Chapter 5 Loop Antennas

→ another simple, cheap antenna that is the basis for many other antennas

→ usually circular loops, but square, rectangle, elliptical ... shapes work

→ usually broken into two categories < electrically small / electrically large

where small means $\underset{\uparrow \atop \#\text{turns}}{N}(\underset{\uparrow \atop \text{circumference}}{C}) < \lambda/10$

and large means $C \sim \lambda$

→ usually used up to UHF band (~3 GHz), but can go higher (tough to make)

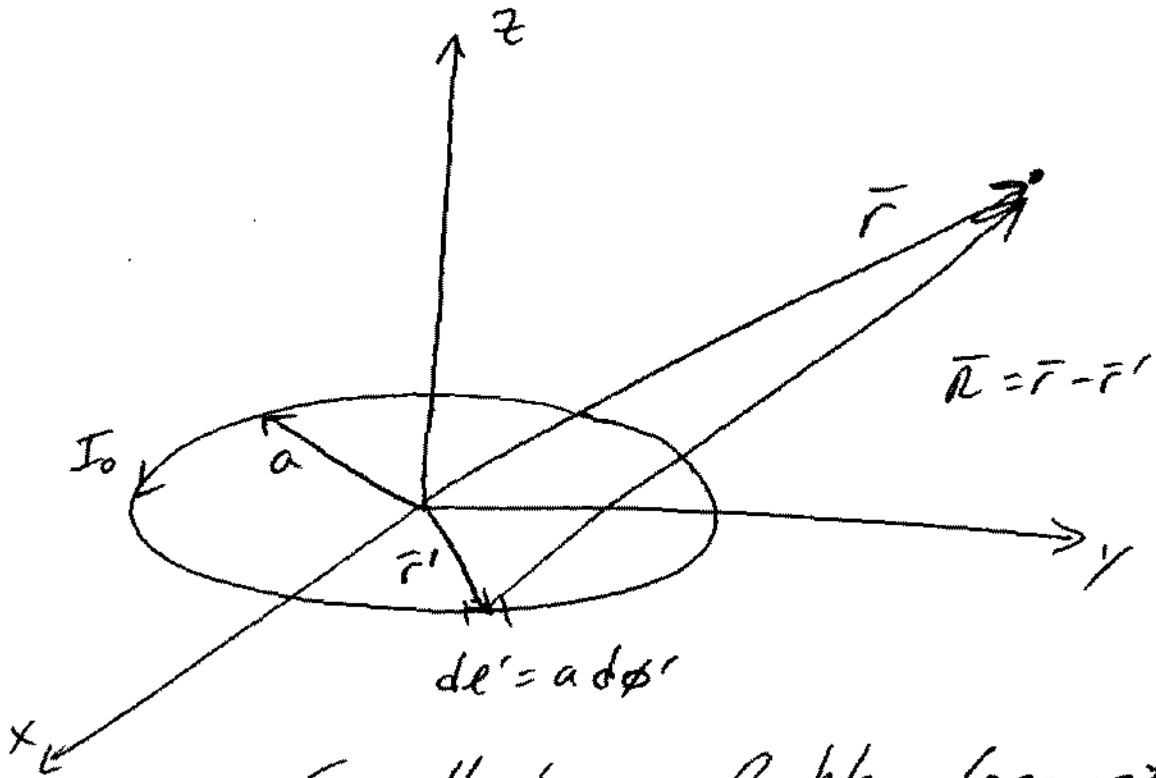
→ Small loops, like small dipoles, have low radiation resistances (can be be smaller than loss resistance). So, they are not used much for transmitting, but do find applications in receive mode. A ferrite core helps increase R_r (boosts $\bar{B} = \mu \bar{H}$) (AM radio a key application)

5.2 Small Circular loop

$N(C) < \lambda/10$

If $N=1$, $C < \lambda/10$ or $a < \frac{\lambda}{20\pi}$

→ assume $\bar{I} = \hat{a}_\phi I_o$ ← constant current



Small Loop Problem Geometry

Again

$$\bar{A} = \frac{\mu}{4\pi} \int_C \bar{I}_e \frac{e^{-jkR}}{R}\, dl'$$

⋮ after much math & approximations

$$\bar{A} \simeq \hat{a}_\phi\, j \frac{k\mu a^2 I_o \sin\theta}{4r} \left[1 + \frac{1}{jkr}\right] e^{-jkr}$$

5.2 cont.

