Chapter 7 Power Dividers and Directional Couplers

→ These are passive microwave components used to divide or combine signals.

→ They may be 3-port or more networks.

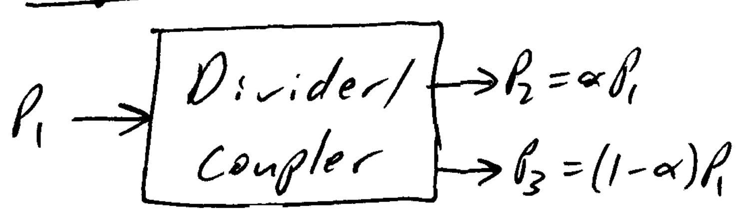
→ Some important performance characteristics are coupling, directivity, isolation, and insertion loss.

→ The MIT Radiation Laboratory developed many of these devices based on waveguides in the 1940s.

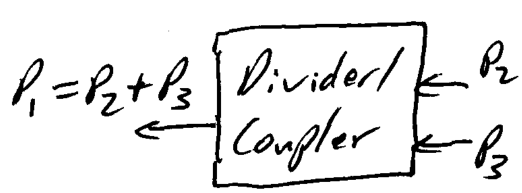
→ From the late 1950s onward many of these designs were transitioned to planar TLs (e.g., stripline and microstrip) and new designs were developed.

→ We will focus on the power dividers and directional couplers that can be implemented on/with planar TLs.

3-port



Power divider $0 \le \alpha \le 1$



Power combiner

Chap 7 cont.

Some uses or applications include:

1) Power division of a single source signal to many receivers. E.g., CATV and antenna arrays in transmit mode.

2) Power combination of several sources into a single signal. E.g., Antenna arrays in receive mode.

3) Separating forward and backward propagating signals, needed for S-parameter test sets and vector network analyzers.

7.1 Basic Properties of Dividers and Couplers

A basic property of three-port networks (T-junction) is that it is NOT possible to make one that is 1) lossless,
2) reciprocal, and
3) matched at all three ports.

For a three-port network, the [S]-matrix takes the form

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \qquad (7.1)$$

7.1 cont.

* If the three-port is matched at every port, then $S_{11} = S_{22} = S_{33} = 0$. This follows from requiring $\Gamma_1 = \Gamma_2 = \Gamma_3 = 0$ when all other ports are terminated with matched loads.

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

* If the three-port is also reciprocal, then $S_{12} = S_{21}$, $S_{13} = S_{31}$, and $S_{23} = S_{32}$.

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

* Next, if the three-port is to be lossless, then [S] must be unitary

$$[S]^t [S^*] = [U] \quad (4.51)$$

$$\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & S_{13}^* \\ S_{12}^* & 0 & S_{23}^* \\ S_{13}^* & S_{23}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This leads to the following set of six equations.

7.1 cont.

$$|S_{12}|^2 + |S_{13}|^2 = 1 \qquad (7.3a)$$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \qquad (7.3b)$$

$$|S_{13}|^2 + |S_{23}|^2 = 1 \qquad (7.3c)$$

$$S_{13}^* S_{23} = 0 \qquad (7.3d)$$

$$S_{23}^* S_{12} = 0 \qquad (7.3e)$$

$$S_{12}^* S_{13} = 0 \qquad (7.3f)$$

To satisfy (7.3d), (7.3e), + (7.3f), we would need at least two of the three S-parameters, S_{13}, S_{12}, + S_{23} to be zero. However, if we do this, it is NOT possible to satisfy equations (7.3a), (7.3b), + (7.3c).

$$\Downarrow$$

We can NOT have a lossless, reciprocal, and matched three-port network.

So, what can we do?

7.1 cont.

1) **Non-reciprocal** three-port, but still **lossless** and **matched** ($S_{11} = S_{22} = S_{33} = 0$).

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

Lossless implies $[S]^t [S^*] = [U]$ leading to

$$S_{31}^* S_{32} = 0 \qquad (7.5a)$$

$$S_{21}^* S_{23} = 0 \qquad (7.5b)$$

$$S_{12}^* S_{13} = 0 \qquad (7.5c)$$

$$|S_{12}|^2 + |S_{13}|^2 = 1 \qquad (7.5d)$$

$$|S_{21}|^2 + |S_{23}|^2 = 1 \qquad (7.5e)$$

$$|S_{31}|^2 + |S_{32}|^2 = 1 \qquad (7.5f)$$

$$\Downarrow$$

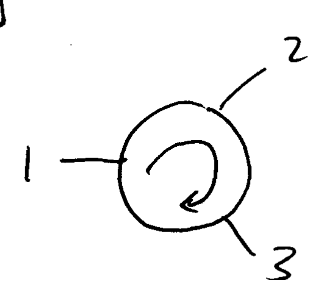
Possibility #1 $\quad S_{12} = S_{23} = S_{31} = 0$ $\quad$ (7.6a)

$\quad$ & $|S_{21}| = |S_{32}| = |S_{13}| = 1$

$$\Downarrow$$

$$[S]_{CW} = \begin{bmatrix} 0 & 0 & |S_{13}| = 1 \\ |S_{21}| = 1 & 0 & 0 \\ 0 & |S_{32}| = 1 & 0 \end{bmatrix}$$

$\Rightarrow$ Clockwise circulator



7.1 cont.

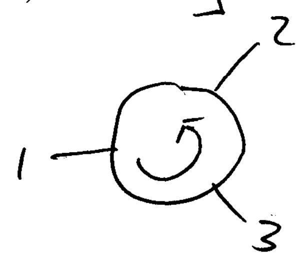
Possibility #2 $\quad S_{21} = S_{32} = S_{13} = 0$

$\quad \& \quad |S_{12}| = |S_{23}| = |S_{31}| = 1 \qquad (7.6b)$

$$\Downarrow$$

$$[S]_{ccw} = \begin{bmatrix} 0 & |S_{12}|=1 & 0 \\ 0 & 0 & |S_{23}|=1 \\ |S_{31}|=1 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ Counterclockwise circulator



2) <u>Reciprocal</u> three-port, <u>lossless</u> <u>but</u> only two of three ports are matched. For example, match ports 1 & 2 ($S_{11} = S_{22} = 0$), but let $S_{33} \neq 0$

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} \qquad (7.7)$$

Lossless implies $[S]^t [S]^* = [U]$ leading to

$$S_{13}^* S_{23} = 0 \qquad (7.8a)$$

$$S_{12}^* S_{13} + S_{23}^* S_{33} = 0 \qquad (7.8b)$$

$$S_{23}^* S_{12} + S_{33}^* S_{13} = 0 \qquad (7.8c)$$

7.1 cont.

$$|S_{12}|^2 + |S_{13}|^2 = 1 \qquad (7.8d)$$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \qquad (7.8e)$$

$$|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = 1 \qquad (7.8f)$$

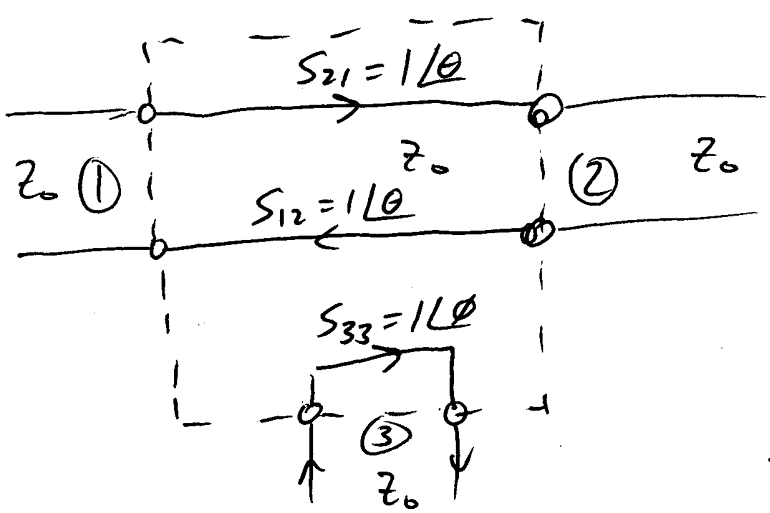
The only way for (7.8d) + (7.8e) to be true is if $|S_{13}| = |S_{23}|$, but then to satisfy (7.8a), we must have $|S_{13}| = |S_{23}| = 0$!

Then, (7.8e) + (7.8f) will require that $|S_{12}| = |S_{33}| = 1$

$$\Downarrow$$

$$[S] = \begin{bmatrix} 0 & 1\angle\theta & 0 \\ 1\angle\theta & 0 & 0 \\ 0 & 0 & 1\angle\phi \end{bmatrix}$$

$$\Downarrow$$



Essentially two separate devices:

1) a matched two-port TL

2) a one-port w/ $|\Gamma| = 1$

$\Rightarrow$ Not too useful

7.1 cont.

3) Lossy three-port can be matched @ all three ports and reciprocal.

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

but $[S]^t [S]^* \neq [U]$

E.g., A resistive divider can meet these requirements

Properties of Four-Port Networks

It is possible to make/have a lossless, matched, and reciprocal four-port network, often called directional couplers.

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

7.1 cont.

Applying the lossless condition that

$$[S]^t[S^*] = [U]$$

leads to

$$S_{14}^*\left(|S_{13}|^2 - |S_{24}|^2\right) = 0 \quad (7.11)$$

and

$$S_{23}\left(|S_{12}|^2 - |S_{34}|^2\right) = 0 \quad (7.13)$$

If we choose to make $S_{14} = S_{23} = 0$,

we get

$$|S_{12}|^2 + |S_{13}|^2 = 1 \quad (7.14a)$$

$$|S_{12}|^2 + |S_{24}|^2 = 1 \quad (7.14b)$$

$$|S_{13}|^2 + |S_{34}|^2 = 1 \quad (7.14c)$$

$$|S_{24}|^2 + |S_{34}|^2 = 1 \quad (7.14d)$$

Equations (7.14a) & (7.14b) imply $|S_{13}| = |S_{24}|$

Equations (7.14b) & (7.14d) imply $|S_{12}| = |S_{34}|$

$\Downarrow$

Select $S_{12} = S_{34} = \alpha$

$S_{13} = \beta\angle\theta$ $\qquad$ α & β are real

$S_{24} = \beta\angle\phi$

Further, $\theta + \phi = \pi \pm 2\pi n \quad (7.16)$

7.1 cont.

All of this leads to two common choices for realizing actual devices

1) Symmetrical Coupler

$\Rightarrow$ let $\theta = \phi = \pi/2$, then, as $1\angle\pi/2 = j$,

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix} \qquad (7.17)$$

$\Rightarrow$ α + β are real

$\Rightarrow$ $\alpha^2 + \beta^2 = 1$ [see eqn (7.14a)]

$\Rightarrow$ We'll see this again in section 7.5

2) Asymmetrical Coupler

$\Rightarrow$ let $\theta = 0$ + $\phi = \pi$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix} \qquad (7.18)$$

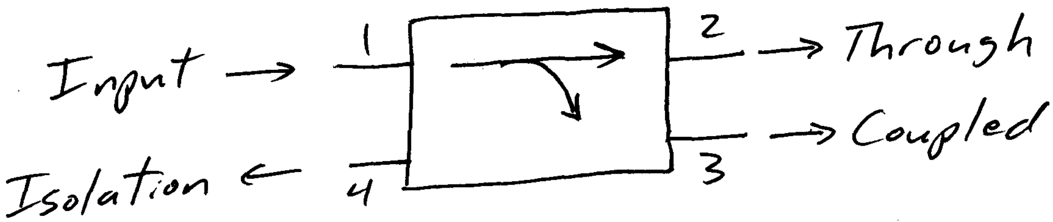
$\Rightarrow$ α + β are real

$\Rightarrow$ $\alpha^2 + \beta^2 = 1$

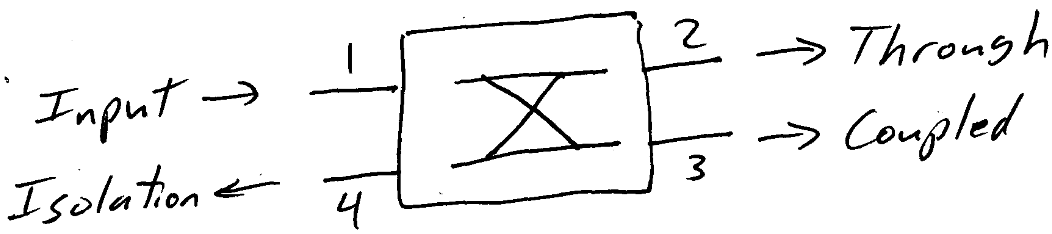
$\Rightarrow$ We'll see an example in Section 7.8

7.1 cont.

Two commonly used circuit symbols are:



and



$\Rightarrow$ arrows indicate power flow directions

From (7.17) & (7.18), we saw the [S] matrices had the form

$$[S] = \begin{bmatrix} 0 & * & * & 0 \\ * & 0 & 0 & * \\ * & 0 & 0 & * \\ 0 & * & * & 0 \end{bmatrix} \qquad * \neq 0$$

From the form of this [S] matrix we can see that if power enters port 1, it will go to ports 2 & 3 since $S_{21} \neq 0$ and $S_{31} \neq 0$ but no power goes to port 4 ($S_{41} = 0$) or is reflected from port 1 ($S_{11} = 0$).

7.1 cont.

$\Rightarrow$ Directional coupler

Similarly, if power entered port 2, we see from the second column of the [S]-matrix that power will go to ports 1 and 4 ($S_{12} \neq 0$ & $S_{42} \neq 0$), but no power goes to port 3 ($S_{32} = 0$) or is reflected ($S_{22} = 0$).

$\Rightarrow$ Directional coupler

Reality - Practical devices will not have $S_{11} = S_{22} = S_{33} = S_{44} = 0$, be perfectly lossless, ...

To characterize the performance of actual directional couplers, we define coupling, directivity, isolation, and insertion loss.

For the definitions, we'll assume port 1 is the input, port 2 is the through port, port 3 is the coupled port, and port 4 is the isolated port (hopefully no power ends up here).

7.1 cont.

$$\text{Coupling} \equiv C = 10\log_{10}\frac{P_1}{P_3} = 10\log_{10}\frac{1}{|S_{31}|^2} \quad (7.20a)$$

(dB)

reciprocal/ Symmetric

$$= 10\log_{10}\frac{1}{|S_{13}|^2}$$

$$= -20\log_{10}|S_{31}| = -20\log_{10}|S_{13}|$$

(7.17) & (7.18)

$$= -20\log_{10}\beta$$

⇒ This tells us what fraction of the input power goes or is coupled to port 3.

$$\text{Directivity} \equiv D = 10\log_{10}\frac{P_3}{P_4} = 10\log_{10}\frac{P_3/P_1}{P_4/P_1} \quad (7.20b)$$

(dB)

$$= 10\log_{10}\frac{|S_{31}|^2}{|S_{41}|^2}$$

$$= 20\log_{10}\frac{|S_{31}|}{|S_{41}|} = 20\log_{10}\frac{|S_{13}|}{|S_{14}|}$$

$$= 20\log_{10}\frac{\beta}{|S_{14}|}$$

⇒ This parameter gives us a measure of how well the coupler isolates the forward and backward propagating waves. Ideally, it is desirable for $D \to \infty$, i.e., be very large.

7.1 cont.

$$\text{Isolation} \equiv I = 10\log_{10}\frac{P_1}{P_4} = 10\log_{10}\frac{1}{P_4/P_1} \quad (7.20c)$$
(dB)

$$= 10\log_{10}\frac{1}{|S_{41}|^2} = 10\log_{10}\frac{1}{|S_{14}|^2}$$

$$= -20\log_{10}|S_{41}| = -20\log_{10}|S_{14}|$$

⇒ This parameter tells us how much input power makes it to the isolated port. Ideally, it is desirable for $I \to \infty$, i.e., be very large.

As it turns out, the coupling, directivity, and isolation are interrelated as

$$I = D + C \qquad (7.21)$$

$$\text{Insertion loss} \equiv L = 10\log_{10}\frac{P_1}{P_2} = 10\log_{10}\frac{1}{P_2/P_1} \quad (7.20d)$$
(dB)

$$= 10\log_{10}\frac{1}{|S_{21}|^2} = 10\log_{10}\frac{1}{|S_{12}|^2}$$

$$= -20\log_{10}|S_{21}| = -20\log_{10}|S_{12}|$$

$$= -20\log_{10}\alpha$$

⇒ This parameter tells us how much of the input power is delivered to the through port (less power to coupled and isolated ports).

7.1 cont.

example - Let's look at a Mini-Circuits BDCA-10-25 directional coupler. From the datasheet, Coupling = $C = 10.1\,dB$, directivity $= D = 22\,dB$, and mainline loss $= 1\,dB$ (this is insertion loss w/ lossy TL, ...)

Per (7.21), Isolation $= I = D + C = 10.1 + 22$

$$\underline{\underline{I = 32.1\,dB}}$$

Per (7.20a), $C = 10.1\,dB = -20\log_{10}\beta$

$$\hookrightarrow \underline{\underline{\beta = 10^{-\frac{10.1}{20}} = 0.3126}}$$

Per (7.20d), $L = 1\,dB = -20\log_{10}\alpha$

$$\hookrightarrow \underline{\underline{\alpha = 10^{-1/20} = 0.89125}}$$

For a lossless coupler, $\alpha^2 + \beta^2 = 1$. Here, we get $\alpha^2 + \beta^2 = 0.89125^2 + 0.3126^2 = 0.892$.

