Example- LPDA Design Procedure

- Based on work of R. E. Carrel.

1. Select or specify design parameters
   a. Desired directivity (gain) 9.5 dBi
   b. Frequency range (f_{high} and f_{low}) $300\text{ MHz} \leq f \leq 900\text{ MHz}$
   c. Desired input impedance R_0 (real) $R_0 = 75\Omega$

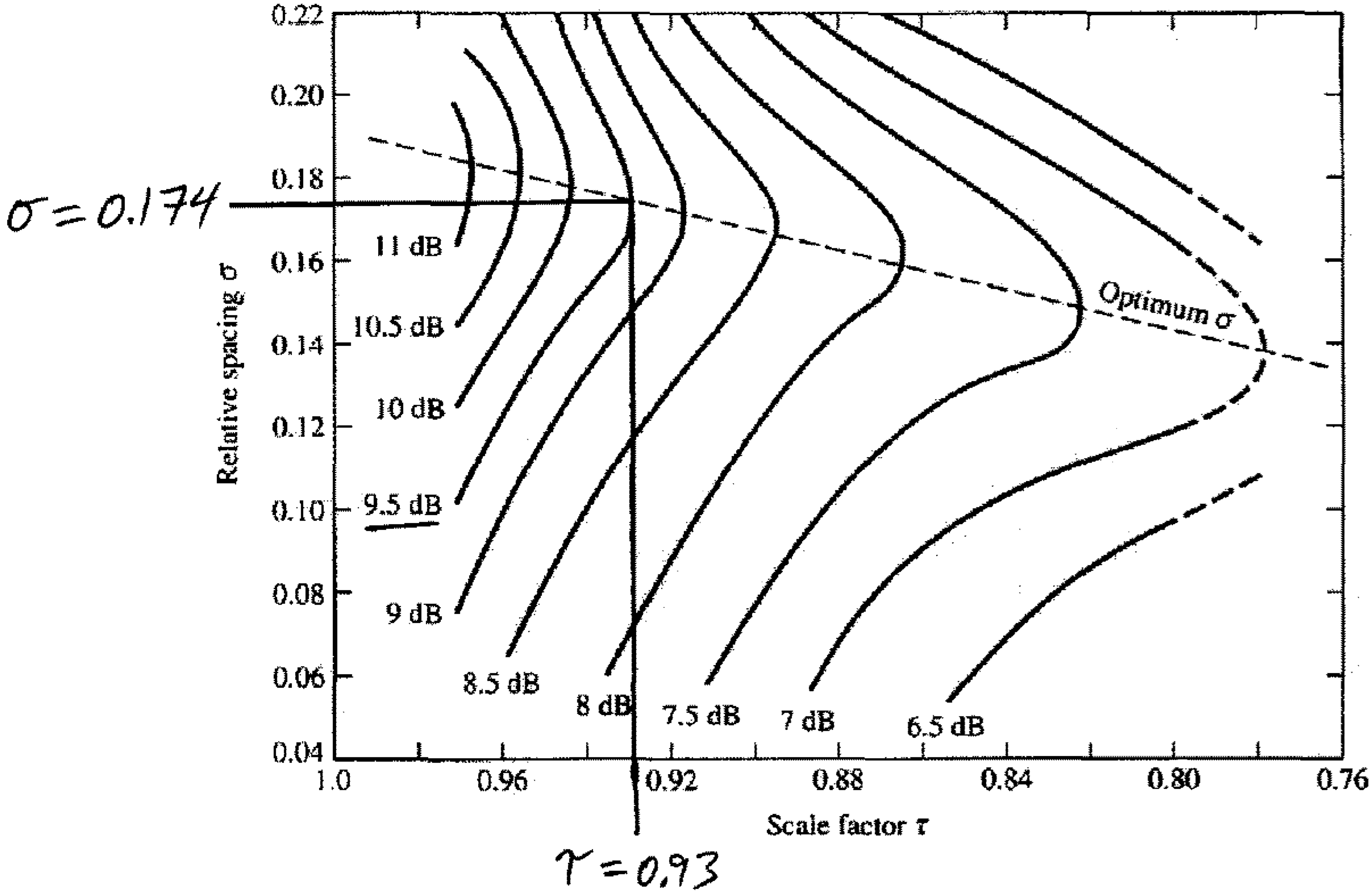
2. Use graph [Balanis, Figure 11.13, p. 561], which shows contours of constant directivity versus σ (relative spacing) and τ (scale factor), to select σ and τ for the desired directivity. $\sigma = 0.174$