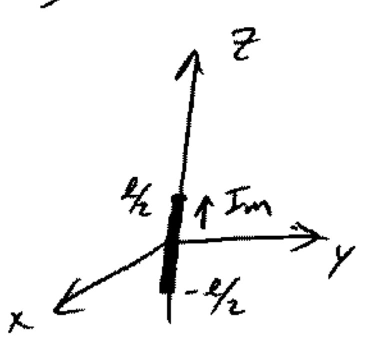
Applying $\bar{H} = \bar{H}_A = \frac{1}{\mu} \bar{\nabla} \times \bar{A}$, yields:

$$\boxed{\begin{aligned} H_r &= j \frac{Ka^2 I \cos\theta}{2r^2}\left[1 + \frac{1}{jKr}\right] e^{-jKr} \\ H_\theta &= -\frac{(Ka)^2 I_0 \sin\theta}{4r}\left[1 + \frac{1}{jKr} - \frac{1}{(Kr)^2}\right] e^{-jKr} \\ H_\phi &= 0 \end{aligned}}$$

using $\bar{\nabla} \times \bar{H} = \cancel{\bar{J}}^{\,0} + j\omega\epsilon \bar{E}$, yields:

$$\boxed{\begin{aligned} E_r &= E_\theta = 0 \\ E_\phi &= \eta \frac{(Ka)^2 I_0 \sin\theta}{4r}\left[1 + \frac{1}{jKr}\right] e^{-jKr} \end{aligned}}$$

These fields very similar to what would obtained for a small magnetic dipole w/ constant magnetic current I_m on a length ℓ along z-axis



5.2 cont.

small magnetic dipole

$$\boxed{\begin{aligned} H_r &= \frac{I_m l \cos\theta}{2\pi\eta r^2}\left[1 + \frac{1}{jkr}\right] e^{-jkr} \\ H_\theta &= j\,\frac{k I_m l \sin\theta}{4\pi\eta r}\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right] e^{-jkr} \\ E_\phi &= -j\,\frac{k I_m l \sin\theta}{4\pi r}\left[1 + \frac{1}{jkr}\right] e^{-jkr} \\ E_r &= E_\theta = H_\phi = 0 \end{aligned}}$$

In fact, a small loop & small magnetic dipole are equivalent provided

$$\boxed{I_m l = j\, S \omega \mu I_o}$$

area of loop

$$\overline{W_{ave}} = \tfrac{1}{2}\,\mathrm{Re}\left\{\bar{E} \times \bar{H}^*\right\}$$

$$= \tfrac{1}{2}\,\mathrm{Re}\left\{-\hat{a}_r E_\phi H_\theta^* + \hat{a}_\theta E_\phi H_r^*\right\}$$

not radiating (falls off as $\frac{1}{r^3}$ or more)

$$\overline{W_{ave}} = \hat{a}_r\,\frac{(ka)^4 |I_o|^2 \sin^2\theta}{32\, r^2}$$

$$P_{rad} = \oiint_{sphere} \overline{W}_{ave} \cdot d\overline{S}_r$$

$$\underline{P_{rad} = \eta \left(\pi/12\right) (ka)^4 |I_0|^2} \leftarrow ka = \frac{2\pi a}{\lambda} = C/\lambda$$

letting $\eta\left(\pi/12\right)(ka)^4 |I_0|^2 = 1/2\, |I_0|^2 R_r$

yields

$$\underline{\underline{R_r = \eta\left(\pi/6\right)(ka)^4 = \eta \frac{2\pi}{3}\Big(\frac{\underbrace{kS}_{\text{area of loop}}}{\lambda}\Big)^2 = \eta\left(\pi/6\right)\left(C/\lambda\right)^4}}$$

What if you have multiple turns of wire?

$$\boxed{R_{r,N} = N^2 R_r}$$

$\leftarrow$ can greatly increase R_r (R_{Loss} goes up however)

ex. Choose $a = \lambda/50$

$$R_r = \eta\left(\pi/6\right)(ka)^4 = 376.73\left(\pi/6\right)\left(\frac{2\pi}{\lambda}\,\frac{\lambda}{50}\right)^4$$

$$R_r = 0.049189\,\Omega$$

$N = 30$

$$R_{r,30} = (0.049189)\,30^2 = \underline{44.27\,\Omega}$$

Far-field Region ($kr \gg 1$)

