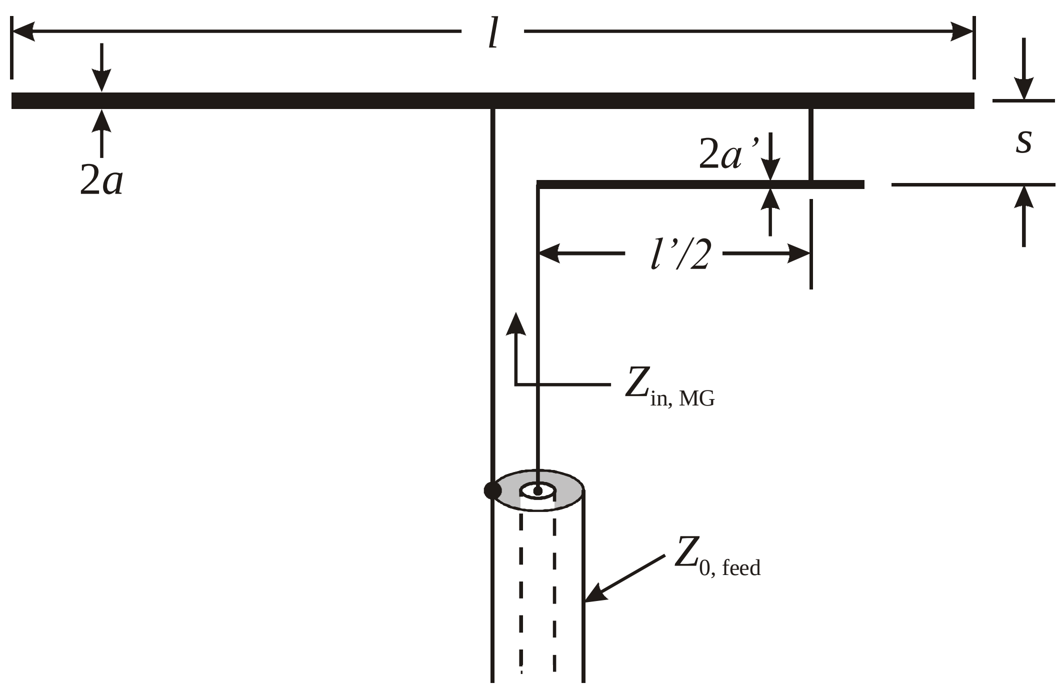
Matching Techniques for Driving Yagi-Uda Antennas: Modified Gamma-Match

(Sections 9.5 & 9.8 of Balanis)

Modified Gamma-Match:



- The modified Gamma-Match also allows the driven element of Yagi-Uda antennas to be fed by coaxial transmission lines (unbalanced). Again, the shield of the coaxial line is connected to the center of the driven element (the axis of symmetry) by "pigtailing" the shield braid. The center conductor is connected to the Gamma-Match.

- Note that it simply omits the series capacitor of the Gamma-Match. This can make matching more difficult, requiring more adjustments of element lengths. However, eliminating the capacitor saves on component costs and simplifies the structure design.

- Therefore, the modified Gamma-Match is simply the T-Match split down the middle. The analysis follows that of the T-Match.

- The input impedance of the modified Gamma-Match $Z_{\text{in, MG}}$ is half that of the T-Match Z_{in}. Conversely, the input admittance of the modified Gamma-Match $Y_{\text{in, MG}}$ is twice that of the T-Match Y_{in}. Or, the input impedance/admittance of the modified Gamma-Match is that of the Gamma-Match minus the reactance/susceptance of the capacitor.

$$\begin{aligned} Y_{\text{in, MG}} &= 2Y_{\text{in}} \\ &= Y_t + \frac{2Y_a}{(1+\alpha)^2} \end{aligned}$$

$$\begin{aligned} Z_{\text{in, MG}} &= \frac{Z_{\text{in}}}{2} \\ &= \frac{(1+\alpha)^2 Z_a Z_t}{(1+\alpha)^2 Z_a + 2Z_t} \end{aligned}$$

where Z_a is the input impedance of the antenna mode of the driven element, α is the current divisor factor, and Z_t is the input impedance of the transmission line mode of the modified Gamma-Match.