Ideally, $|S_{14}| = |S_{41}| = 0$. Here, using (7.20b)

$$D = 22\,dB = 20\log_{10}\frac{\beta}{|S_{14}|} \Rightarrow |S_{14}| = 0.0248 = -32.1\,dB = -I$$

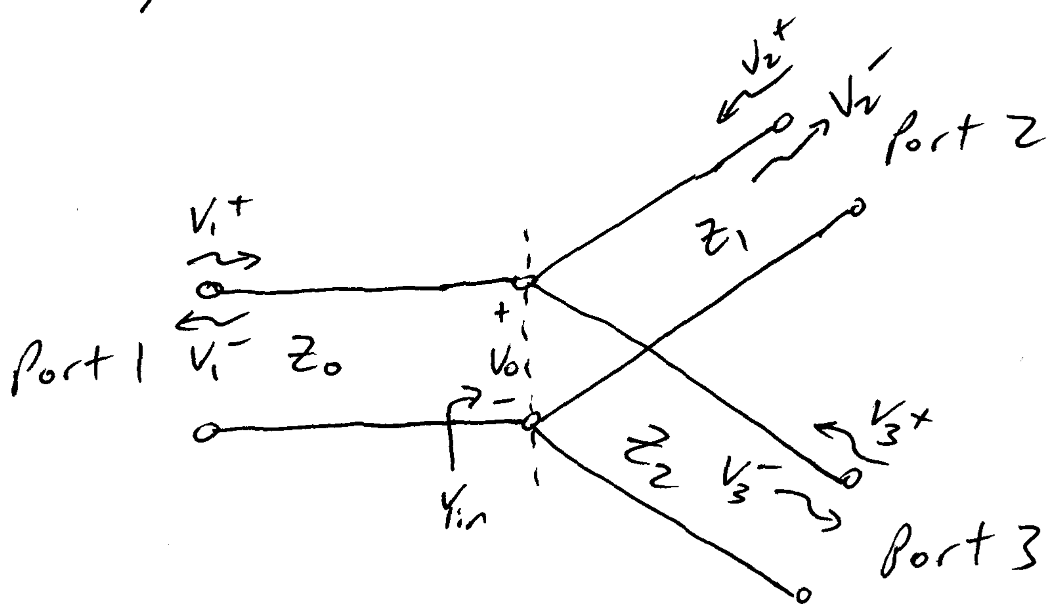
Ideally, $|S_{12}|^2 = \alpha^2 = 1 - \beta^2 = 1 - 0.3126^2$

$$\hookrightarrow \alpha = 0.94988$$ (earlier we got 0.89125)

$\Rightarrow$ losses ☹

7.2 The T-Junction Power Divider

→ Easy to make



Design Constraints

1) Input (Port 1) should be matched

↳ $Y_{in} = \frac{1}{Z_0} = \frac{1}{Z_1} + \frac{1}{Z_2}$ (7.25)

Note: Text adds a lumped susceptance jB in parallel at junction to account for junction discontinuity effects. Then,

$$Y_{in} = \frac{1}{Z_0} = jB + \frac{1}{Z_1} + \frac{1}{Z_2} \quad (7.24)$$

2) Power division between ports 2 & 3

⇒ we don't always pick a 50%-50% split

7.2 cont.

Define power division as X : Y where

→ $\frac{X}{X+Y} \times 100\%$ is delivered to one output port (e.g., Port 2)

→ $\frac{Y}{X+Y} \times 100\%$ is delivered to the other output port (e.g., Port 3)

Example

1 : 1 ⇒ $\frac{1}{1+1} \times 100 = 50\%$ even split

$\frac{1}{1+1} \times 100 = 50\%$

2 : 1 ⇒ $\frac{2}{1+2} \times 100\% = 66.\bar{6}\%$ Two thirds

$\frac{1}{1+2} \times 100\% = 33.\bar{3}\%$ One third

So, how do we select Z_1 & Z_2 ?

Input (Port 1) $P_{in} = \frac{|V_1^+|^2}{2Z_0} = \frac{|V_0|^2}{2Z_0}$ (matched, No reflection)

7.2 cont.

Assuming ports 2 & 3 are matched to their connecting TLs / loads,

$$P_2 = \frac{|V_0|^2}{2Z_1} \text{ and } P_3 = \frac{|V_0|^2}{2Z_2}$$

We can now compute power ratios

$$\frac{P_1}{P_{in}} = \frac{|V_0|^2/2Z_1}{|V_0|^2/2Z_0} = \frac{Z_0}{Z_1} \Rightarrow Z_1 = \frac{Z_0}{P_2/P_{in}}$$

$$\frac{P_2}{P_{in}} = \frac{|V_0|^2/2Z_2}{|V_0|^2/2Z_0} = \frac{Z_0}{Z_2} \Rightarrow Z_2 = \frac{Z_0}{P_3/P_{in}}$$

Because we have no losses

$$\frac{P_2}{P_{in}} + \frac{P_3}{P_{in}} = 1$$

$$\hookrightarrow \frac{Z_0}{Z_1} + \frac{Z_0}{Z_2} = 1$$

$$\hookrightarrow \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0} \quad \text{same as (7.25)}$$

7.2 cont.

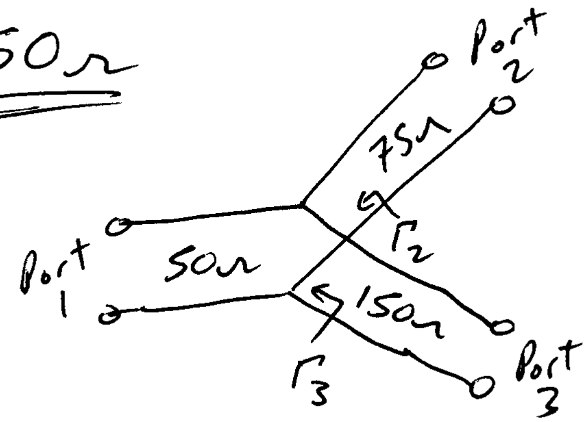
Example - For a 50Ω feed, design a lossless T-junction power divider for a 2:1 ratio.

Port 2 $\frac{P_2}{P_{in}} = \frac{2}{2+1} \times 100\% = 66.\bar{6}\%$

Port 3 $\frac{P_3}{P_{in}} = \frac{1}{2+1} \times 100\% = 33.3\%$

$$Z_1 = \frac{Z_0}{P_2/P_{in}} = \frac{50}{0.\bar{66}} = \underline{\underline{75\Omega}}$$

$$Z_2 = \frac{Z_0}{P_3/P_{in}} = \frac{50}{0.\bar{33}} = \underline{\underline{150\Omega}}$$



Check $\frac{1}{Z_1} + \frac{1}{Z_2} \stackrel{?}{=} \frac{1}{Z_0}$

$$\frac{1}{75} + \frac{1}{150} \stackrel{?}{=} \frac{1}{50}$$

$\underline{0.02S = 0.02S}$ $\therefore$ Input is matched ($S_{11} = 0$)

What about Ports 2 + 3?

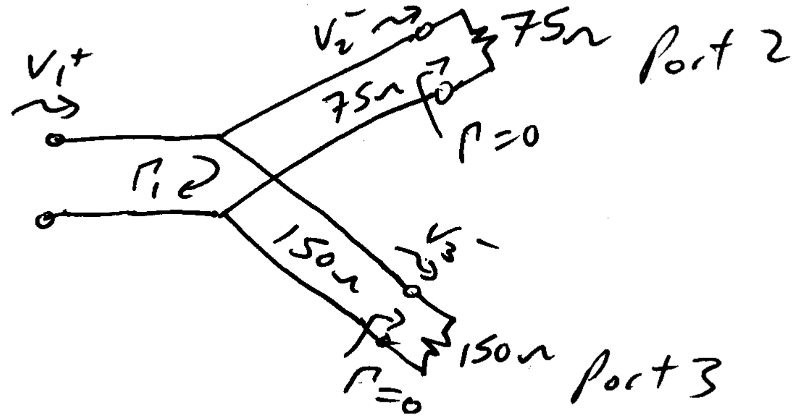
$$S_{22} = \Gamma_2 = \frac{50//150 - 75}{50//150 + 75} = \frac{37.5 - 75}{37.5 + 75} = -0.\bar{3}$$

$$S_{33} = \Gamma_3 = \frac{50//75 - 150}{50//75 + 150} = \frac{30 - 150}{30 + 150} = -0.\bar{6}$$

7.2 cont.

Let's find remaining [S]-parameters

$$\boxed{S_{12} = S_{21}}$$



Per TL theory, $V_1^+(1+\Gamma_1) = V_2^-$

$$\left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=V_3^+=0} = 1 + \cancel{\Gamma_1}^{\to 0} = 1$$

To get $S_{12} = S_{21}$, we need to account for differing line impedances by using generalized [S]-parameters, i.e.,

$$S_{ij} = \left.\frac{V_i^- \sqrt{Z_{0,j}}}{V_j^+ \sqrt{Z_{0,i}}}\right|_{V_k^+=0}$$

$$\Downarrow$$

$$S_{12} = S_{21} = \frac{V_2^-}{V_1^+}\frac{\sqrt{50}}{\sqrt{75}} = 1(0.8165)$$

$$\underline{S_{12} = S_{21} = 0.8165}$$

7.2 cont.

$\boxed{S_{13} = S_{31}}$ Per TL theory, $V_1^+ (1 + \Gamma_1) = V_3^-$

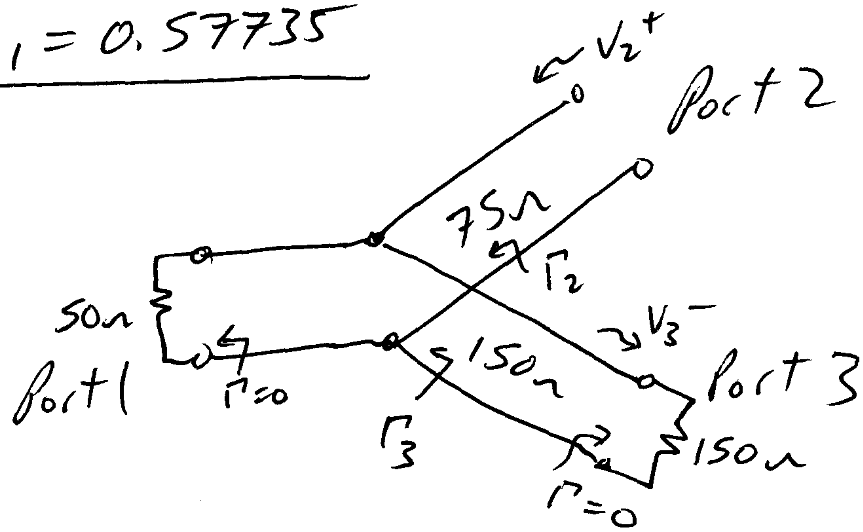
$$\left.\frac{V_3^-}{V_1^+}\right|_{V_2^+ = V_3^+ = 0} = 1 + \Gamma_1^{\nearrow 0} = 1$$

and

$$S_{13} = S_{31} = \frac{V_3^-}{V_1^+}\frac{\sqrt{50}}{\sqrt{150}} = 1(0.57735)$$

$$\underline{S_{13} = S_{31} = 0.57735}$$

$\boxed{S_{32} = S_{23}}$



Per TL theory

$$V_2^+ (1 + \Gamma_2) = V_3^-$$

$$\frac{V_3^-}{V_2^+} = 1 + \Gamma_2 = 1 - 0.\overline{3} = 0.\overline{66}$$

$$S_{32} = S_{23} = \frac{V_3^-}{V_2^+}\frac{\sqrt{75}}{\sqrt{150}} = 0.\overline{6}(0.7071)$$

$$\underline{S_{32} = S_{23} = 0.4714}$$

7.2 cont.

$$[S]_{2:1\ T\text{-jun.}} = \begin{bmatrix} 0 & 0.8165 & 0.57735 \\ 0.8165 & -0.\overline{33} & 0.4714 \\ 0.57735 & 0.4714 & -0.\overline{66} \end{bmatrix}$$

Lossless $[S]^t[S^*] \overset{?}{=} [U]$

$$\begin{bmatrix} 0 & 0.8165 & 0.57735 \\ 0.8165 & -0.\overline{33} & 0.4714 \\ 0.57735 & 0.4714 & -0.\overline{66} \end{bmatrix} \begin{bmatrix} 0 & 0.8165 & 0.57735 \\ 0.8165 & -0.\overline{33} & 0.4714 \\ 0.57735 & 0.4714 & -0.\overline{66} \end{bmatrix} \overset{?}{=} [U]$$

Check row 1-column 1 $\quad 0^2 + 0.8165^2 + 0.57735^2 \overset{?}{=} 1$

$$\underline{1 = 1} \therefore$$

Check row 1 - column 2 $\quad 0(0.8165) + (0.8165)(-0.\overline{3}) + 0.57735(0.4714) \overset{?}{=} 0$

$$\underline{0 = 0} \therefore$$

Note: $S_{23} = S_{32} = 0.4714$

implies that ports 2 & 3

are not well isolated

7.2 cont

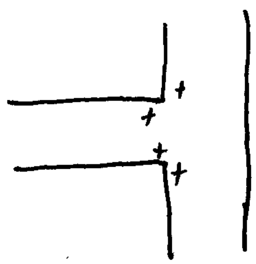
Practical Issues w/ lossless T-junctions

1) Little isolation between ports.

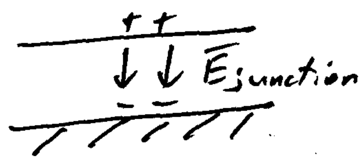
 ⇒ Have to live with this issue.

2) Junction discontinuity effects

 ⇒ Charges tend to build up at corners in microstrip or stripline



Top View



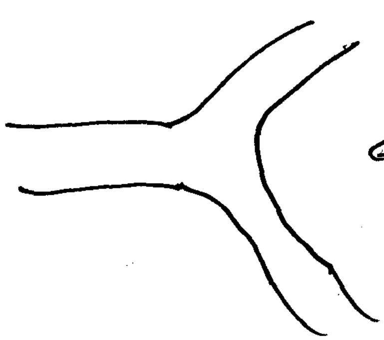
Side View

⇓

Energy stored in $\bar{E}_{junction}$ can be modeled as a parasitic capacitance.

⇒ account for in model

⇒ minimize by junction design



← rounded corners

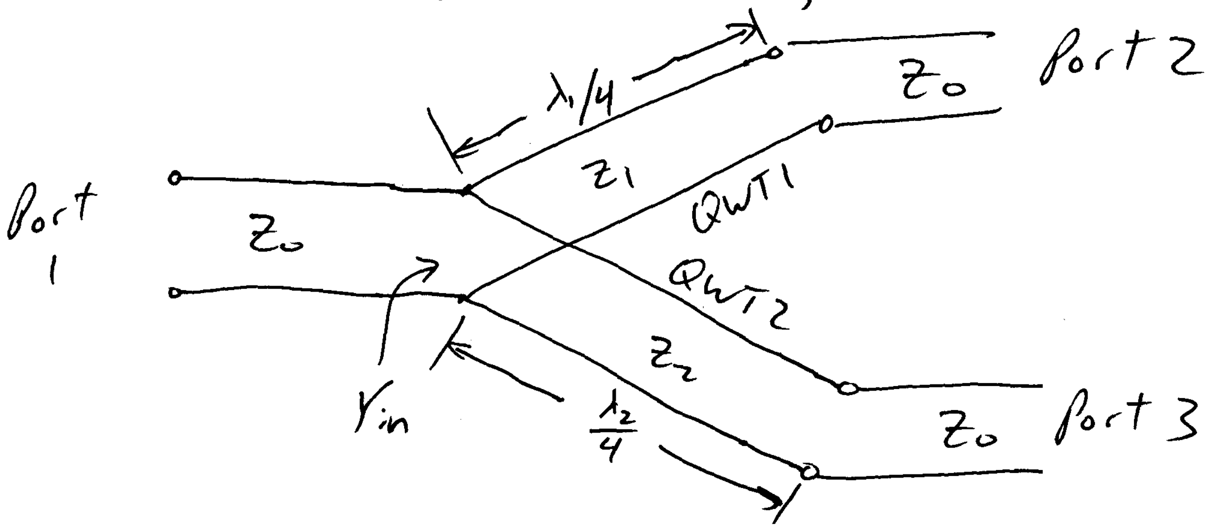
⇒ less charge build up

7.2 cont.

3) Mismatch at ports 2 & 3 if we want to use a common Z_0 for our circuit, a typical design preference

Option 1 – Add an impedance transformer (i.e., QWT) to ends of ports 2 & 3
⇒ Narrowband solution! ☹

Option 2 – Use QWTs for Z_1 & Z_2 sections. I.e.,



QWT1 transforms Z_0 to $Z_{in,1}$ ⇒ $Z_1 = \sqrt{Z_0 Z_{in,1}}$

QWT2 transforms Z_0 to $Z_{in,2}$ ⇒ $Z_2 = \sqrt{Z_0 Z_{in,2}}$

We require: see (2.63)

$$Y_{in} = \frac{1}{Z_0} = \frac{1}{Z_{in,1}} + \frac{1}{Z_{in,2}}$$

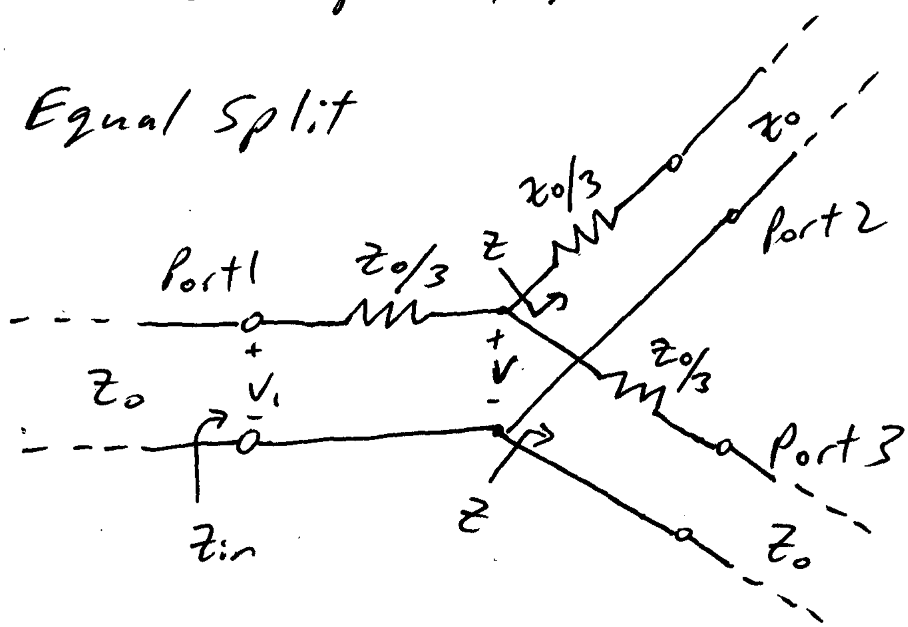
↳ $\frac{P_2}{P_{in}} = \frac{Z_0}{Z_{in,1}}$ and $\frac{P_3}{P_{in}} = \frac{Z_0}{Z_{in,2}}$

7.2 cont.