→ only keep field components proportional to $1/r$

$$\boxed{\begin{aligned} H_\theta &\simeq -\frac{(ka)^2 I_0 \sin\theta}{4r} e^{-jkr} = -\frac{\pi S I_0 \sin\theta}{\lambda^2 r} e^{-jkr} \\ E_\phi &\simeq \eta \frac{(ka)^2 I_0 \sin\theta}{4r} e^{-jkr} = \eta \frac{\pi S I_0 \sin\theta}{\lambda^2 r} e^{-jkr} \\ H_r &\simeq H_\phi = E_r = E_\theta = 0 \end{aligned}}$$

Again $Z_{wave} = \frac{-E_\phi}{H_\theta} = \eta$

orthogonal to each other + direction of propagation

Radiation Intensity

$$\boxed{\begin{aligned} U &= r^2 W_r = \frac{\eta}{2}\left(\frac{k^2a^2}{4}\right)^2 |I_0|^2 \sin^2\theta = \eta\frac{(ka)^4}{32}|I_0|^2\sin^2\theta \\ U &= \frac{r^2}{2\eta}|E_\phi|^2 \\ U_{max}\Big|_{\theta=\pi/2} &= \eta\frac{(ka)^4}{32}|I_0|^2 \end{aligned}}$$

$$\boxed{D_0 = \frac{4\pi U_{max}}{P_{rad}} = 3/2 = 1.7609\,dB_i \qquad A_{em} = \left(\frac{\lambda^2}{4\pi}\right) D_0 = \frac{3\lambda^2}{8\pi}}$$

→ all very similar to infinitesimal dipole

Ohmic Losses

→ Single turn ($N=1$) $R_L = R_{hf} = \underbrace{\frac{\overbrace{l}^{2\pi a}}{p}}_{p\,=\,2\pi b} \sqrt{\frac{\omega \mu_0}{2\sigma}} = \frac{a}{b}\sqrt{\frac{\omega\mu_0}{2\sigma}}$

→ multiple turns

* Current not uniformly distributed

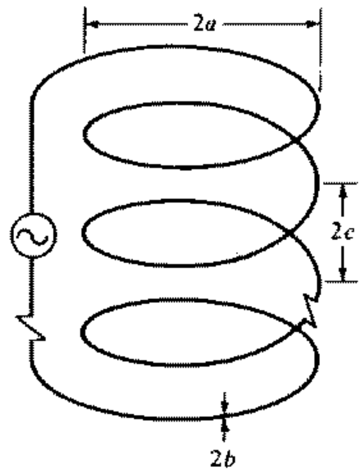
$$R_L = R_{ohmic} = \frac{Na}{b}\underbrace{\sqrt{\frac{\omega \mu_0}{2\sigma}}}_{R_S}\left(\frac{R_p}{R_0} + 1\right)$$

R_S Surface impedance of conductor

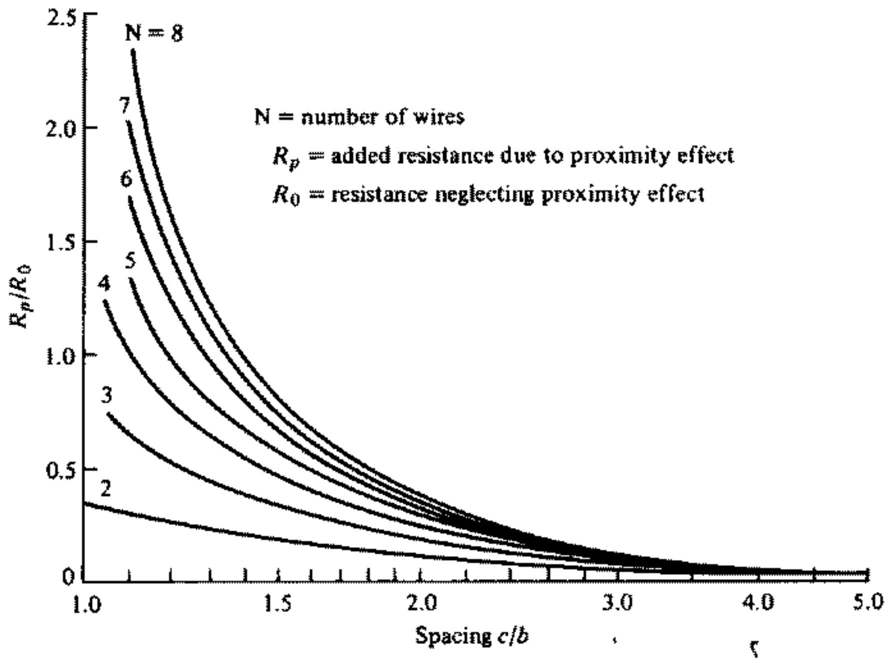
$R_p \equiv$ ohmic resistance per unit length due to proximity effect