   $\tau = 0.93$

3. Calculate the apex half angle α using-

$$\alpha = \tan^{-1}\left(\frac{1-\tau}{4\sigma}\right) = \tan^{-1}\left(\frac{1-0.93}{4(0.174)}\right) = \underline{5.7432^\circ}$$

$$\underline{2\alpha = 11.4864^\circ}$$



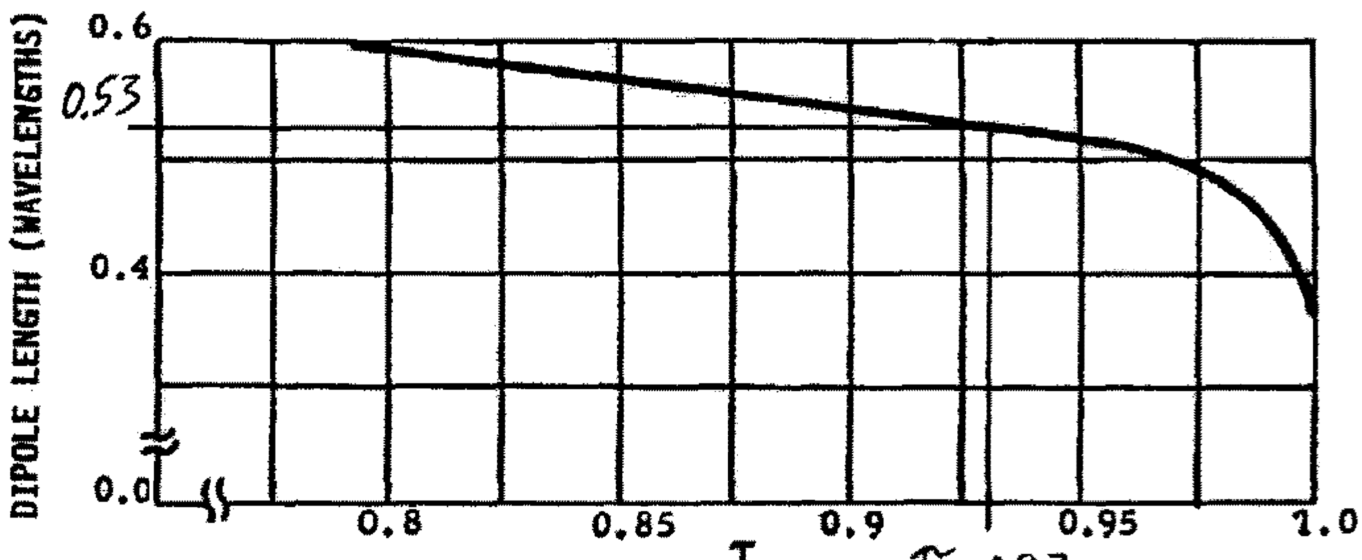
Computed Contours of constant directivity versus σ and τ for log periodic dipole arrays. [Balanis, Figure 11.13, p. 561]

4. Find length l_1 (Note: start count of elements with longest) of the **longest** element $\lambda_{max} = \frac{3\times10^8 \, m/s}{300\times10^6 \, Hz} = 1m$

- take length in wavelengths from graph if using optimum σ and τ;

- else, use $\lambda_{max}/2$ where $\lambda_{max} = c/f_{low}$ is the wavelength at the lowest frequency in the desired frequency range. $l_1 = 0.53\,\lambda_{max} = 0.53m$