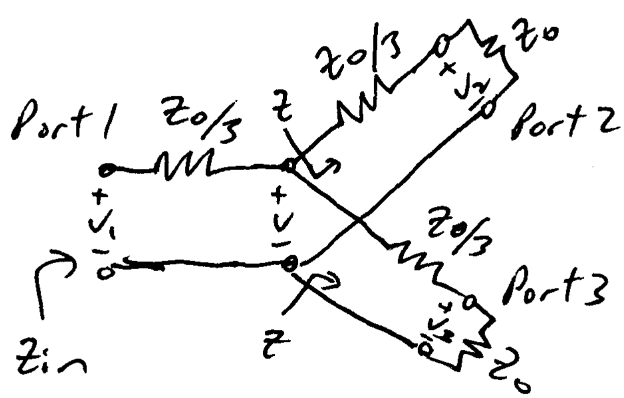
Resistive Divider

→ Lossy but able to match all three ports and is reciprocal/symmetric

Equal Split



⇒ Three-fold symmetry



with ports 2 + 3 terminated w/ loads of Z_0, the impedance looking from the center junction toward ports 2 + 3 is $Z = \frac{Z_0}{3} + Z_0 = \frac{4Z_0}{3}$

7.2 cont.

The input impedance looking into port 1 is

$$Z_{in} = \frac{Z_o}{3} + \frac{4Z_o}{3} \,//\, \frac{4Z_o}{3} = \frac{Z_o}{3} + \frac{2Z_o}{3}$$

$\underline{Z_{in} = Z_o}$ (will get same result @ ports 2 + 3)

$$\Rightarrow \underline{S_{11} = S_{22} = S_{33} = 0}$$

By voltage division

$$V = V_1 \frac{4Z_o/3 \,//\, \frac{4Z_o}{3}}{Z_o} = V_1 \frac{\frac{2Z_o}{3}}{Z_o}$$

$V = \frac{2}{3} V_1$ at center junction

By voltage division

$$V_2 = V_3 = V \frac{Z_o}{\frac{Z_o}{3} + Z_o} = \frac{2}{3} V_1 \left(\frac{3}{4}\right)$$

$$V_2 = V_3 = \frac{1}{2} V_1$$

Since we are matched, $V_1 = V_1^+$, $V_2 = V_2^-$, and $V_3 = V_3^-$

$$S_{21} = \frac{V_2^-}{V_1^+} = \frac{\frac{1}{2}V_1^+}{V_1^+} = \frac{1}{2} = S_{12} \text{ (symmetry)}$$

$$S_{31} = \frac{V_3^-}{V_1^+} = \frac{\frac{1}{2}V_1^+}{V_1^+} = \frac{1}{2} = S_{13} \text{ (symmetry)}$$

Similarly $S_{23} = S_{32} = \frac{1}{2}$

7.2 cont.

$$[S] = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix} = 0.5 \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad (7.30)$$

What about power?

$$P_{in} = \frac{|V_1^+|^2}{2Z_0} = \frac{|V_1|^2}{2Z_0} \quad (7.31)$$

$$P_2 = P_3 = \frac{|0.5 V_1^+|^2}{2Z_0} = \frac{|V_1^+|^2}{8Z_0} = 0.25 P_{in} \quad (7.32)$$

$$\frac{P_2}{P_{in}} = \frac{P_3}{P_{in}} = 0.25$$

$$\Downarrow$$

25% of input power to Port 2

25% of input power to Port 3

50% of input power dissipated by the three $Z_0/3$ resistors

$\Rightarrow$ Many resources on internet for unequal power division and alternate layouts (e.g., can do a Δ- or Delta-configuration).

7.3 The Wilkinson Power Divider

The power dividers of the previous section have drawbacks. The lossless T-junction is not matched at all ports and has limited isolation between outputs. The resistive divider is matched, but lossy, and also does not have significant isolation between outputs. Theory says that, if we allow for losses, we can match all ports and have isolation

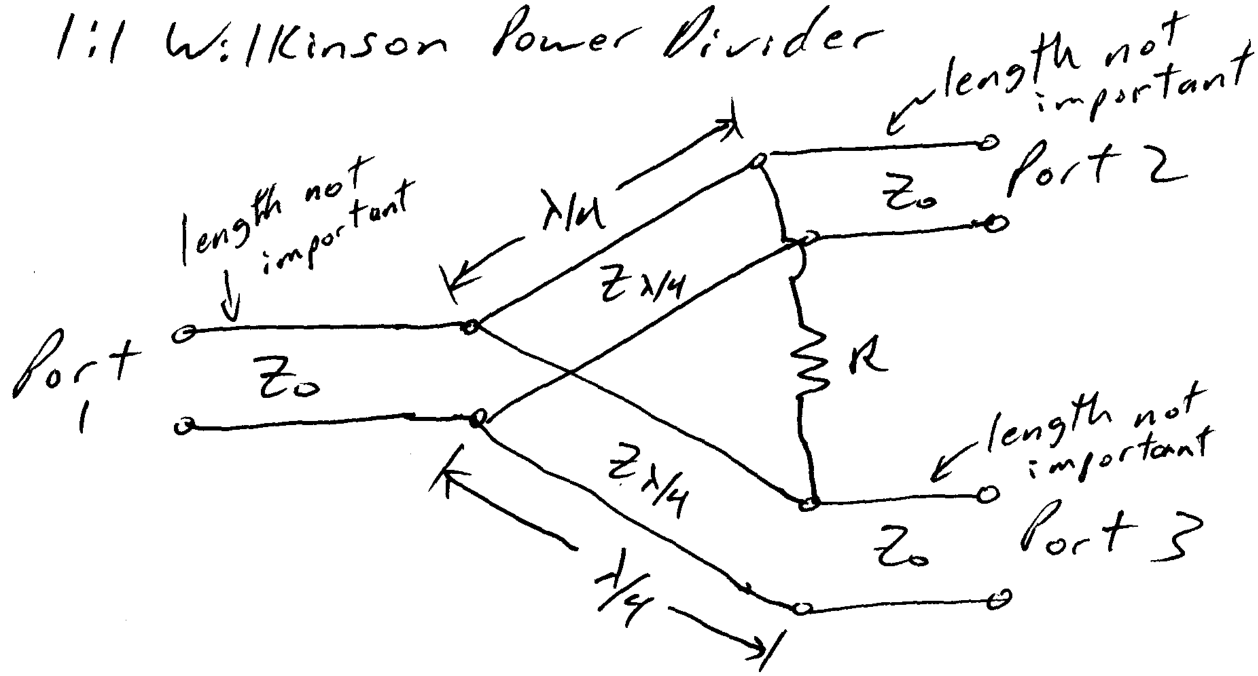
$\Downarrow$

Wilkinson Power Divider

- → Easy to construct
- → Reciprocal
- → Large isolation
- → lossless when output ports are matched (does have resistance)
- → can be made w/ arbitrary power division, but equal (1:1) split is most common.

7.3 cont.

1:1 Wilkinson Power Divider



⇒ For a 1:1 split, $Z_{\lambda/4} = \sqrt{2}\,Z_o$ and $R = 2Z_o$

Even-Odd Mode Analysis

(or Symmetric and anti-Symmetric excitation)

→ Technique is used for antenna analysis (e.g., folded dipoles) as well as in other areas.

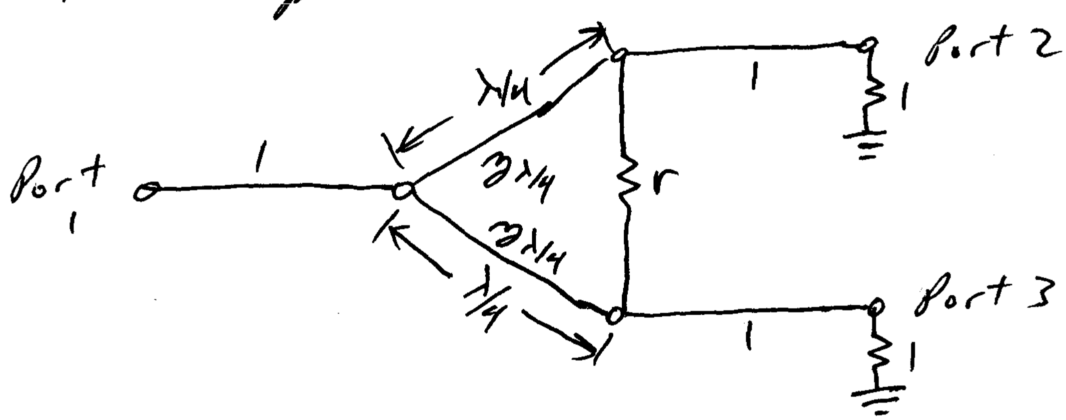
To simplify the drawings, we will:

1) Normalize all impedances by Z_o
2) Not draw return line for TLs

e.g. 

7.3 cont.

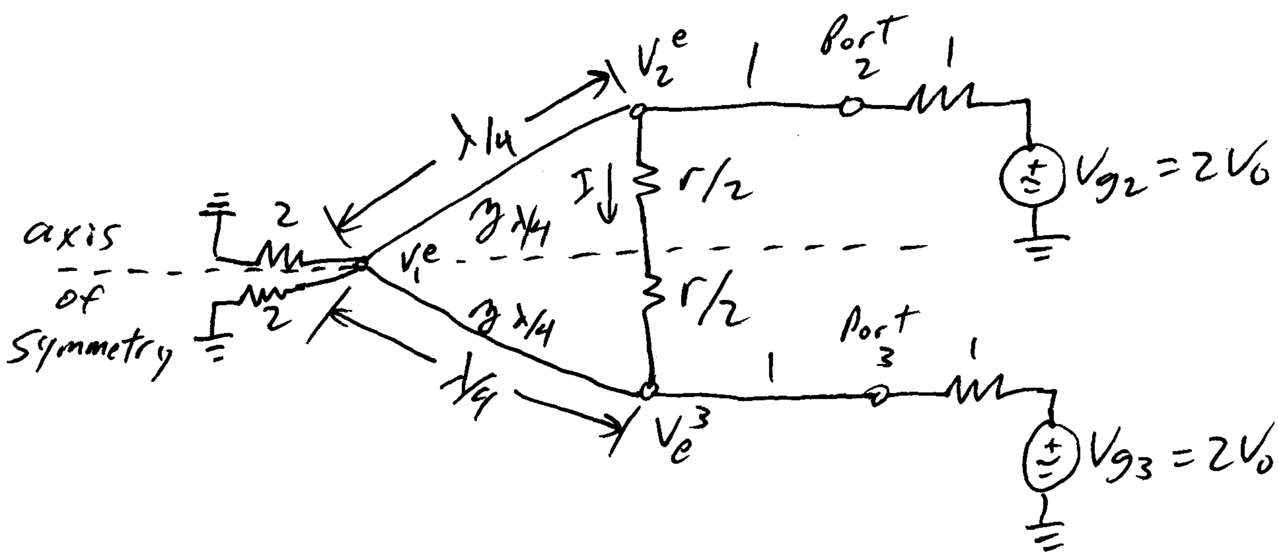
So, with matched loads at ports 2 + 3, the Wilkinson power divider becomes



Even Mode (Symmetric excitation)

→ Drive both ports 2 + 3 w/ $2V_0$ matched sources

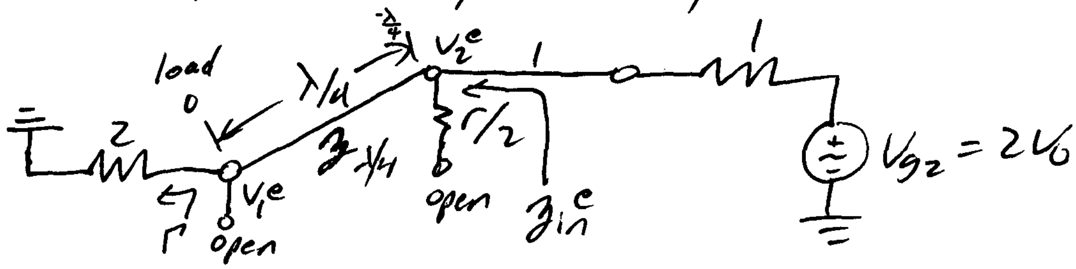
→ Terminate port 1 w/ matched load constructed with parallel equivalent normalized impedances, e.g., 1 = 2//2.



From symmetry, $V_2^e = V_3^e \Rightarrow I = \dfrac{V_2^e - V_3^e}{r} = 0$

7.3 cont.

⇒ We can bisect the even mode circuit, placing open circuits at the dividing line or axis of symmetry



⇒ we have a QWT with a load of 2.

$$(2.63)\quad Z_{in} = \frac{Z_{\lambda/4}^2}{Z_L} \xrightarrow{\text{normalized}} z_{in} = \frac{z_{\lambda/4}^2}{z_L}$$

Here, $z_{in}^e = \dfrac{z_{\lambda/4}^2}{2}$ (7.33) (ignore $r/2$ as it is in series w/ ∞)

$\hookrightarrow z_{\lambda/4} = \sqrt{2 z_{in}^e}$

To match the even mode, choose $z_{in}^e = 1$.

Then, $z_{\lambda/4} = \sqrt{2(1)} = \sqrt{2} \Rightarrow \underline{\underline{Z_{\lambda/4} = \sqrt{2}\, Z_0}}$

With a matched load + TL, remember the voltage magnitude is unchanged along TL and the input impedance is $Z_0 \Rightarrow 1$ (normalize)

So, $$V_2^e = V_{g2} \frac{z_{in}^e}{z_{in}^e + 1} = 2V_0 \frac{1}{1+1} = V_0$$

If we assume the TL is short, we can ignore phase shift.

7.3 cont.

To find our load voltage V_1^e, we'll use TL theory. At the port 2 input,

$$V(-\lambda/4) = V_2^e = V_0 = V^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)\Big|_{z=-\lambda/4}$$

Remembering $\beta = \frac{2\pi}{\lambda}$,

$$V_0 = V^+\left[e^{+j\frac{2\pi}{\lambda}\lambda/4} + \Gamma e^{-j\frac{2\pi}{\lambda}\lambda/4}\right]$$

$$= V^+\left[\underbrace{e^{+j\pi/2}}_{\to j} + \Gamma \underbrace{e^{-j\pi/2}}_{\to -j}\right]$$

$$V_0 = jV^+(1-\Gamma) \rightarrow V^+ = \frac{V_0}{j(1-\Gamma)} \quad (A)$$

At the load,

$$V_1^e = V(0) = V^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)\Big|_{z=0}$$

$$V_1^e = V^+(1+\Gamma) \quad (B)$$

The load reflection coefficient is

$$\Gamma = \frac{2 - z_{\lambda/4}}{2 + z_{\lambda/4}} = \frac{2-\sqrt{2}}{2+\sqrt{2}} \quad (C)$$

Substitute (A) + (C) into (B)

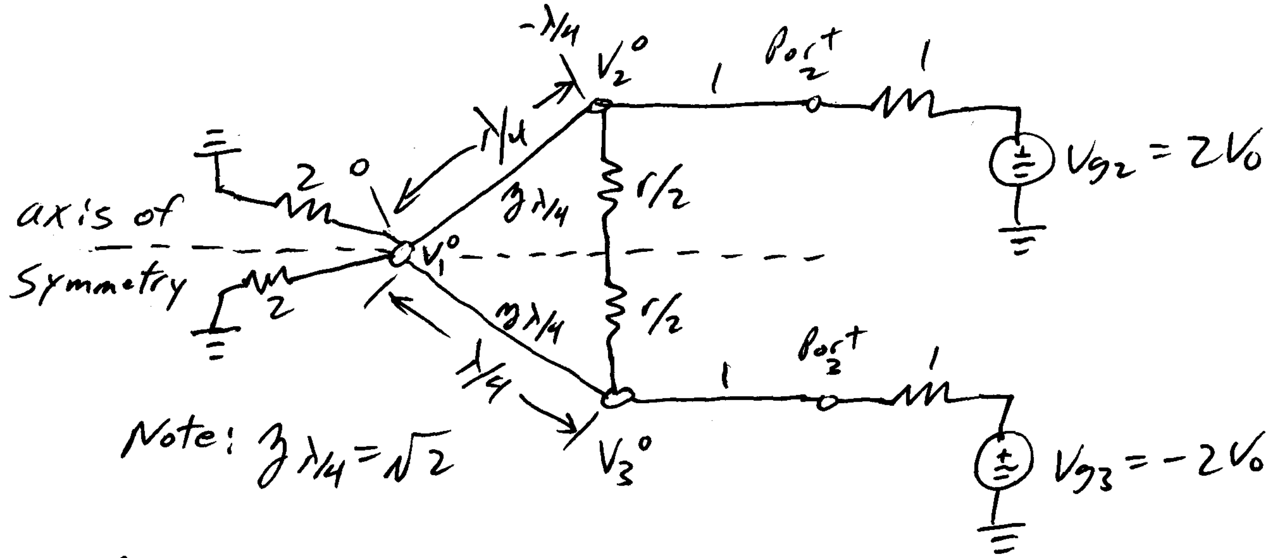
$$V_1^e = \frac{V_0}{j(1-\Gamma)}(1+\Gamma)\Big|_{\Gamma = \frac{2-\sqrt{2}}{2+\sqrt{2}}} = -jV_0\sqrt{2}$$

7.3 cont.

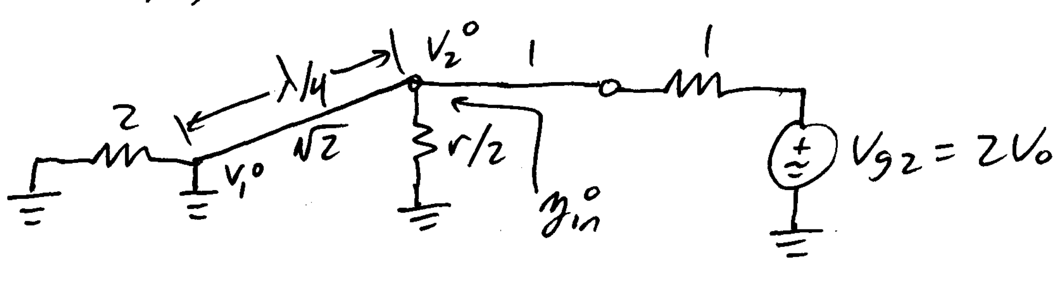
Odd Mode (anti-symmetric excitation)

→ Drive ports 2 & 3 w/ $V_{g2} = 2V_0$ & $V_{g3} = -2V_0$ matched sources.