$R_0 \equiv$ ohmic skin effect resistance per unit length

→ see graph for values of $\frac{R_p}{R_0}$ vs loop spacing



(a) N-turn circular loop



(b) Ohmic resistance due to proximity (after G. N. Smith)

Figure 5.2 N-turn circular loop and ohmic resistance due to proximity effect. (SOURCE: G. S. Smith, "Radiation Efficiency of Electrically Small Multiturn Loop Antennas," *IEEE Trans. Antennas Propagat.*, Vol. AP-20, No. 5, pp. 656-657, Sept. 1972© (1972) IEEE).

5.3 Circular Loop of Constant Current

→ constant current approx. good up to

$$C \lesssim \lambda/5 \quad \text{or} \quad a \lesssim \frac{\lambda}{10\pi} = \lambda/31.4$$

(analogous to short dipole w/ triangular current distribution)

Here, we assume $R \simeq r - a\sin\theta\cos\phi'$ for phase (i.e., e^{-jkR})

& $R \simeq r$ for amplitude (i.e., $1/R$) for the far-field.

Then,

$$\bar{A} \simeq \hat{a}_\phi \, j \frac{a\mu I_0 e^{-jkr}}{2r} J_1(ka\sin\theta)$$

Bessel function of first kind, order 1

and

$$\boxed{\begin{aligned} &E_r \simeq E_\theta = 0 \\ &E_\phi \simeq \frac{ak\eta I_0 e^{-jkr}}{2r} J_1(ka\sin\theta) \\ &H_r \simeq H_\phi = 0 \\ &H_\theta = -\frac{akI_0 e^{-jkr}}{2r} J_1(ka\sin\theta) \end{aligned}}$$

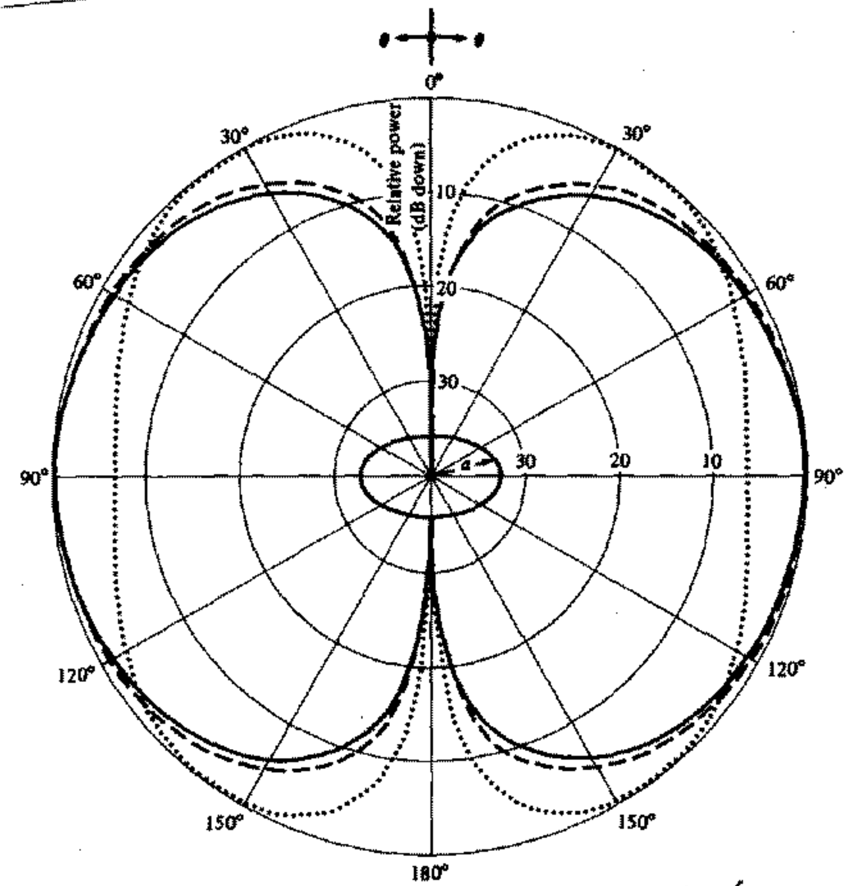
5.3 cont.