$= 53cm$

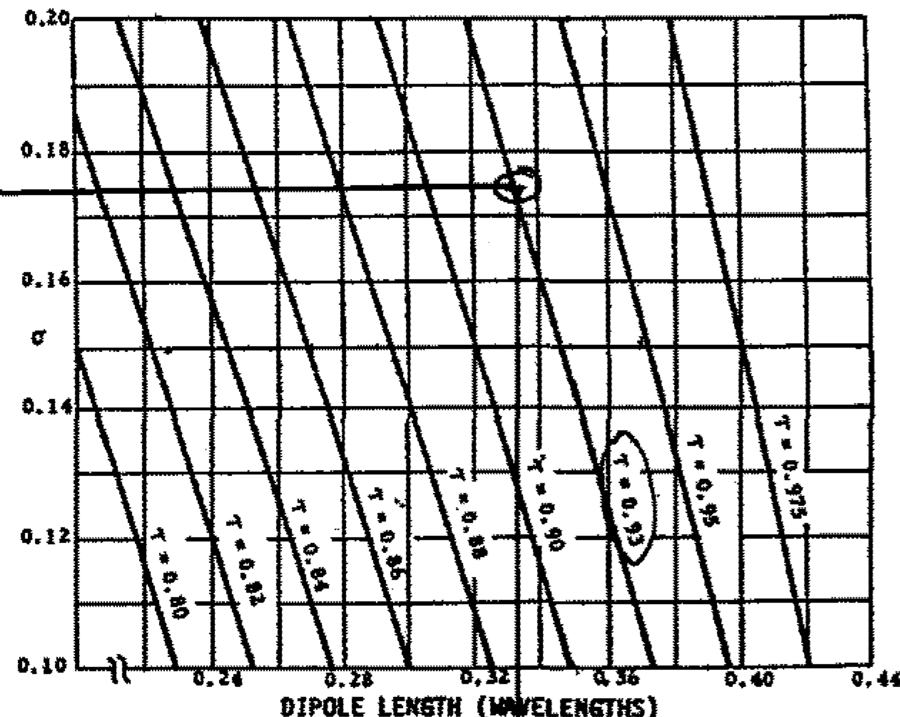


Measured length, normalized by λ_{max}, of longest dipole in LPDA versus optimum σ and τ.

5. Find length l_N of the **shortest** element $\lambda_{min} = \dfrac{3\times10^8\ \text{m/s}}{900\times10^6\ \text{Hz}} = 0.\overline{33}\ \text{m}$

- take length in wavelengths from graph where $\lambda_{min} = c / f_{high}$ is the wavelength at the highest frequency in the desired frequency range. This length will be used to know when to truncate (stop) the LPDA. It may or may not be the actual length of the smallest element.

$\sigma = 0.174$



0.333

$l_N = 0.333(0.\overline{33}\ \text{m})$

$= 0.111\ \text{m}$

$= 11.1\ \text{cm}$

Estimated length, normalized by λ_{min}, of shortest dipole in LPDA versus σ and τ.

6. Calculate location R_1 of **longest** element (as measured from the apex)-

$$R_1 = \frac{l_1}{2}\cot(\alpha) = \frac{53\text{cm}}{2}\,\frac{1}{\tan(5.7432°)} = \underline{\underline{263.485\text{cm}}}$$

7. Calculate the total bandwidth B_s, includes additional bandwidth B_{ar} due to active region, using the specified bandwidth $B = f_{high} / f_{low} = \frac{900\text{MHz}}{300\text{MHz}} = \underline{3}$

$$B_{ar} = 1.1 + 7.7\,(1\text{-}\tau)^2\cot(\alpha) = 1.1 + 7.7(1-0.93)^2\frac{1}{\tan(5.7432°)} = 1.47514$$

$$B_s = B_{ar}\cdot B = [1.1 + 7.7\,(1\text{-}\tau)^2\cot(\alpha)]\,B = 1.47514(3) = \underline{4.4254}$$

8. Calculate the approximate number N of elements required for design

$$N = 1 + \log_{10}(B_s)/\log_{10}(1/\tau) = 1 + \frac{\log 4.4254}{\log\left(\frac{1}{0.93}\right)} = 21.5 \simeq \underline{22}$$

9. Calculate the approximate distance L_T between the longest and shortest elements.

$$L_T = \frac{l_1}{2}(1-1/B_s)\cot(\alpha) = \frac{\lambda_{max}}{4}(1-1/B_s)\cot(\alpha)$$

$$= \frac{53\text{cm}}{2}\left(1-\frac{1}{4.4254}\right)\frac{1}{\tan 5.7432°} = \underline{203.946\text{ cm}}$$

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10. Calculate the location R_2 (from the apex) and length l_2 of the second longest element using the scale factor τ, R_1, and l_1 -

$$\text{e.g.,}\quad R_2 = R_1\,\tau \quad \& \quad l_2 = l_1\,\tau$$

11. Recursively calculate the location R_{n+1} and length l_{n+1} of the n+1th element(s) using the scale factor τ, R_n, and l_n –

$$\text{e.g.,}\quad R_{n+1} = R_n\,\tau \quad \& \quad l_{n+1} = l_n\,\tau.$$

Stop when l_{n+1} is less than or equal to l_N (calculated in step 5.).

12. Count actual number of elements and calculate actual length of LPDA (compare to approximate calculations in steps 8. & 9.).

13. Select a length to diameter ratio $K = l/d$ for the elements of the LPDA. This choice is a compromise between mechanical strength for the largest and smallest elements, available tubing sizes, and the selected diameter of the boom.