→ Same termination for port 1.



By the symmetry of the circuit, we get $V_2^o = -V_3^o$. Therefore, along the axis of symmetry, the voltage is zero ⇒ 0V ⇒ ground! Bisecting the odd mode circuit and placing grounds along the axis of symmetry, yields



7.3 cont.

To find z_{in}^{o}, start at the load

$$2 \parallel 0 = 0$$

go $\lambda/4$ down a TL from 0 (think 180° around a Smith chart) to get ∞

$$z_{in}^{o} = \infty \parallel r/2 = r/2$$

For a match in the odd mode, set

$$z_{in}^{o} = 1 = r/2 \Rightarrow r = 2 \Rightarrow \underline{\underline{R = 2Z_0}}$$

By voltage division

$$V_2^{o} = \frac{r/2}{r/2+1} 2V_0 = \frac{1}{1+1} 2V_0$$

$$V_2^{o} = V_0$$

$V_1^{o} = 0$ as it's on axis of symmetry

from our even and odd mode analysis, we have selected

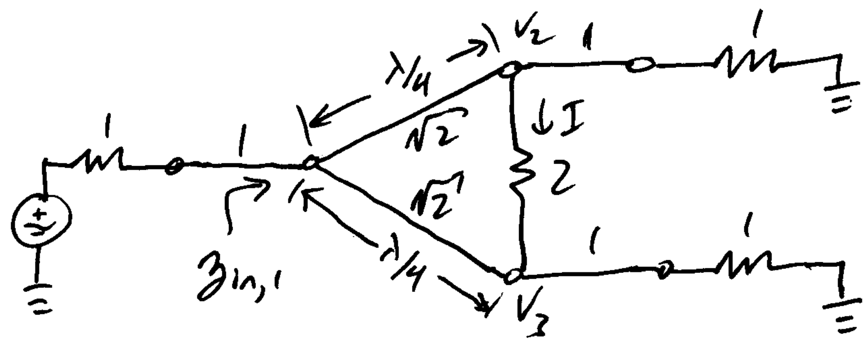
$$Z_{\lambda/4} = \sqrt{2}\, Z_0$$

and $R = 2Z_0$

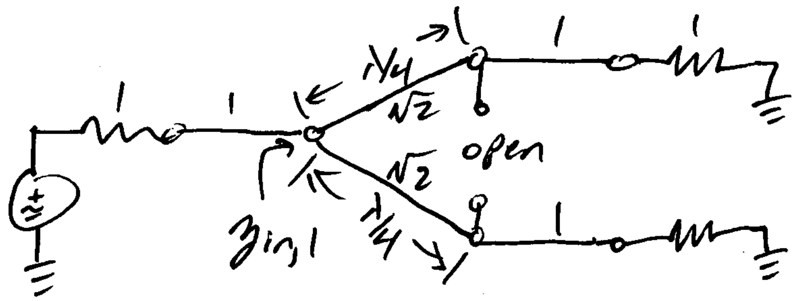
what happens at the input?

7.3 cont.

Put matched source on port 1 and matched loads/terminations at ports 2 & 3.



By symmetry, $V_2 = V_3$. Therefore, $I = 0$, i.e., $R = 2$ looks like an open circuit.



Looking into each QWT, we see from

$$(2.63) \quad Z_{in} = \frac{Z_{\lambda/4}^2}{Z_L} \Rightarrow z_{in} = \frac{\sqrt{2}^2}{1} = 2$$

Therefore, $z_{in,1} = z_{in} \| z_{in} = 2 \| 2$

$$z_{in,1} = 1 \Rightarrow Z_{in,1} = Z_0$$

(Matched!)

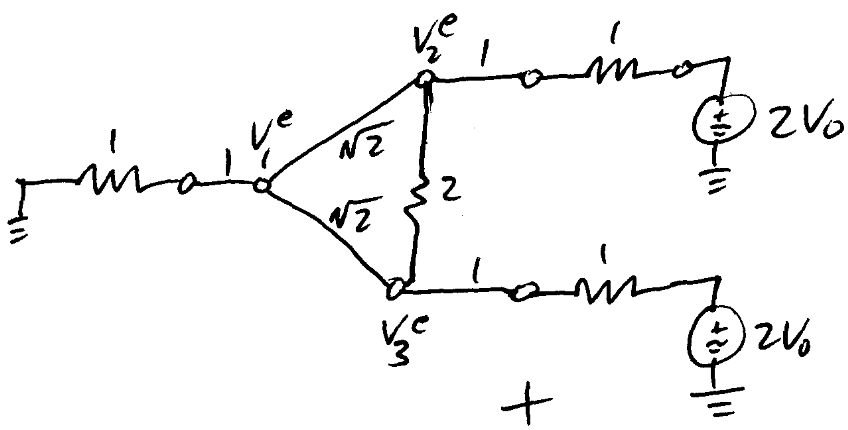
7.3 cont.

Also, for the matched source and matched loads at ports 2 + 3 situation, since $I = 0$ through $R \Rightarrow$ No losses!!

Now, time to find the [S]-parameters for our 1:1 Wilkinson power divider.

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+ = V_3^+ = 0} = \underline{\underline{0}} \quad \text{(matched)}$$

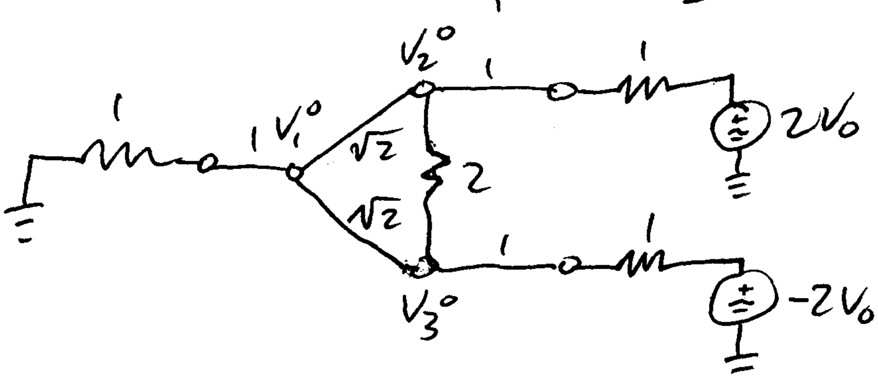
To get $S_{22} = S_{33}$, let's go back to the even + odd mode circuits and add



$V_2^e = V_3^e = V_0$

$V_1^e = -jV_0\sqrt{2}$

+



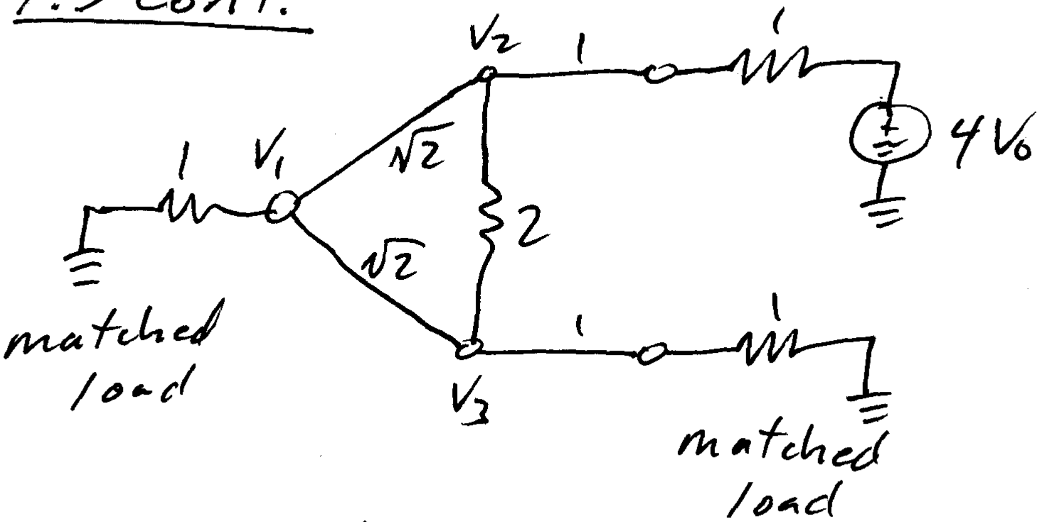
$V_2^o = V_0$

$V_3^o = -V_0$

$V_1^o = 0$

||

7.3 cont.



$V_1 = -jV_0\sqrt{2}$

$V_2 = V_2^e + V_2^o$
$= 2V_0$

$V_3 = 0$

$$S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{V_1^+ = V_3^+ = 0}$$ ← ok, we have matched loads @ ports 1 & 3

$$= \frac{V_2^{e-} + V_2^{o-}}{V_2^{e+} + V_2^{o+}}$$

Remember, in the even mode, we picked $z_{\lambda/4} = \sqrt{2}$ to ensure $z_{in} = 1$ (matched)
$\Rightarrow V_2^{e-} = 0$

AND

In the odd mode, we picked $r = 2$ to ensure $z_{in} = 1$ (matched)
$\Rightarrow V_2^{o-} = 0$

$$= \frac{0 + 0}{V_2^+} = 0$$

$$\underline{\underline{S_{22} = S_{33} = 0}}$$

7.3 cont.

Next, let's determine $S_{12} = S_{21}$ (no anisotropic materials). Using the circuit we got when finding S_{22}, we can find S_{12}.

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{V_1^+ = V_3^+ = 0}$$ ok, ports 1 & 3 are matched

Looking back at the even & odd mode circuits (before bisecting), port 1 was terminated in a matched load $\Rightarrow V_1^{e+} = V_1^{o+} = 0$. Therefore, $V_1 = V_1^e + V_1^o = V_1^- = -jV_0\sqrt{2}$.

We already determined that port 2 is matched for both even & odd modes $\Rightarrow V_2^{e-} = V_2^{o-} = 0$. Therefore,

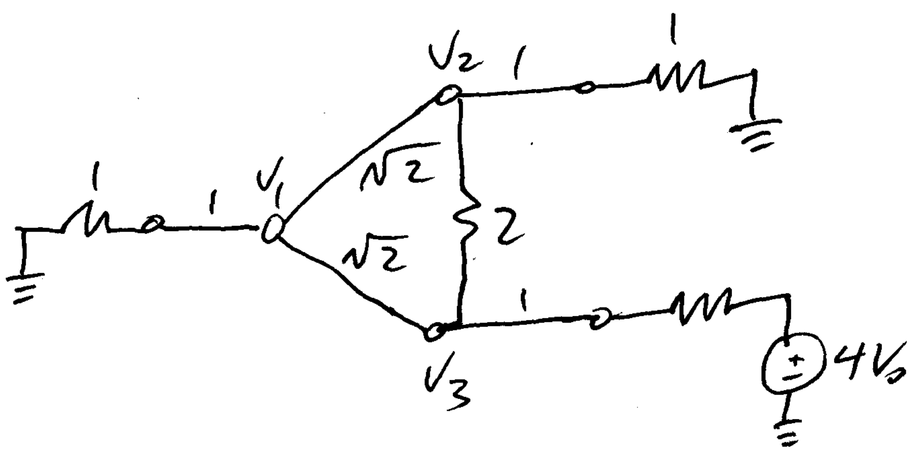
$$V_2 = V_2^e + V_2^o = V_2^+ = 2V_0$$

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{V_1^+ = V_3^+ = 0} = \frac{-jV_0\sqrt{2}}{2V_0}$$

$$S_{12} = S_{21} = \frac{-j}{\sqrt{2}}$$

7.3 cont.

Finding $S_{13} = S_{31}$, can be done by reversing the polarity of the sources in the odd mode analysis. This would lead to



$V_1 = -j V_0 \sqrt{2}$

$V_2 = 0$

$V_3 = 2V_0$

Therefore, following the same chain of reasoning leads to

$$\underline{\underline{S_{13} = S_{31} = \frac{-j}{\sqrt{2}}}}$$

Lastly, we find $S_{23} = S_{32}$.

$$S_{32} = \left.\frac{V_3^-}{V_2^+}\right|_{V_1^+ = V_3^+ = 0}$$

Using our even mode $V_2^{e-} = V_3^{e-}$

Using our odd mode $V_2^{o-} = -V_3^{o-}$

7.3 cont.

$$S_{32} = \frac{V_3^{e-} + V_3^{o-}}{V_2^{e+} + V_2^{o+}} = \frac{V_2^{e-} + (-V_2^{o-})}{V_2^{+}}$$

We earlier showed that $V_2^{+} = V_2 = 2V_0$. With matched input impedances for both even & odd modes, we get

$$S_{32} = \frac{0}{2V_0} \Rightarrow \underline{\underline{S_{32} = S_{23} = 0}}$$

$\Rightarrow$ ports 2 & 3 are isolated from each other!!

$$\boxed{[S]_{\substack{\text{Wilkinson}\\1:1}} = \begin{bmatrix} 0 & \frac{-j}{\sqrt{2}} & \frac{-j}{\sqrt{2}} \\ \frac{-j}{\sqrt{2}} & 0 & 0 \\ \frac{-j}{\sqrt{2}} & 0 & 0 \end{bmatrix}}$$

Much of this design relied on symmetry. It is very important to build these devices w/ as near perfect symmetry, i.e., mirror image, as possible.

7.3 cont.

example – Using the Rogers Corporation Microwave Impedance Calculator (MWI), design a 50Ω 1:1 Wilkinson power divider for use at 2.4 GHz. on a R4003C substrate that is 0.06" = 1.524 mm thick with 1oz copper cladding using microstrip.

From MWI, $Z_0 = 50\Omega \Rightarrow \underline{\underline{W = 3.475\text{ mm}}}$

$v_p = 1.784 \times 10^8 \text{ m/s}$

$\lambda = 7.4\overline{33}\text{ cm}$

$Z_{\lambda/4} = \sqrt{2}\,50 = 70.7107\,\Omega$

$\Rightarrow \underline{\underline{W = 1.8888\text{ mm}}}$

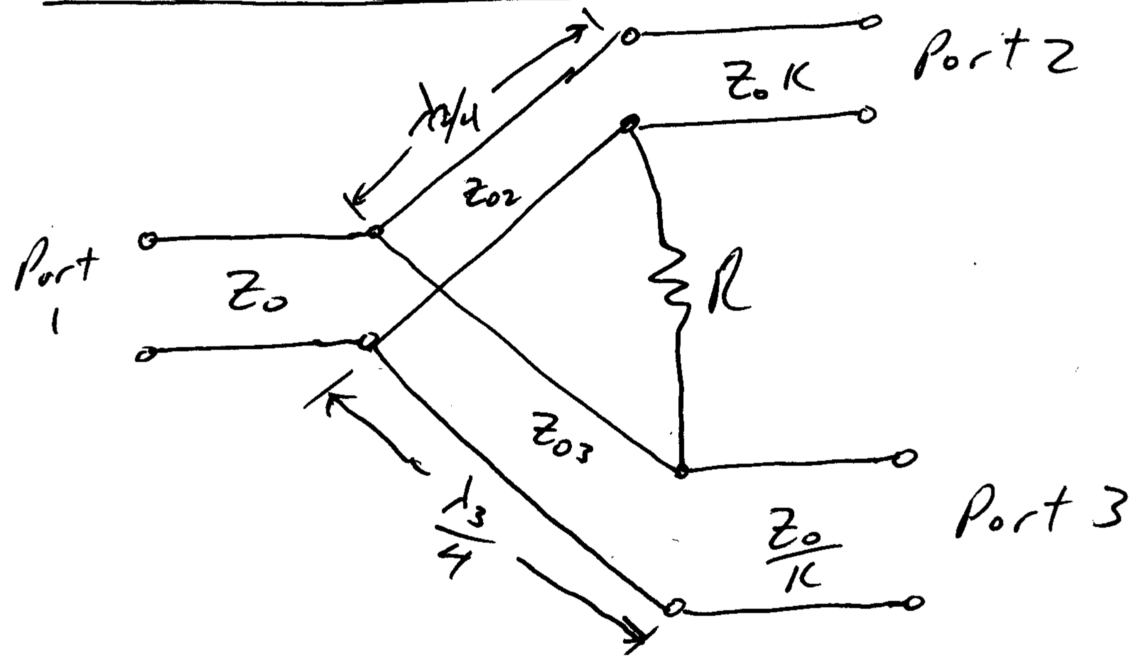
$v_p = 1.828 \times 10^8 \text{ m/s}$

$\lambda = 7.61\bar{6}\text{ cm} \Rightarrow \underline{\underline{\lambda/4 = 1.9041\bar{6}\text{ cm}}}$

$\underline{\underline{R = 2Z_0 = 100\,\Omega}}$

7.3 cont.