$$\overline{W}_{ave} = \hat{a}_r \frac{1}{2\eta} |E_\phi|^2 = \hat{a}_r \frac{(ka)^2 \eta^2 |I_0|^2}{\eta \, 8 r^2} J_1^2(ka \sin\theta)$$

$$\overline{W}_{ave} = \hat{a}_r \frac{(a\omega\mu)^2 |I_0|^2}{8 \eta r^2} J_1^2(ka \sin\theta)$$

$$U = r^2 W_r = \frac{(ka)^2 \eta |I_0|^2}{8} J_1^2(ka \sin\theta)$$

$$= \frac{(a\omega\mu)^2 |I_0|^2}{8\eta} J_1^2(ka \sin\theta)$$



——— $a = 0.1\lambda$ $ka = C/\lambda = 0.63$ ← borderline

– – – $a = 0.2\lambda$ $ka = C/\lambda = 1.26$ } small loop constant I assumption not met

······ $a = 0.5\lambda$ $ka = C/\lambda = 3.1$

Figure 5.6 Elevation plane amplitude patterns for a circular loop of constant current ($a = 0.1\lambda$, 0.2λ, and 0.5λ).

[Balanis, Ant. Theory (2nd edn), p. 220]

5.4 Circular Loop w/ Nonuniform Current

What about loops where $C > \lambda/5$?
$(a > 0.03\lambda)$

→ when $C = 2\pi a \simeq \lambda$, or $(ka = 1)$, the loop radiates **on axis** $\begin{pmatrix}\theta = 0^\circ \\ \text{or } \theta = \pi\end{pmatrix}$ instead of broadside

↳ useful for Yagi-Uda array, basis of helical antenna, ...

* Here $a = \lambda/2\pi \simeq 0.159\lambda$ and the current is no longer constant

↳ Sometimes the current is approximated as

$$I(\phi') = I_0 + 2\sum_{n=1}^{M} I_n \cos(n\phi')$$

(a Fourier series)

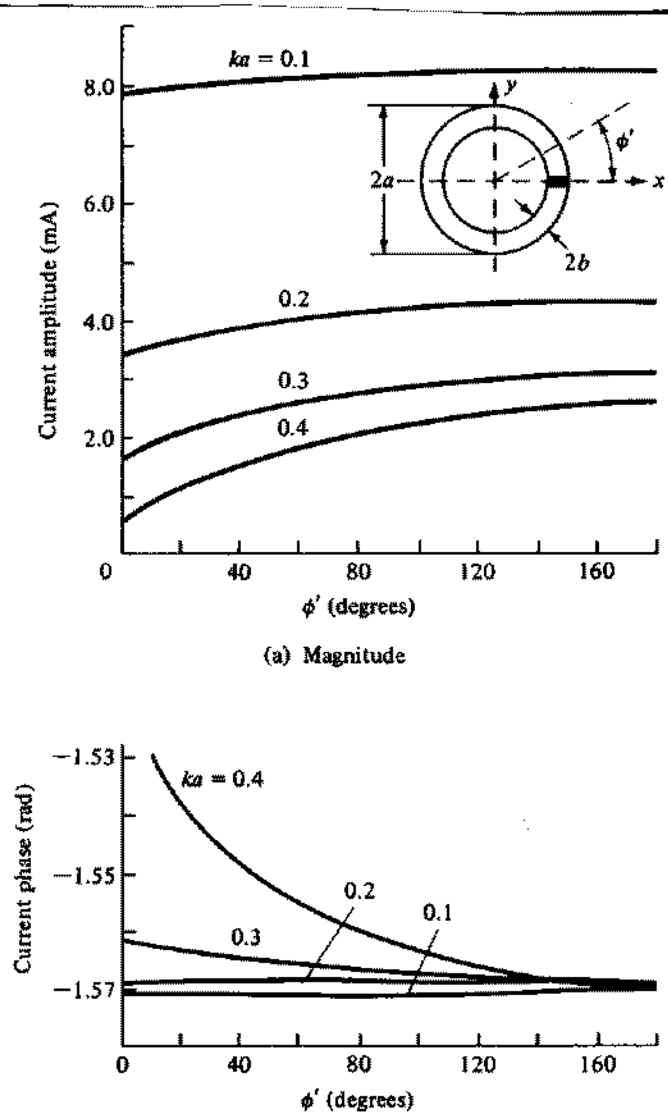
→ easier/more accurate to due numerical analysis