Choose boom diameter of 7/8" = $\underline{2.223\text{ cm} = D}$

if $d_1 = 5/8" = 1.5875\text{ cm}$ $\quad K_1 = \frac{53\text{ cm}}{1.5875\text{ cm}} = 33.38$ $\Rightarrow$ Choose $\underline{K = 33.38}$

$d_{23} = \frac{l_{23}}{K} = \frac{10.74\text{ cm}}{33.38} = 0.322\text{ cm} \sim 1/8"$ $\underline{\text{OK}}$

Steps 10. & 11. **Calculate: $R_{n+1} = R_n$ * tau and $l_{n+1} = l_n$ * tau where tau = 0.93**

n	ln (cm)	Rn (cm)
1	53.00	263.485
2	49.29	245.04
3	45.84	227.89
4	42.63	211.94
5	39.65	197.10
6	36.87	183.30
7	34.29	170.47
8	31.89	158.54
9	29.66	147.44
10	27.58	137.12
11	25.65	127.52
12	23.86	118.60
13	22.19	110.29
14	20.63	102.57
15	19.19	95.39

n	ln (cm)	Rn (cm)
16	17.85	88.72
17	16.60	82.51
18	15.43	76.73
19	14.35	71.36
20	13.35	66.36
21	12.41	61.72
22	11.55	57.40
23	10.74	53.38

Stop since $l_{23} < l_N = 11.11$ cm

Step 12. Actual # of elements & antenna length

***N* actual = 23**

***L* actual = *R*1 - *R*23 = 210.10 cm**

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14. Calculate the diameter d_n for each element. Then, select the closest available tube/pipe/rod diameter to the calculated value.

$$d_n = l_n / K$$

15. Calculate the actual length to diameter ratio K_n for each element and the average length to diameter ratio K_{ave} after quantization. Check for unusually large deviations from desired K (may want to go back to step 13. and select another value of K). $K_{ave} = 33.743$

16. Calculate the approximate average characteristic impedance of the active region elements-

$$Z_a = 60 \ln(2 X K_{ave} / \pi) = 60 \ln\left(\frac{2(0.67076)33.743}{\pi}\right)$$

$$= 160.07\,\Omega$$

where $X = 8\,\tau\sigma/(1+\tau)$.

$$= \frac{8(0.93)0.174}{1+0.93} = 0.67076$$

Step 13. $K = 33.38$

Step 14. & 15. $d\text{n} = l\text{n} / K$ $K\text{actual} = l\text{n} / d\text{ quantized}$

		Exact	Quantized		Actual
n	ln (cm)	dn (cm)	dn (cm)	dn (in)	Kn
1	53.00	1.588	1.5875	5/8	33.386
2	49.29	1.477	1.4290	9/16	34.493
3	45.84	1.373	1.4290	9/16	32.078
4	42.63	1.277	1.2700	1/2	33.568
5	39.65	1.188	1.1110	7/16	35.686
6	36.87	1.105	1.1110	7/16	33.188
7	34.29	1.027	1.0320	13/32	33.227
8	31.89	0.955	0.9525	3/8	33.480
9	29.66	0.888	0.8730	11/32	33.972
10	27.58	0.826	0.7940	5/16	34.738
11	25.65	0.768	0.7940	5/16	32.306
12	23.86	0.715	0.7140	9/32	33.411
13	22.19	0.665	0.6350	1/4	34.938
14	20.63	0.618	0.6350	1/4	32.492
15	19.19	0.575	0.5560	7/32	34.511
16	17.85	0.535	0.5560	7/32	32.096
17	16.60	0.497	0.4760	3/16	34.866
18	15.43	0.462	0.4760	3/16	32.425
19	14.35	0.430	0.3970	5/32	36.156
20	13.35	0.400	0.3970	5/32	33.625
21	12.41	0.372	0.3970	5/32	31.271
22	11.55	0.346	0.3175	1/8	36.364
23	10.74	0.322	0.3175	1/8	33.819

Average K = 33.743

17. Find the characteristic impedance of the unloaded transmission line Z_0 for the desired input impedance R_0-

$$Z_0 = \frac{R_0^{\,2}}{4Z_a X} + R_0\sqrt{\left(\frac{R_0}{4Z_a X}\right)^2 + 1} = \frac{75^2}{4(160.07)0.671} + 75\sqrt{\left(\frac{75}{4(160)0.671}\right)^2 + 1}$$

$$= 13.0974 + 76.135 = \underline{\underline{89.232\,\Omega}}$$

18. Calculate the **center-to-center** spacing S of the booms using the unloaded, cylindrical, twin-lead transmission line formula-

$$S = D\cosh(Z_0/120) \qquad D = 7/8'' = 2.223\text{ cm}$$

where D is the diameter of the booms (assumed to be identical). The air gap Δ_{gap} between the inner surfaces of the booms is $\Delta_{gap} = S - D$.

$$S = 2.223\text{ cm} \cosh\left(\frac{89.232}{120}\right)$$

$$\underline{\underline{S = 2.866\text{ cm}}}$$

$$\Delta_{gap} = 2.866\text{ cm} - 2.223\text{ cm} = \underline{\underline{0.643\text{ cm} = 6.43\text{ mm}}}$$