Unequal Power Division



For $\frac{P_3}{P_2} = K^2$, $Z_{03} = Z_0\sqrt{\frac{1+K^2}{K^3}}$ (7.37a)

$Z_{02} = K^2 Z_{03} = Z_0\sqrt{K(1+K^2)}$ (7.37b)

$R = Z_0\left(K + \frac{1}{K}\right)$ (7.37c)

Drawbacks – Ports 2 + 3 are NOT matched to Z_0 (unless $K=1$)

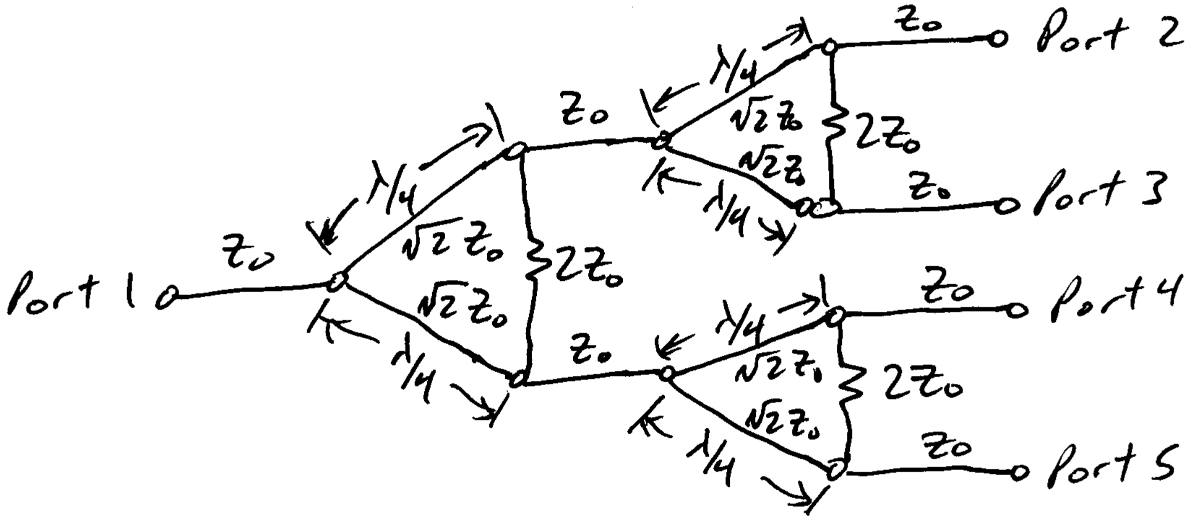
– will need impedance transformers to convert to Z_0 (narrows BW)

7.3 cont.

N-way Power Division

The easiest possibility is powers of 2, e.g., $2^2 = 4$, where we feed the outputs of one Wilkinson 1:1 power divider into others.

For example, a 4-way equal split using simplified drawing technique



⇒ Note, <u>three</u> 1:1 Wilkinson Power Dividers were used to achieve a 4-way split

⇒ Seven Wilkinson Power Dividers would be needed for a $2^3 = 8$-way split.

7.3 cont.

For an arbitrary N-way split ($N \geq 3$) Wilkinson Power Divider, the architecture shown in Figure 7.14 will provide a matched output (Z_0) for all output ports with isolation between the outputs.

However, note the resistors must crossover output TLs $\Rightarrow$ Harder to fabricate using microstrip or stripline.

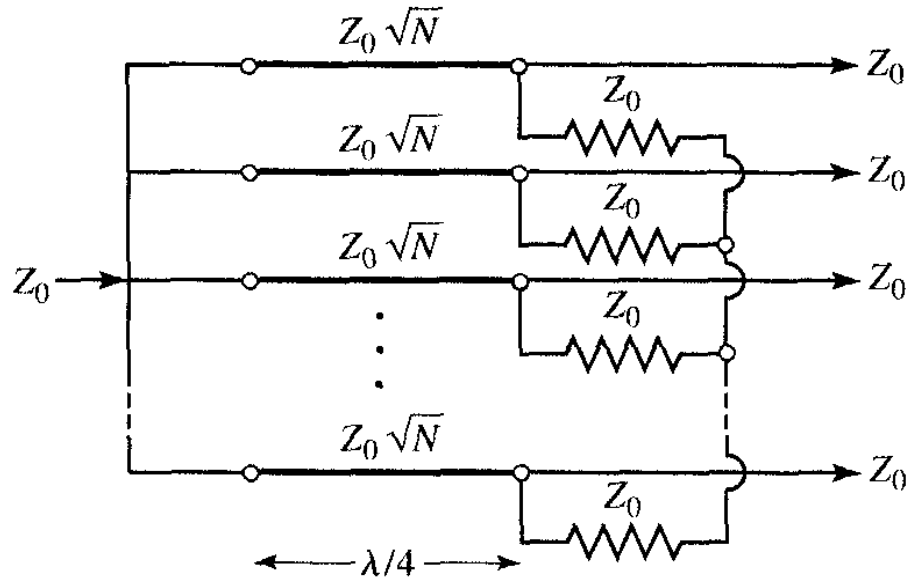


FIGURE 7.14 An N-way, equal-split Wilkinson power divider.

Microwave Engineering (Fourth Edition), Pozar, Wiley, 2012, ISBN 978-0-470-63155-3.

7.5 The Quadrature (90°) Hybrid

AKA: Quadrature coupler, Quad coupler
Branch-line hybrid, branch-line coupler

→ 3 dB directional coupler (i.e., 1:1 split at output ports) when ports are matched.

$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix} \qquad (7.61)$$

Note: $S_{11} = S_{22} = S_{33} = S_{44} = 0$ (all ports matched)

$$S_{21} = \frac{-j}{\sqrt{2}} \quad \& \quad S_{31} = \frac{-1}{\sqrt{2}}$$

⇒ even power division to output ports 2 & 3, but 90° (j) phase shift

$S_{41} = 0$ (port 4 isolated)

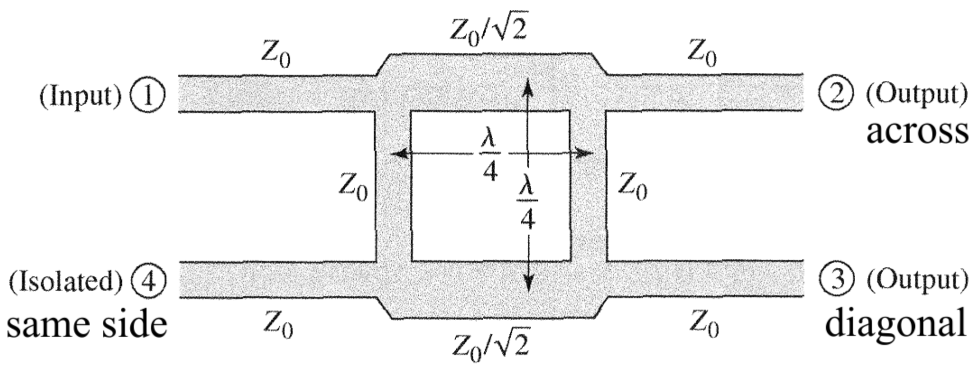


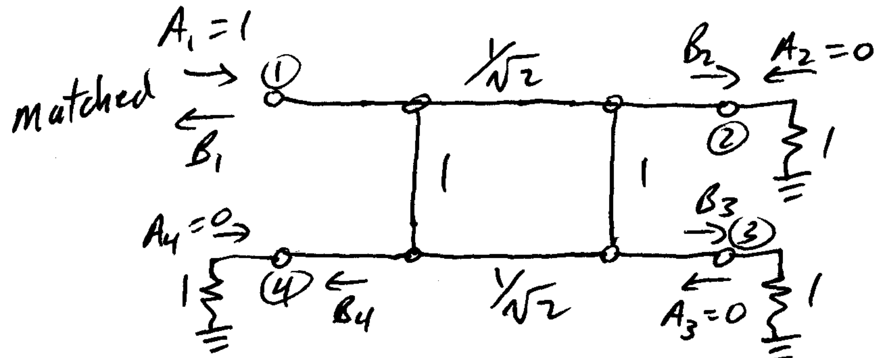
FIGURE 7.21 Geometry of a branch-line coupler.

Microwave Engineering (Fourth Edition), Pozar, Wiley, 2012, ISBN 978-0-470-63155-3.

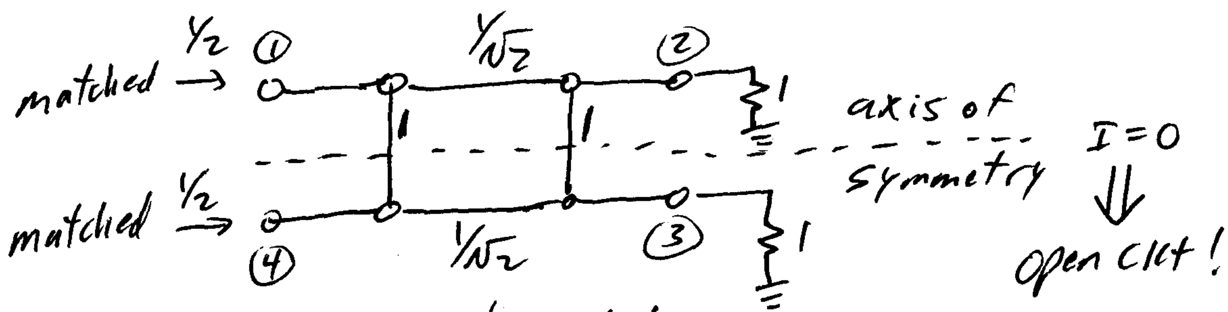
7.5 cont.

Even-Odd Mode Analysis

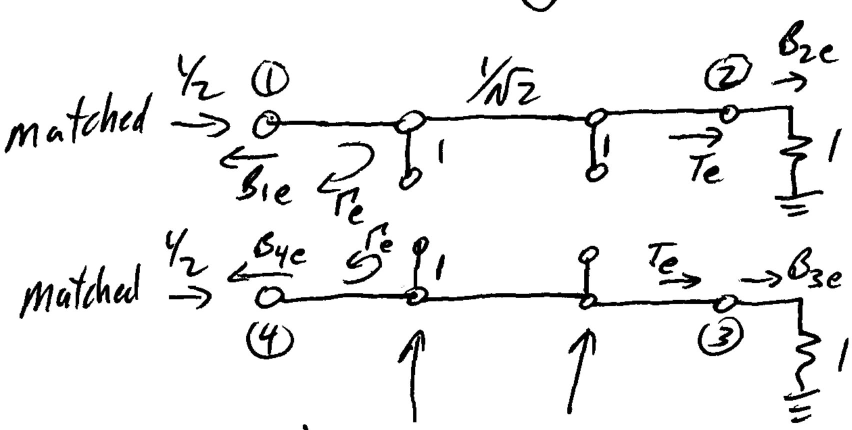
- → single line representation of TLs
- → normalize impedances by Z_0
- → Terminate ports 2, 3, + 4 w/ matched loads
- → Input wave w/ amplitude $A_1 = 1$ @ port 1



Even Mode



⇓ split



λ/8 open ckt stubs

$$B_{1e} = \Gamma_e (1/2)$$

$$B_{2e} = T_e (1/2)$$

$$B_{3e} = T_e (1/2)$$

$$B_{4e} = \Gamma_e (1/2)$$

7.5 cont.

Looking at the top half, we can model this as the cascade connection of three 2-ports, i.e., a shunt admittance, a $\lambda/4$-long $Z_0/\sqrt{2}$ TL, and another shunt admittance.

From Chapter 2, (2.46c) shows for an open circuit stub of length $\lambda/8$

$$Z_{in} = -j\, Z_0 \cot \beta l = -j\, Z_0 \cot\left(\frac{2\pi}{\lambda}\frac{\lambda}{8}\right)$$

$$= -j\, Z_0 \underbrace{\cot(\pi/4)}_{1} = -j\, Z_0$$

$$z_{in} = \frac{Z_{in}}{Z_0} = -j \Rightarrow y_{in} = \frac{1}{-j} = j$$

Per Table 4.1 Shunt Y

$$[ABCD]_Y = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

normalize

$$[ABCD]_{Y_{norm}} = \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix}$$

Per Table 4.1 Lossless TL

$\beta l = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \pi/2$

a $Z_0/\sqrt{2}$ or $Y_0\sqrt{2}$

$$[ABCD]_{TL} = \begin{bmatrix} \cos\beta l & j\frac{Z_0}{\sqrt{2}}\sin\beta l \\ j\sqrt{2}\sin\beta l & \cos\beta l \end{bmatrix}$$

$$[ABCD]_{TL_{norm}} = \begin{bmatrix} \cos\pi/2 & \frac{j}{\sqrt{2}}\sin\pi/2 \\ j\sqrt{2}\sin\pi/2 & \cos\pi/2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & j/\sqrt{2} \\ j\sqrt{2} & 0 \end{bmatrix}$$

7.5 cont.

$$[ABCD]_{e,norm} = \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix} \begin{bmatrix} 0 & j/\sqrt{2} \\ j\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & j/\sqrt{2} \\ j\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix} = \begin{bmatrix} \frac{-1}{\sqrt{2}} & j/\sqrt{2} \\ j\sqrt{2} - j/\sqrt{2} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & j \\ j & -1 \end{bmatrix} \qquad (7.63)$$

$\Downarrow$ convert to $[S]_e$

Per Table 4.2

$$S_{11_e} = \Gamma_e = \frac{\overset{\text{unitless}}{A} + \overset{\text{ohms}}{B/Z_0} - \overset{\text{siemens}}{C} Z_0 - \overset{\text{unitless}}{D}}{A + B/Z_0 + CZ_0 + D}$$

(matched ports)

$$= \frac{A + B - C - D}{A + B + C + D}$$ using normalized ABCD

$$= \frac{-1/\sqrt{2} + j/\sqrt{2} - j/\sqrt{2} + 1/\sqrt{2}}{-1/\sqrt{2} + j/\sqrt{2} + j/\sqrt{2} - 1/\sqrt{2}} = \frac{0}{-\sqrt{2} + j\sqrt{2}}$$

$$\underline{\Gamma_e = 0} \qquad (7.64a)$$

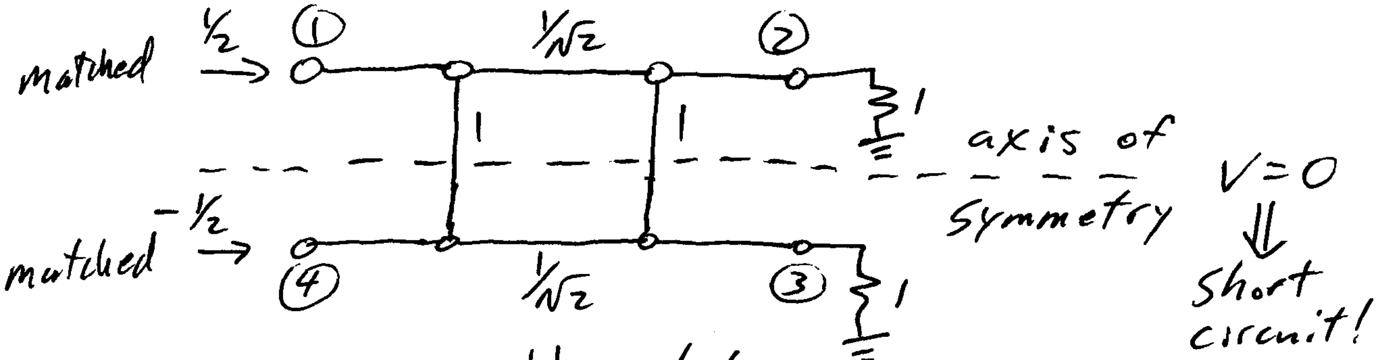
$$S_{21_e} = T_e = \frac{2}{A + B + C + D} = \frac{2}{-\sqrt{2} + j\sqrt{2}} = \frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$$

(matched ports)

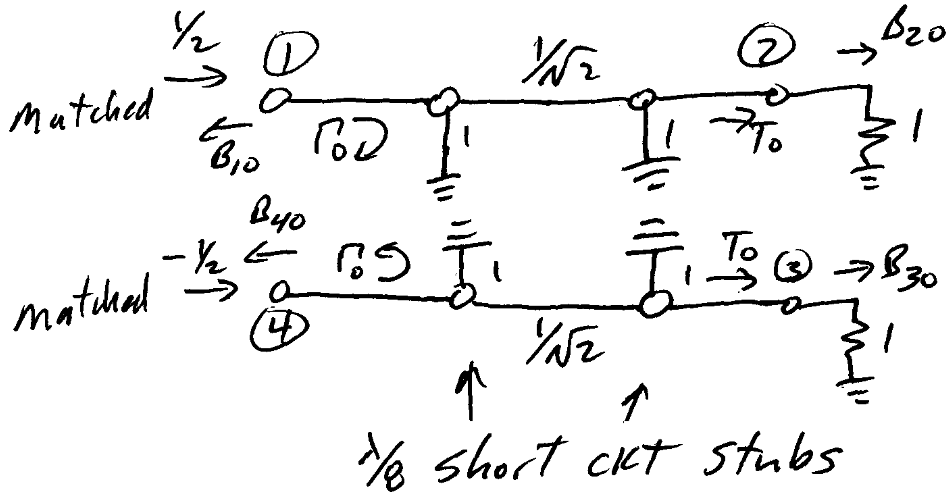
$$\underline{T_e = \frac{-1}{\sqrt{2}}(1 + j)} \qquad (7.64b)$$

7.5 cont.

Odd Mode



⇓ split



$$B_{10} = \Gamma_o(1/2)$$
$$B_{20} = T_o(1/2)$$
$$B_{30} = T_o(-1/2)$$
$$B_{40} = \Gamma_o(-1/2)$$

Here, each path is a cascade of three 2-ports, i.e., shunt admittance, $\lambda/4$-long $Z_0/\sqrt{2}$ TL, and shunt admittance.

From Chapter 2, (2.45c) shows for a short circuit stub of length $\lambda/8$

$$Z_{in} = jZ_0 \tan\beta l = jZ_0 \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{8}\right) = jZ_0$$

$$z_{in} = j \Rightarrow y_{in} = \frac{1}{z_{in}} = -j \Rightarrow [ABCD]_{Y_{norm}} = \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix}$$

7.5 cont.