→ Note: Many texts/articles quantify the relative thickness of the wire as

$$\Omega = 2\ln\left(\frac{2\pi a}{b}\right)$$

circumference

wire radius



(a) Magnitude

(b) Phase

Figure 5.9 Current magnitude and phase distributions on small circular loop antennas. (SOURCE: J. E. Storer, "Impedance of Thin-Wire Loop Antennas," *AIEE Trans.*, Vol. 75, November 1956. © 1956 IEEE)

[Balanis, Ant. Theory (2nd Edn), p 225]

Note: Current progressively becomes non-uniform as ka increases.

5.4 cont

→ Directivity of loop significantly larger than comparable λ/2 dipole (which is 2.15 dBi)

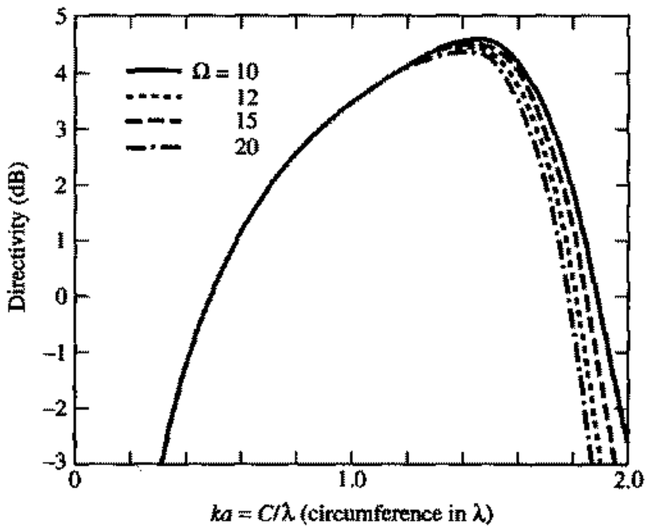


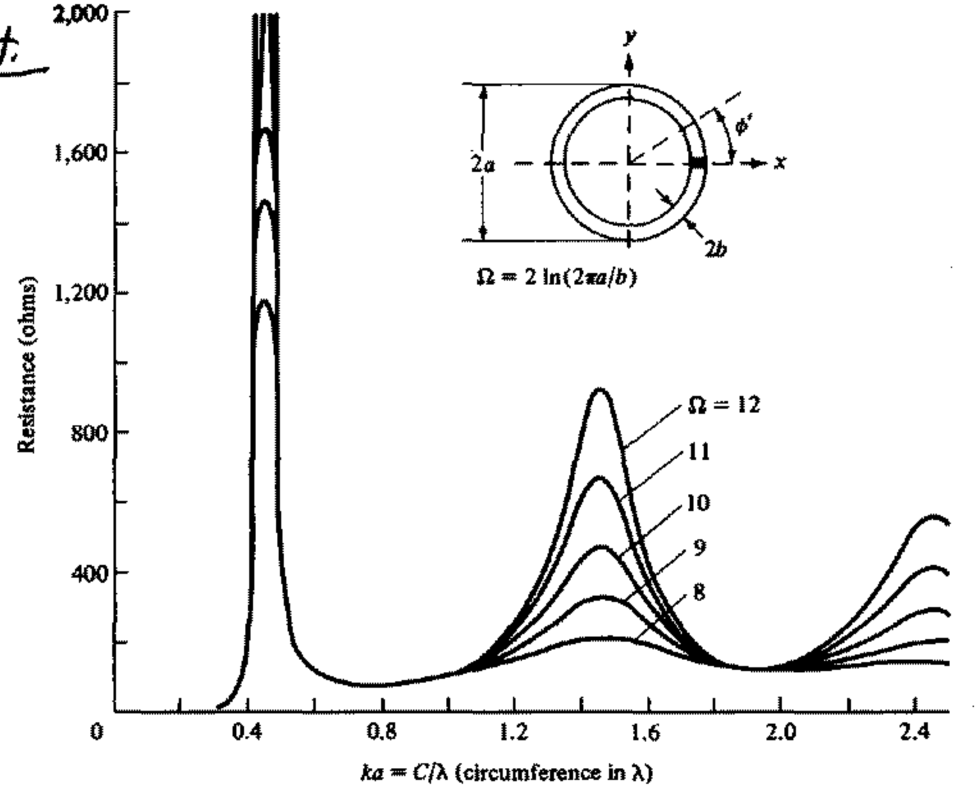
Figure 5.10 Directivity of circular-loop antenna for $\theta = 0, \pi$ versus electrical size (circumference/wavelength). (SOURCE: G. S. Smith, "Loop Antennas," copyright © McGraw-Hill, Inc. Permission by McGraw-Hill, Inc.)