The current divisor factor α is calculated using

$$\alpha = \frac{\cosh^{-1}\left(\dfrac{v^2 - u^2 + 1}{2v}\right)}{\cosh^{-1}\left(\dfrac{v^2 + u^2 - 1}{2vu}\right)} \qquad \begin{aligned} &\alpha > 1 \text{ when } a > a' \\ &\alpha = 1 \text{ when } a = a' \\ &\alpha < 1 \text{ when } a < a' \end{aligned}$$

where $u = \dfrac{a}{a'}$ and $v = \dfrac{s}{a'}$. The current divisor factor α has a big impact on the magnitude of $Z_{\text{in, MG}}$ (i.e., when α increases $|Z_{\text{in, MG}}|$ increases and vice versa). The current divisor factor α is inversely related to the spacing s (i.e., if s decreases α increases and vice versa).

To find input impedance of the antenna mode Z_a, insert the driven element into the Yagi-Uda antenna, if applicable, and modify the driven element to have an equivalent radius a_e over the length $l'/2$ corresponding to the modified Gamma-Match. The equivalent radius a_e is calculated using

$$\ln(a_e) \approx \ln(a') + \frac{1}{(1+u)^2}\left(u^2 \ln u + 2u \ln v\right)$$

To find the input impedance Z_t of the transmission line mode, use

$$Z_t = j Z_0 \tan\left(kl'/2\right)$$

where $k = \beta = 2\pi/\lambda$ and Z_0 is the characteristic impedance of the transmission line portion of the modified Gamma-Match. Z_0 is given by

$$Z_0 = \frac{\eta}{2\pi}\cosh^{-1}\left(\frac{s^2 - a^2 - a'^2}{2aa'}\right)$$

where $\eta = \sqrt{\dfrac{\mu}{\varepsilon}}$.

Note: If Z_a has an inductive reactance (i.e., $X_a > 0$), it may not be possible to achieve a realizable match using a modified Gamma-Match. However, if used for the driven element in a Yagi-Uda antenna, a match is usually feasible if the lengths of the reflector l_1, driven l_2, and first director l_3 elements are varied slightly in addition to the length of the modified Gamma-Match $l'/2$.

Design Process:

- We desire to match a given Yagi-Uda antenna to a transmission line characteristic impedance $Z_{0,\text{feed}}$. Usually a specification in terms of the VSWR is given.

1) Select a driven element length l_2 so that $l_1 < l_2 < l_3$, a', s, and l' (usually $l'/2 < l_2/4$). These values may be changed later.
2) Calculate the characteristic impedance Z_0 of the transmission line mode of the modified Gamma-Match.
3) Calculate the transmission line mode input impedance Z_t.
4) Calculate the parameters u, v, and α.
5) Calculate the equivalent radius a_e of the T-Match section.
6) Find input impedance of the antenna mode Z_a.
7) Find overall input impedance $Z_{\text{in, MG}}$.
8) Determine if $Z_{\text{in, MG}}$ meets your specification. If so, stop design process. If not, try changing $l'/2$ to

$$l'/2 = \frac{1}{k}\tan^{-1}\left[\frac{1}{Z_0 \operatorname{Im}\left(\frac{2Y_a}{(1+\alpha)^2}\right)}\right]$$

to better offset the antenna mode reactance, and repeat steps 2) - 8). If necessary, l_2, s, l_1, l_3, and a' can be varied (try in that order). Remember, the magnitude of the input impedance is greatly affected by α (i.e., when α increases $|Z_{\text{in, MG}}|$ increases and vice versa). Also, α is inversely related to s (i.e., if s decreases α increases and vice versa).