The TL section is the same as for even mode. So, the overall odd mode $[ABCD]_o$ is

$$[ABCD]_o = \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} \begin{bmatrix} 0 & j/\sqrt{2} \\ j\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & j/\sqrt{2} \\ j\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & j/\sqrt{2} \\ j\sqrt{2} - j/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix} \quad (7.65)$$

$\Downarrow$ convert to $[S]_o$

Per Table 4.2,

w/ B + C normalized

$$S_{11_o} = \Gamma_o = \frac{A + B - C - D}{A + B + C + D} = \frac{1/\sqrt{2} + j/\sqrt{2} - j/\sqrt{2} - 1/\sqrt{2}}{1/\sqrt{2} + j/\sqrt{2} + j/\sqrt{2} + 1/\sqrt{2}}$$

(matched ports)

$$= \frac{0}{\sqrt{2} + j\sqrt{2}}$$

$$\underline{\Gamma_o = 0} \quad (7.66a)$$

$$S_{21_o} = T_o = \frac{2}{A + B + C + D} = \frac{2}{\sqrt{2} + j\sqrt{2}}$$

(matched ports)

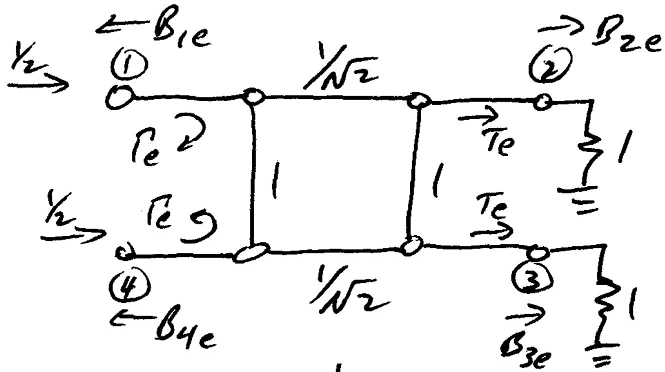
$$= \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

$$\underline{T_o = \frac{1}{\sqrt{2}}(1 - j)} \quad (7.66b)$$

7.5 cont.

Add even & odd mode circuits to get back our original circuit driven at port 1 w/ all other ports matched.

Even



$B_{1e} = \Gamma_e (1/2)$

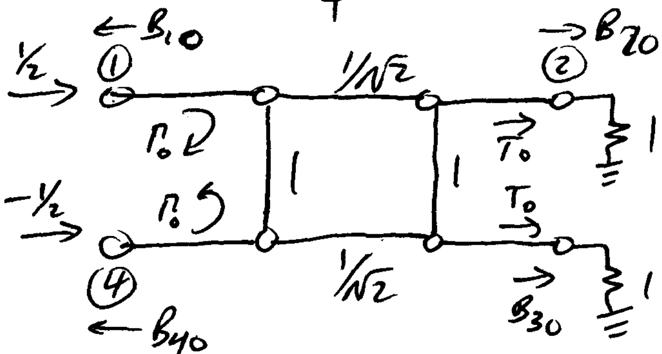
$B_{2e} = T_e (1/2)$

$B_{3e} = T_e (1/2)$

$B_{4e} = \Gamma_e (1/2)$

+

Odd



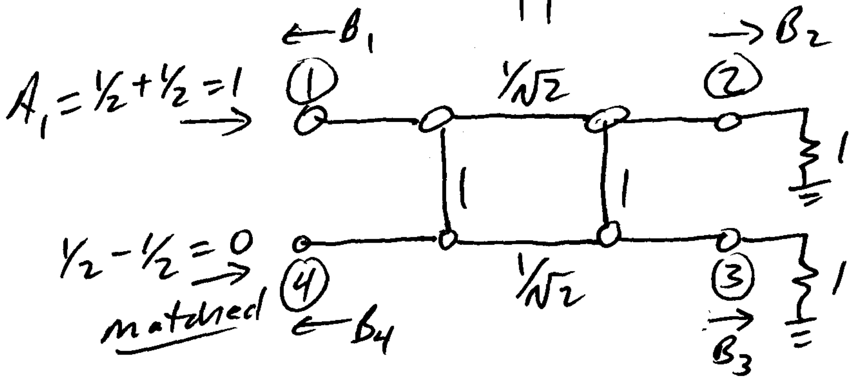
$B_{1o} = \Gamma_o (1/2)$

$B_{2o} = T_o (1/2)$

$B_{3o} = T_o (-1/2)$

$B_{4o} = \Gamma_o (-1/2)$

=



$$B_1 = B_{1e} + B_{1o} = \tfrac{1}{2}\Gamma_e + \tfrac{1}{2}\Gamma_o \quad (7.62a)$$

$$B_2 = B_{2e} + B_{2o} = \tfrac{1}{2}T_e + \tfrac{1}{2}T_o \quad (7.62b)$$

$$B_3 = B_{3e} + B_{3o} = \tfrac{1}{2}T_e - \tfrac{1}{2}T_o \quad (7.62c)$$

$$B_4 = B_{4e} + B_{4o} = \tfrac{1}{2}\Gamma_e - \tfrac{1}{2}\Gamma_o \quad (7.62d)$$

7.5 cont.

We can now use (7.64a), (7.64b), (7.66a) & (7.66b) to get

$$B_1 = \tfrac{1}{2}(0) + \tfrac{1}{2}(0) = 0 \qquad (7.67a)$$

$$B_2 = \tfrac{1}{2}\tfrac{-1}{\sqrt{2}}(1+j) + \tfrac{1}{2}\tfrac{1}{\sqrt{2}}(1-j) = \frac{-j}{\sqrt{2}} \qquad (7.67b)$$

$$B_3 = \tfrac{1}{2}\tfrac{-1}{\sqrt{2}}(1+j) - \tfrac{1}{2}\tfrac{1}{\sqrt{2}}(1-j) = \frac{-1}{\sqrt{2}} \qquad (7.67c)$$

$$B_4 = \tfrac{1}{2}(0) - \tfrac{1}{2}(0) = 0 \qquad (7.67d)$$

Since $A_1 = 1$ and ports 2, 3, & 4 are matched (i.e., $A_2 = A_3 = A_4 = 0$), for generalized [S]-parameters, we get:

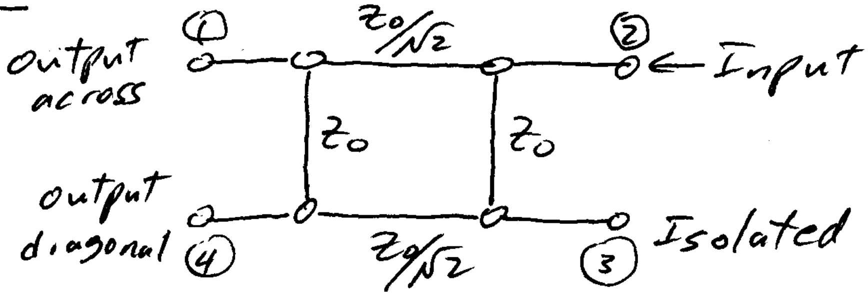
$$\left.\begin{aligned} S_{11} &= \frac{B_1}{A_1} = 0 \\ S_{21} &= B_2/A_1 = -j/\sqrt{2} \\ S_{31} &= B_3/A_1 = -1/\sqrt{2} \\ S_{41} &= B_4/A_1 = 0 \end{aligned}\right\}$$

First column of [S]-matrix! (7.61)

Given the perfect symmetry of the quad coupler, we can get the remaining columns of [S] by simply treating ports 2, 3, & 4 as the inputs in turn.

7.5 cont.

Column 2



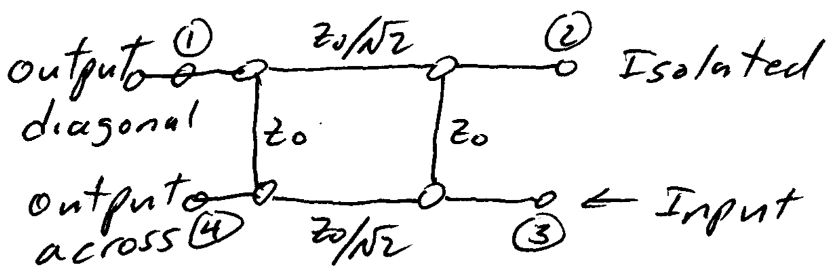
output across $S_{12} = S_{21} = -j/\sqrt{2}$

input $S_{22} = S_{11} = 0$

isolated $S_{32} = S_{41} = 0$

output diagonal $S_{42} = S_{31} = -1/\sqrt{2}$

Column 3



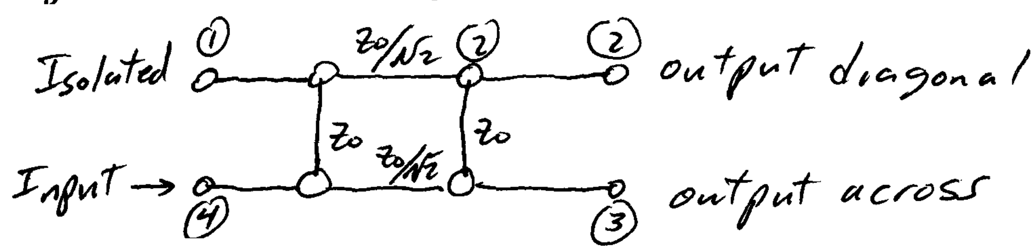
output diagonal $S_{13} = S_{31} = -1/\sqrt{2}$

Isolated (same side) $S_{23} = S_{41} = 0$

input $S_{33} = S_{11} = 0$

output across $S_{43} = S_{21} = -j/\sqrt{2}$

Column 4



Isolated $S_{14} = S_{41} = 0$

output diagonal $S_{24} = S_{31} = -1/\sqrt{2}$

output across $S_{34} = S_{21} = -j/\sqrt{2}$

input $S_{44} = S_{11} = 0$

7.5 cont.

What do these results mean?

Input $S_{11} = 0 \Rightarrow$ Port 1 is matched

Output across $S_{21} = \frac{-j}{\sqrt{2}} \Rightarrow |S_{21}|^2 = 1/2$ so half of power into port 1 goes to port 2

$\Rightarrow -j$ means there is a $-90°$ phase shift from port 1 to port 2 (or $90°$ phase delay)

Output diagonal $S_{31} = \frac{-1}{\sqrt{2}} \Rightarrow |S_{31}|^2 = 1/2$ so other half of power into port 1 goes to port 2

$\Rightarrow -1$ means there is a $-180°$ phase shift from port 1 to port 3 (or $180°$ phase delay)

$\Rightarrow$ Note there is a $90°$ difference between ports 2 and 3.

Isolated $S_{41} = 0 \Rightarrow$ No power to port 4, i.e., it is isolated!

Practical note – Quad coupler bandwidth is relatively narrow (10–20%) due to $\lambda/4$ lengths of branches.

7.5 cont.

example- Design a quad coupler for 100Ω microstrip system on Rogers 4003C substrate that is 0.032" = 0.812 mm thick for operation at 2.4 GHz w/ 1oz copper.

For $Z_0 = 100\Omega$, from Rogers Corp. MWI

$\underline{W = 0.4588 \text{ mm}}$

$v_p = 1.874 \times 10^8 \text{ m/s}$

$$\lambda = \frac{v_p}{f} = \frac{1.874 \times 10^8}{2.4 \times 10^9} = 7.808\overline{3} \text{ cm}$$

$\underline{\lambda/4 = 1.952 \text{ cm}}$

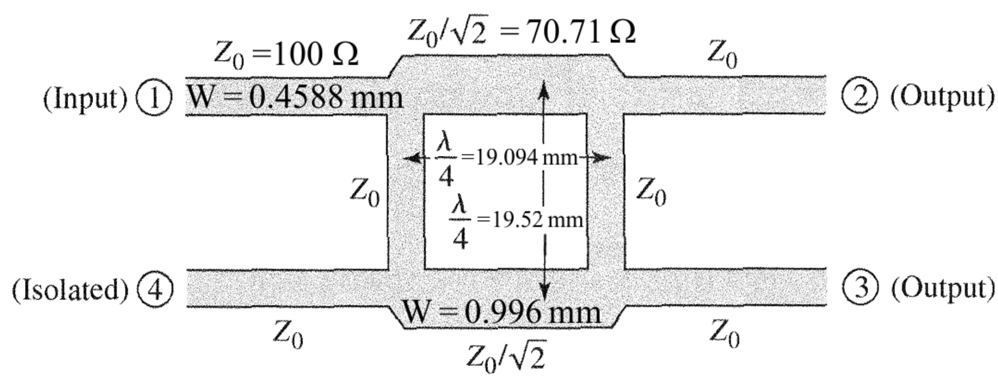
For $Z_0/\sqrt{2} = 100/\sqrt{2} = 70.7107\,\Omega$, using MWI

$\underline{W = 0.996 \text{ mm}}$

$v_p = 1.833 \times 10^8 \text{ m/s}$

$$\lambda = \frac{1.833 \times 10^8}{2.4 \times 10^9} = 7.6375 \text{ cm}$$

$\underline{\lambda/4 = 1.9094 \text{ cm}}$

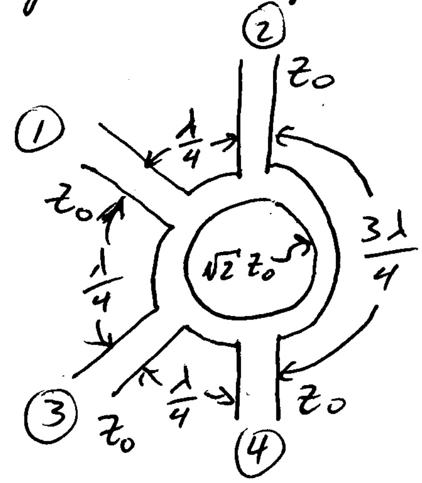


7.8 The 180° Hybrid

AKA: Ring hybrid or rat-race

→ Easy to construct w/ microstrip and stripline

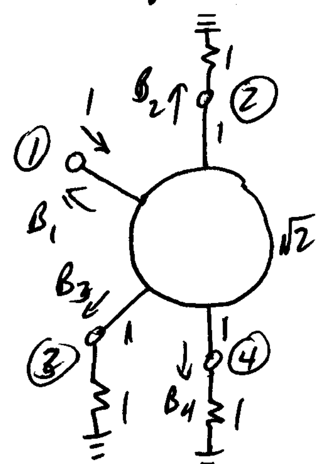
→ Versions available w/ waveguide (magic-T) and tapered coupled lines



Even-Odd Mode Analysis of the Ring Hybrid

→ Use one-line representation of TLs

→ Normalize impedances by Z_0.



$A_1 = 1$

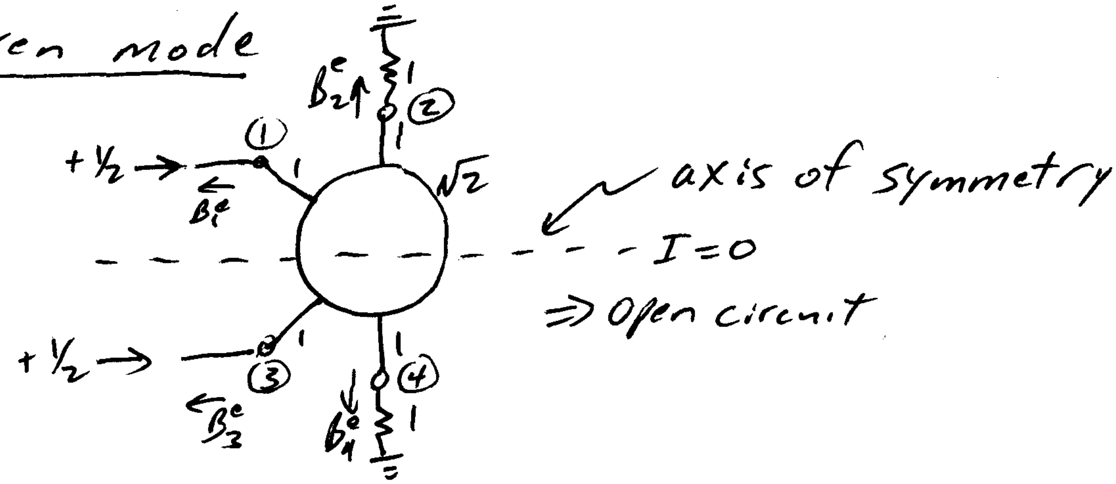
$A_2 = A_3 = A_4 = 0$
(matched)

Want to determine B_1, B_2, B_3, & B_4

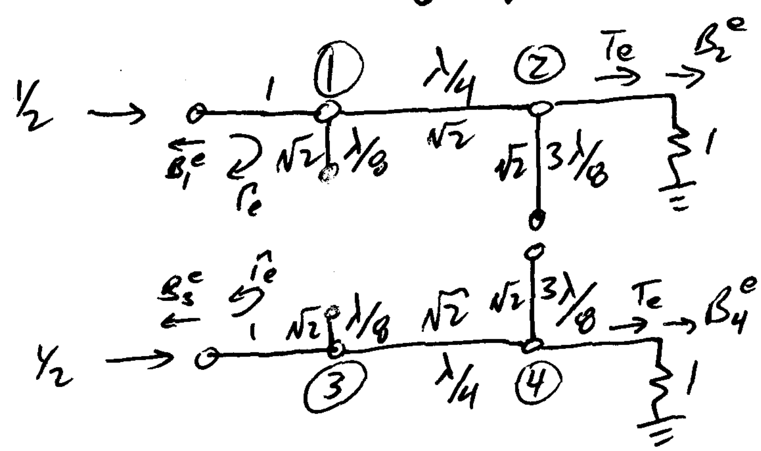
⇒ Driving port 1 while terminating other ports

7.8 cont.

Even mode



⇓ split along axis of symmetry



$$B_1^e = B_3^e = \tfrac{1}{2}\Gamma_e$$

$$B_2^e = B_4^e = \tfrac{1}{2}T_e$$

We will represent these even mode circuits with a cascade of three [ABCD]-matrices: shunt Y for $\lambda/8$-long $\sqrt{2}$ open circuit stub, lossless $\lambda/4$-long $\sqrt{2}$ TL, and shunt Y for $3\lambda/8$-long $\sqrt{2}$ open circuit stub.

Using (2.46c), we can find the input impendance for an open circuit stub

$$Z_{in} = -jZ_0 \cot \beta l$$

7.8 cont.