[Balanis, Ant. Theory (2nd Edn), p 226]

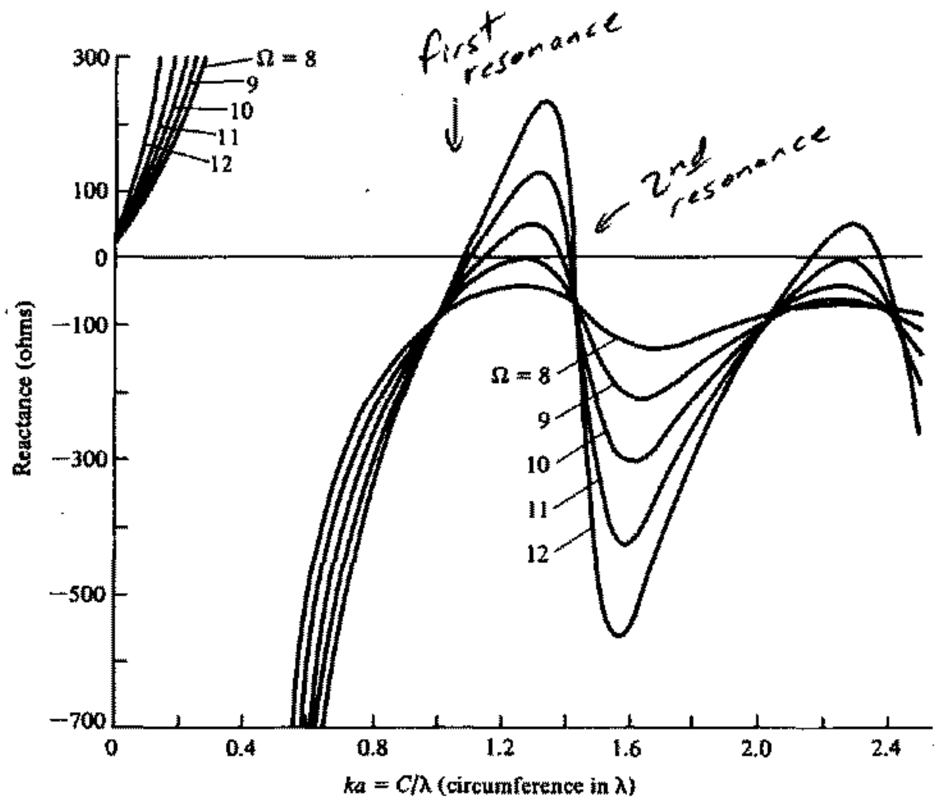
@ $ka = 1$ $\quad D_0 \simeq 3.4$ dBi (most often used)

@ $ka \simeq 1.4$ $\quad D_0 \simeq 4.5$ dBi (not used so much → matching)

Remember $\Omega = 2 \ln\left(\frac{2\pi a}{b}\right)$



(a) Resistance



(b) Reactance

Figure 5.11 Input impedance of circular loop antennas. (SOURCE: J. E. Stover, "Impedance of Thin-Wire Loop Antennas," *AIEE Trans.*, Vol. 75, November 1956. © 1956 IEEE).

[Balanis, Ant Theory (2nd Edn), p. 227]