$\boxed{\ell = \lambda/8}$ $\quad z_{in} = -j\sqrt{2}\, Z_o \underbrace{\cot\left(\frac{2\pi}{\lambda}\,\lambda/8\right)}_{1} = -j\sqrt{2}\, Z_o$

$z_{in} = -j\sqrt{2}$ and $y_{in} = j/\sqrt{2}$

$\boxed{\ell = \frac{3\lambda}{8}}$ $\quad Z_{in} = -j\sqrt{2}\, Z_o \cot\left(\frac{2\pi}{\lambda}\,\frac{3\lambda}{8}\right) = -j\sqrt{2}\, Z_o \underbrace{\cot\left(\frac{3\pi}{4}\right)}_{-1}$

$= j\sqrt{2}\, Z_o$

$z_{in} = j\sqrt{2}$ and $y_{in} = \frac{-j}{\sqrt{2}}$

Per Table 4.1
Shunt Y $\quad [ABCD]_Y = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \Rightarrow [ABCD]_{Y_{norm}} = \begin{bmatrix} 1 & 0 \\ y & 1 \end{bmatrix}$

$$[ABCD]_{Y_{\lambda/8,\,norm,\ o.c.}} = \begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix}$$

$$[ABCD]_{Y_{\frac{3\lambda}{8},\,norm,\ o.c.}} = \begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix}$$

Per Table 4.1
$\lambda/4$-long $\sqrt{2}\, Z_o$
or $\frac{Y_o}{\sqrt{2}}$
lossless TL
$\beta\ell = \frac{2\pi}{\lambda}\,\lambda/4 = \pi/2$

$$[ABCD]_{TL} = \begin{bmatrix} \cos\beta\ell & jZ_o\sin\beta\ell \\ jY_o\sin\beta\ell & \cos\beta\ell \end{bmatrix}$$

$$[ABCD]_{TL_{norm}} = \begin{bmatrix} \cos\pi/2 & j\sqrt{2}\sin\pi/2 \\ \frac{j}{\sqrt{2}}\sin\pi/2 & \cos\pi/2 \end{bmatrix} = \begin{bmatrix} 0 & j\sqrt{2} \\ \frac{j}{\sqrt{2}} & 0 \end{bmatrix}$$

Overall,

$$[ABCD]^e_{norm} = [ABCD]_{Y_{\lambda/8,\,norm,\ o.c.}}\,[ABCD]_{TL_{norm}}\,[ABCD]_{Y_{\frac{3\lambda}{8},\,norm,\ o.c.}}$$

7.8 cont.

$$[ABCD]^{e}_{norm} = \begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix}$$

$$[ABCD]^{e}_{norm} = \begin{bmatrix} +1 & j\sqrt{2} \\ j\sqrt{2} & -1 \end{bmatrix}$$

$\Downarrow$ Convert to $[S]_e$ using Table 4.2

normalized

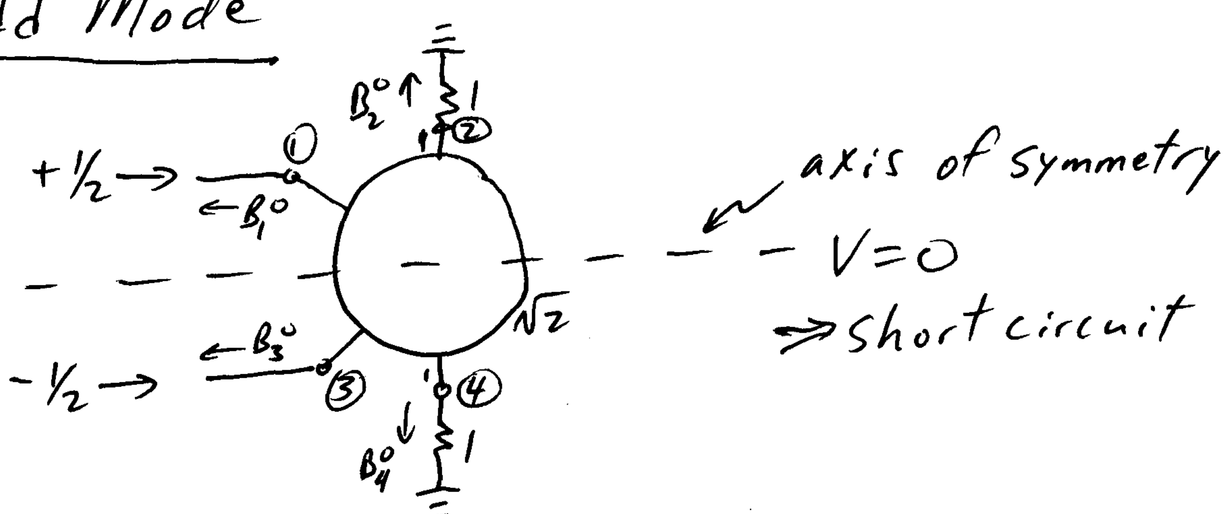
$$S_{11e} = \Gamma_e = \frac{A + B - C - D}{A + B + C + D} = \frac{1 + j\sqrt{2} - j\sqrt{2} + 1}{1 + j\sqrt{2} + j\sqrt{2} - 1} = \frac{2}{j2\sqrt{2}}$$

(matched ports)

$$\underline{\Gamma_e = -j/\sqrt{2}}$$

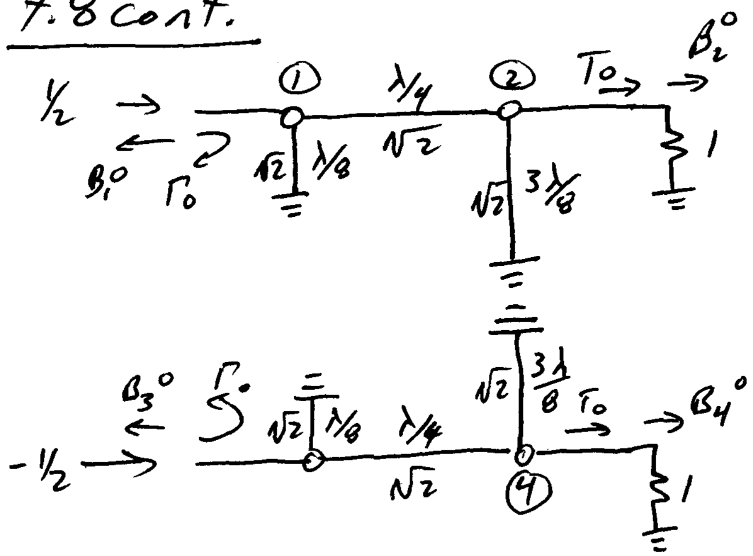
$$S_{21e} = T_e = \frac{2}{A+B+C+D} = \frac{2}{j2\sqrt{2}} \Rightarrow \underline{T_e = -j/\sqrt{2}}$$

Odd Mode



$\Downarrow$ split along axis of symmetry

7.8 cont.



$B_1^o = \frac{1}{2}\Gamma_o$

$B_2^o = \frac{1}{2} T_o$

$B_3^o = -\frac{1}{2}\Gamma_o$

$B_4^o = -\frac{1}{2} T_o$

Again, we will represent these odd mode circuits with a cascade of three [ABCD]-matrices: shunt Y for $\lambda/8$-long $\sqrt{2}$ short circuit stub, lossless $\lambda/4$-long $\sqrt{2}$ TL, and shunt Y for $3\lambda/8$-long $\sqrt{2}$ short circuit stub.

As before, $[ABCD]_{TL_{norm}} = \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix}$

From (2.45c) in chapter 2, a short circuit stub has input impedance

$$Z_{in} = j Z_o \tan\beta\ell$$

$\boxed{\ell = \lambda/8}$ $\quad Z_{in} = j\sqrt{2}\, Z_o \underbrace{\tan(\pi/4)}_{1} = j\sqrt{2}\, Z_o \Rightarrow z_{in} = j\sqrt{2}$

$y_{in} = -j/\sqrt{2}$

$\boxed{\ell = 3\lambda/8}$ $\quad Z_{in} = j\sqrt{2}\, Z_o \underbrace{\tan(3\pi/4)}_{-1} = -j\sqrt{2}\, Z_o \Rightarrow z_{in} = -j\sqrt{2}$

$y_{in} = +j/\sqrt{2}$

7.8 cont.

Using Table 4.1, our normalized shunt Y [ABCD]-matrices are:

$$[ABCD]_{Y_{\lambda/8}\,\text{norm}\atop \text{s.c.}} = \begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix}$$

$$[ABCD]_{Y_{3\lambda/8}\,\text{norm}\atop \text{s.c.}} = \begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix}$$

Overall,

$$[ABCD]^{o}_{norm} = [ABCD]_{Y_{\lambda/8}\,\text{norm}\atop \text{s.c.}} [ABCD]_{TL_{norm}} [ABCD]_{Y_{3\lambda/8}\,\text{norm}\atop \text{sc}}$$

$$= \begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix}\begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix}$$

$$[ABCD]^{o}_{norm} = \begin{bmatrix} -1 & j\sqrt{2} \\ j\sqrt{2} & 1 \end{bmatrix}$$

$\Downarrow$ convert to $[S]_o$ using Table 4.2

$$S_{11_o} = \Gamma_o = \frac{A+B-C-D}{A+B+C+D} = \frac{-1+j\sqrt{2}-j\sqrt{2}-1}{-1+j\sqrt{2}+j\sqrt{2}+1} = \frac{-2}{j2\sqrt{2}}$$

(matched ports)

$$\Leftrightarrow \underline{\Gamma_o = +j/\sqrt{2}}$$

$$S_{21_o} = T_o = \frac{2}{A+B+C+D} = \frac{2}{j2\sqrt{2}} \Rightarrow \underline{T_o = -j/\sqrt{2}}$$

(matched ports)

7.8 cont.

Once again, adding our even- and odd-mode circuits yields the original circuit driven at port 1 w/ matched terminations at ports 2, 3, & 4. Therefore,

$$B_1 = B_1^e + B_1^o = \tfrac{1}{2}\Gamma_e + \tfrac{1}{2}\Gamma_o \quad (7.102a)$$

$$B_2 = B_2^e + B_2^o = \tfrac{1}{2}T_e + \tfrac{1}{2}T_o \quad (7.102b)$$

$$B_3 = B_3^e + B_3^o = \tfrac{1}{2}\Gamma_e - \tfrac{1}{2}\Gamma_o \quad (7.102c)$$

$$B_4 = B_4^e + B_4^o = \tfrac{1}{2}T_e - \tfrac{1}{2}T_o \quad (7.102d)$$

where we have found $\Gamma_e = {-j}/{\sqrt{2}}$ (7.104a), $T_e = {-j}/{\sqrt{2}}$ (7.104b), $\Gamma_0 = {j}/{\sqrt{2}}$ (7.104c), and $T_o = {-j}/{\sqrt{2}}$ (7.104d). This yields

$$B_1 = \tfrac{1}{2}\left({-j}/{\sqrt{2}}\right) + \tfrac{1}{2}\left({+j}/{\sqrt{2}}\right) = 0 \quad (7.105a)$$

$$B_2 = \tfrac{1}{2}\left({-j}/{\sqrt{2}}\right) + \tfrac{1}{2}\left({-j}/{\sqrt{2}}\right) = {-j}/{\sqrt{2}} \quad (7.105b)$$

$$B_3 = \tfrac{1}{2}\left({-j}/{\sqrt{2}}\right) - \tfrac{1}{2}\left({j}/{\sqrt{2}}\right) = {-j}/{\sqrt{2}} \quad (7.105c)$$

$$B_4 = \tfrac{1}{2}\left({-j}/{\sqrt{2}}\right) - \tfrac{1}{2}\left({-j}/{\sqrt{2}}\right) = 0 \quad (7.105d)$$

Since $A_1 = 1$ and $A_2 = A_3 = A_4 = 0$ w/ ports 2, 3, & 4 matched, we can find the generalized [S]-parameters for the first column.

7.8 cont.

$$S_{11} = B_1/A_1 = \frac{0}{1} = 0$$

$$S_{21} = B_2/A_1 = \frac{-j/\sqrt{2}}{1} = -j/\sqrt{2}$$

$$S_{31} = B_3/A_1 = \frac{-j/\sqrt{2}}{1} = -j/\sqrt{2}$$

$$S_{41} = B_4/A_1 = 0/1 = 0$$

We can use the symmetry of the Ring Hybrid to get column 3 of the [S]-matrix

1 → 3 $\qquad$ 4 → 2 $\qquad$ $A_1 \to A_3 = 1$

3 → 1 $\qquad$ 2 → 4 $\qquad$ $A_1 = A_2 = A_4 = 0$

$$S_{13} = \frac{\cancel{B_1} \to B_3}{A_3} = \frac{-j/\sqrt{2}}{1} = -j/\sqrt{2}$$

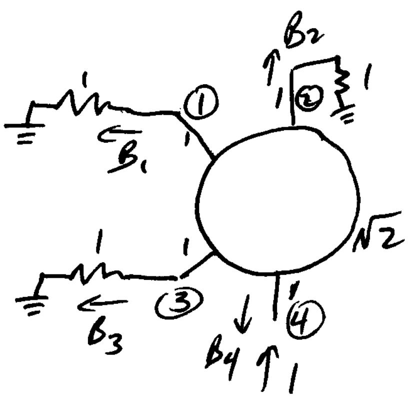
$$S_{23} = \frac{\cancel{B_2} \to B_4}{A_3} = \frac{0}{1} = 0$$

$$S_{33} = \frac{\cancel{B_3} \to B_1}{A_3} = \frac{0}{1} = 0$$

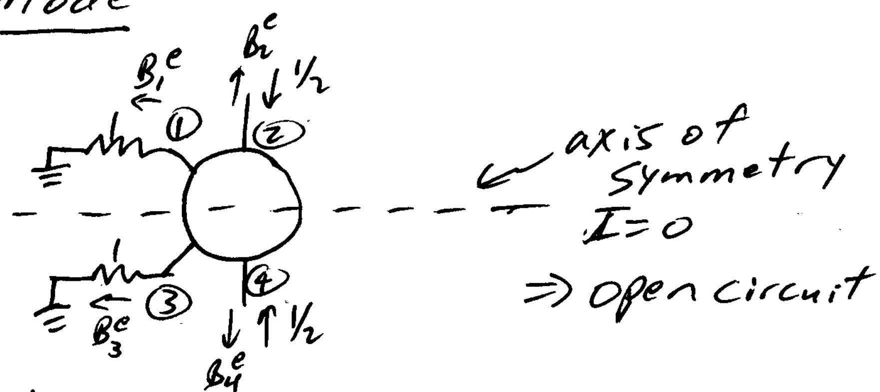
$$S_{43} = \frac{\cancel{B_4} \to B_2}{A_3} = \frac{-j/\sqrt{2}}{1} = -j/\sqrt{2}$$

We still need columns 2 & 4 for the [S]-matrix. To get them, drive port 4 with a wave of 1 while terminating others,

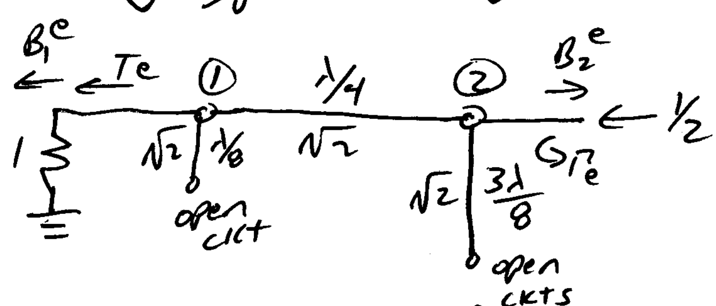
7.8 cont.



Even Mode

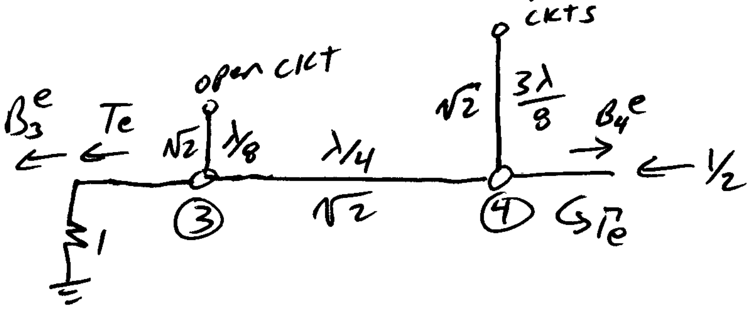


$\Downarrow$ split along axis of symmetry



$B_1^e = \frac{1}{2} T_e$

$B_2^e = \frac{1}{2} \Gamma_e$



$B_3^e = \frac{1}{2} T_e$

$B_4^e = \frac{1}{2} \Gamma_e$

We will represent each path using a cascade of [ABCD]-matrices: Shunt Y for $\sqrt{2}$ $3\lambda/8$-long open circuit stub, lossless $\sqrt{2}$ $\lambda/4$ TL, and $\sqrt{2}$ $\lambda/8$-long open circuit stub.

7.8 cont.

This time, the overall [ABCD]-matrix is

$$[ABCD]^{e}_{norm} = [ABCD]_{Y_{\frac{3\lambda}{8}} \, norm \, oc} \; [ABCD]_{TL_{norm}} \; [ABCD]_{Y_{\lambda/8} \, norm \, o.c}$$

$$= \begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix}$$

$$[ABCD]^{e}_{norm} = \begin{bmatrix} -1 & j\sqrt{2} \\ j\sqrt{2} & 1 \end{bmatrix} \qquad (7.107a)$$

$\Downarrow$ convert to $[S]_e$ using Table 4.2

$$S_{44e} = \Gamma_e = \frac{A+B-C-D}{A+B+C+D} = \frac{-1+j\sqrt{2}-j\sqrt{2}-1}{-1+j\sqrt{2}+j\sqrt{2}+1}$$

(matched ports)

$$= \frac{-2}{j2\sqrt{2}} \Rightarrow \Gamma_e = \frac{+j}{\sqrt{2}} \quad (7.108a)$$

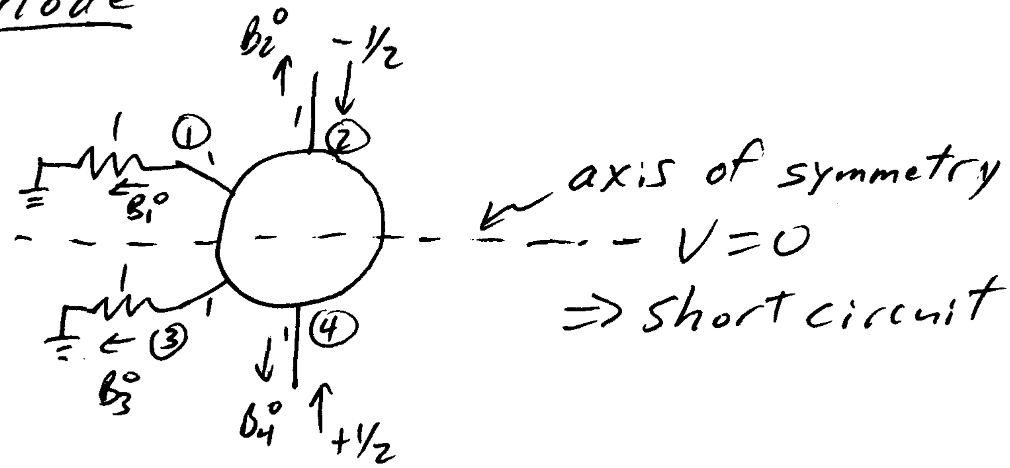
$$S_{34e} = T_e = \frac{2}{A+B+C+D} = \frac{2}{j2\sqrt{2}} \Rightarrow T_e = \frac{-j}{\sqrt{2}}$$

(matched ports) (7.108b)

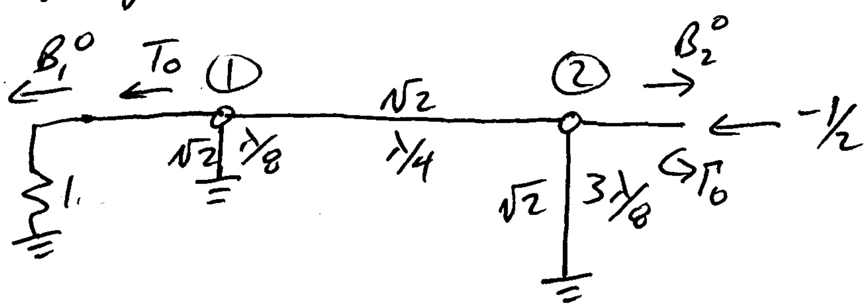
Note: Used the S_{11} formula for S_{44e} as port 4 is the input and the S_{21} formula for S_{34e} as port 3 is the output w/ port 4 input.

7.8 cont.

Odd Mode

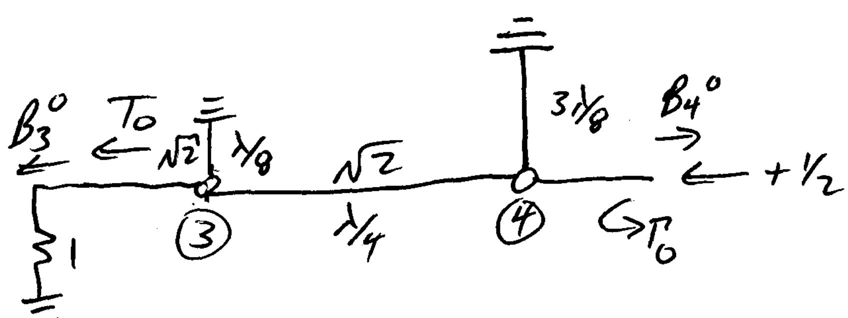


⇓ Split along axis of symmetry



$B_1^o = -\frac{1}{2}\Gamma_o$

$B_2^o = -\frac{1}{2}\Gamma_o$



$B_3^o = \frac{1}{2}\Gamma_o$

$B_4^o = \frac{1}{2}\Gamma_o$

Represent each path using a cascade of [ABCD]-matrices: Shunt Y for $\sqrt{2}$ $3\lambda/8$-long short circuit stub, lossless $\sqrt{2}$ $\lambda/4$-long TL, and $\sqrt{2}$ $\frac{\lambda}{8}$-long short circuit stub. Can 'recycle' previous matrices.

7.8 cont.

$$[ABCD]^{o}_{norm} = [ABCD]_{Y_{\frac{3\lambda}{8}\,norm\,SC}} [ABCD]_{TL_{norm}} [ABCD]_{Y_{\frac{\lambda}{8}\,norm\,S.C}}$$

$$= \begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix}$$

$$[ABCD]^{o}_{norm} = \begin{bmatrix} 1 & j\sqrt{2} \\ j\sqrt{2} & -1 \end{bmatrix} \quad (7.107b)$$

$\Downarrow$ convert to $[S]_o$ using Table 4.2

$$S_{44_o} = \Gamma_o = \frac{A+B-C-D}{A+B+C+D} = \frac{1+j\sqrt{2}-j\sqrt{2}+1}{1+j\sqrt{2}+j\sqrt{2}-1} = \frac{2}{j2\sqrt{2}}$$

$$\Rightarrow \underline{\Gamma_o = \frac{-j}{\sqrt{2}}} \quad (7.108c)$$

$$S_{34_o} = T_o = \frac{2}{A+B+C+D} = \frac{2}{j2\sqrt{2}}$$

$$\Rightarrow \underline{T_o = \frac{-j}{\sqrt{2}}} \quad (7.108d)$$

7.8 cont.

Again, add the even- and odd-mode circuits to get the original circuit driven at port 4 w/ matched terminations at ports 1, 2, & 3.

Therefore,

$$\mathcal{B}_1 = \mathcal{B}_1^e + \mathcal{B}_1^o = \tfrac{1}{2}T_e - \tfrac{1}{2}T_o \quad (7.106a)$$

$$\mathcal{B}_2 = \mathcal{B}_2^e + \mathcal{B}_2^o = \tfrac{1}{2}\Gamma_e - \tfrac{1}{2}\Gamma_o \quad (7.106b)$$

$$\mathcal{B}_3 = \mathcal{B}_3^e + \mathcal{B}_3^o = \tfrac{1}{2}T_e + \tfrac{1}{2}T_o \quad (7.106c)$$

$$\mathcal{B}_4 = \mathcal{B}_4^e + \mathcal{B}_4^o = \tfrac{1}{2}\Gamma_e + \tfrac{1}{2}\Gamma_o \quad (7.106d)$$

where $\Gamma_e = j/\sqrt{2}$ (7.108a), $T_e = -j/\sqrt{2}$ (7.108b),
$\Gamma_o = -j/\sqrt{2}$ (7.108c), and $T_o = -j/\sqrt{2}$ (7.108d).

Therefore

$$\mathcal{B}_1 = \tfrac{1}{2}\left(-j/\sqrt{2}\right) - \tfrac{1}{2}\left(-j/\sqrt{2}\right) = 0 \quad (7.109a)$$

$$\mathcal{B}_2 = \tfrac{1}{2}\left(j/\sqrt{2}\right) - \tfrac{1}{2}\left(-j/\sqrt{2}\right) = j/\sqrt{2} \quad (7.109b)$$

$$\mathcal{B}_3 = \tfrac{1}{2}\left(-j/\sqrt{2}\right) + \tfrac{1}{2}\left(-j/\sqrt{2}\right) = -j/\sqrt{2} \quad (7.109c)$$

$$\mathcal{B}_4 = \tfrac{1}{2}\left(j/\sqrt{2}\right) + \tfrac{1}{2}\left(-j/\sqrt{2}\right) = 0 \quad (7.109d)$$

7.8 cont.

Since $A_4 = 1$ and $A_1 = A_2 = A_3 = 0$ (ports 1, 2, & 3 are matched), we can find the generalized [S]-parameters for column 4 as

$$S_{14} = \frac{B_1}{A_4} = \frac{0}{1} = 0$$

$$S_{24} = \frac{B_2}{A_4} = \frac{j/\sqrt{2}}{1} = j/\sqrt{2}$$

$$S_{34} = B_3/A_4 = \frac{-j/\sqrt{2}}{1} = -j/\sqrt{2}$$

$$S_{44} = B_4/A_4 = 0/1 = 0$$

Using symmetry, we can get column 2 of the [S]-matrix

(4) → (2) (3) → (1) so $A_2 = A_4 = 1$

(2) → (4) (1) → (3) $A_1 = A_3 = A_4 = 0$

$$S_{12} = \frac{\cancel{B_1} \to B_3}{A_2} = \frac{-j/\sqrt{2}}{1} = -j/\sqrt{2}$$

$$S_{22} = \frac{\cancel{B_2} \to B_4}{A_2} = \frac{0}{1} = 0$$

$$S_{32} = \frac{\cancel{B_3} \to B_1}{A_2} = \frac{0}{1} = 0$$

$$S_{42} = \frac{\cancel{B_4} \to B_2}{A_2} = \frac{j/\sqrt{2}}{1} = j/\sqrt{2}$$

7.8 cont.

Putting our results together, the [S]-matrix for our ring hybrid is

$$\boxed{[S] = \frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \qquad (7.101)}$$

How can we intepret these results?

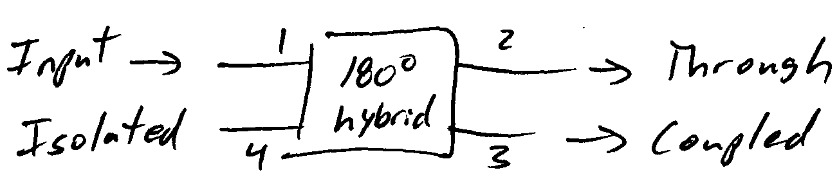
① Driving port 1 (Input). The ring hybrid acts as a 1:1 splitter (in-phase) when remaining ports are matched.

$S_{11} \Rightarrow$ Port 1 is matched

$S_{21} = S_{31} = \frac{-j}{\sqrt{2}} \Rightarrow |S_{21}|^2 = |S_{31}|^2 = \frac{1}{2}$ each gets half the input power

$\Rightarrow$ Ports 2 + 3 are in-phase, both experience $-90°$ phase shift ($90°$ phase delay)

$S_{41} = 0 \Rightarrow$ Port 4 is isolated



7.8 cont.

(2) Driving port 4 (Input). The ring hybrid acts as a 1:1 splitter (out-of-phase) when remaining ports are matched.

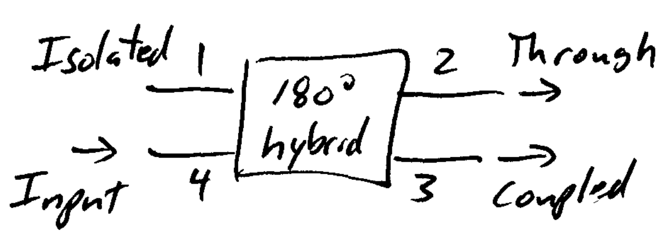
$S_{14} = 0 \Rightarrow$ Port 1 is isolated

$S_{24} = \frac{+j}{\sqrt{2}} \Rightarrow |S_{24}|^2 = \frac{1}{2}$ port 2 gets half of input power

$S_{34} = \frac{-j}{\sqrt{2}} \Rightarrow |S_{34}|^2 = \frac{1}{2}$ port 3 gets half of input power

$\Rightarrow +j - (-j) \Rightarrow 90° + 90° = 180°$
ports 2 + 3 are 180° out-of-phase with respect to each other

$S_{44} = 0 \Rightarrow$ Port 4 is matched



7.8 cont.

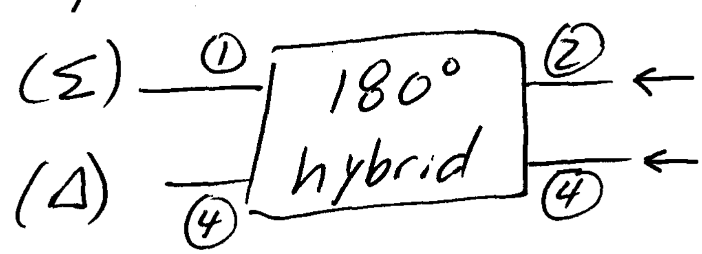
(3) Putting input signals into ports 2 + 3 results in a <u>power combiner</u> at port 1 (sum) and a difference signal at port 4.

$S_{22} = S_{33} = 0 \Rightarrow$ Ports 2 + 3 are matched

$S_{12} = S_{13} = 1 \Rightarrow$ Ports 2 + 3 signals both arrive @ port 1 + are <u>in-phase</u> $\Rightarrow$ Sum (Σ)

$S_{42} = -1$ while $S_{43} = 1 \Rightarrow$ Get a difference (Δ) signal since they are 180° out-of-phase

To annotate these properties, the ring hybrid symbol is:



Practical note – Due the $\lambda/4$ and $\frac{3\lambda}{4}$ section lengths, the bandwidth of ring hybrids is typically ~20-30%

7.6 Coupled Line Directional Couplers

→ If two unshielded TLs are close to one another power & signals can/will be coupled from one to the other ⇒ coupled TLs.

→ Not an issue for coaxial TLs as they are shielded.

→ Can be an issue (or a blessing) for microstrip, co-planar waveguide, stripline. Some examples are shown below.

→ For closely spaced lands/traces on PCBs (analog &/or digital, this can be bad (cross talk)

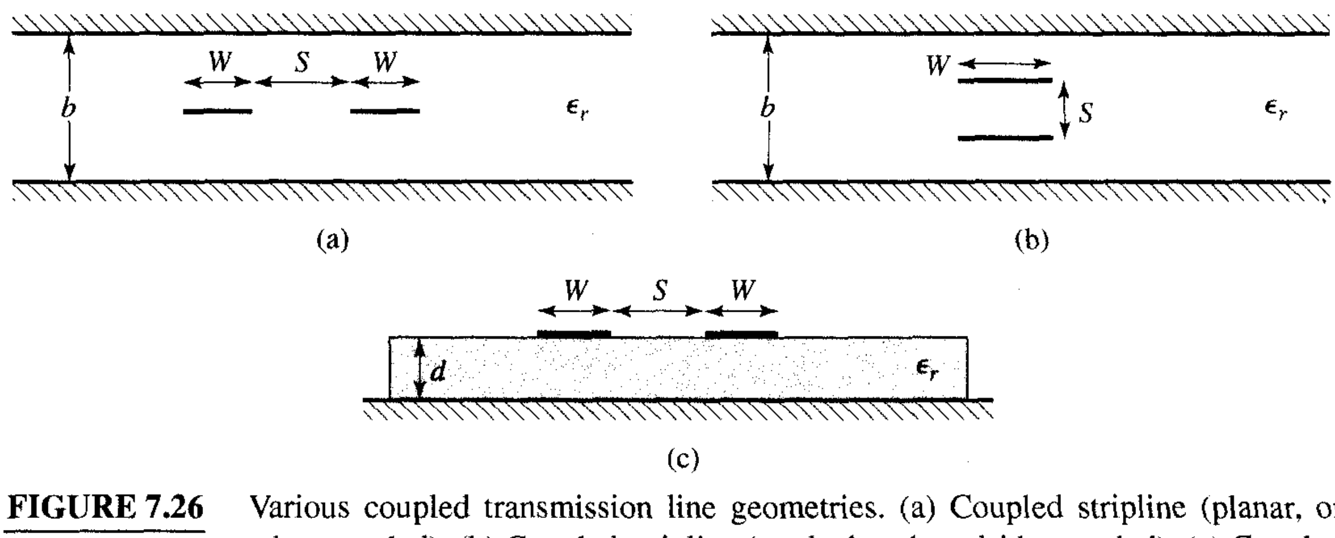


FIGURE 7.26 Various coupled transmission line geometries. (a) Coupled stripline (planar, or edge-coupled). (b) Coupled stripline (stacked, or broadside-coupled). (c) Coupled microstrip lines.

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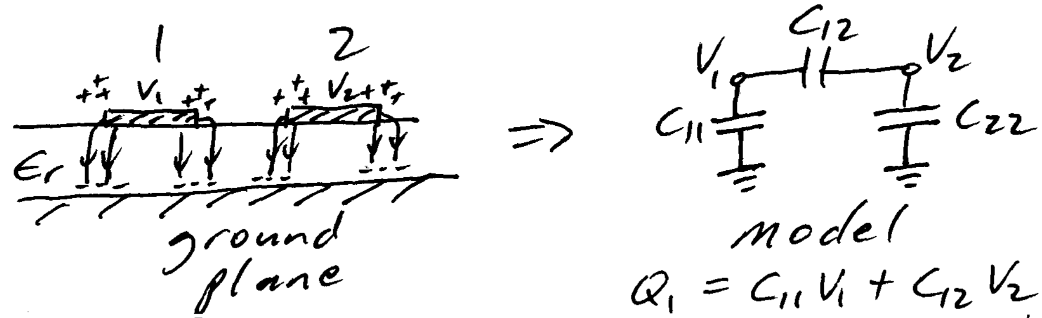
7.6 cont.

Coupled Line Theory

Let's consider the geometry of Fig 7.26c as an example. Assuming some voltages are applied to the conductors, charge distributions will be induced ⇒ electric fields ⇒ capacitances



$$Q_1 = C_{11}V_1 + C_{12}V_2$$
$$Q_2 = C_{21}V_1 + C_{22}V_2$$

→ C_{11} ≡ capacitance of conductor 1 w/ conductor 2 present but grounded

→ C_{22} ≡ capacitance of conductor 2 w/ conductor 1 present but grounded

→ C_{12} ≡ mutual capacitance between conductors 1 & 2, except for very unusual circumstance, i.e., anisotropic materials, $C_{12} = C_{21}$.

* Typically, $C_{11} = C_{22}$ as the conductors are often identical.

7.6 cont.

Assuming TEM or quasi-TEM modes, the impedance is $Z_0 = \sqrt{L/C} = \frac{\sqrt{LC}}{C} = \frac{1}{v_p C}$

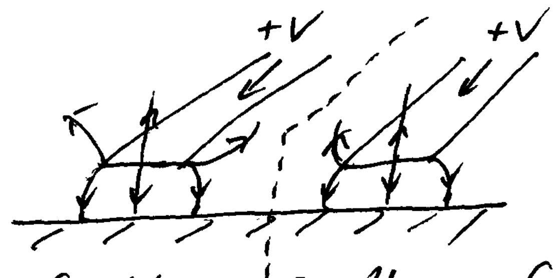
⇒ We need capacitance and wave velocity, don't need L.

⇒ TEM/quasi-TEM, $v_p = \frac{c}{\sqrt{\epsilon_{r,e}}}$ and $\beta = \omega / v_p$.

Even-Odd Mode Analysis

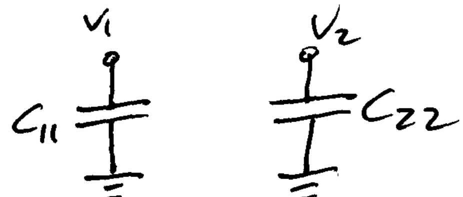
→ Assume identical conductors

Even Mode



Note: No electric field lines between the conductors.

⇒ Two TLs/traces are decoupled.



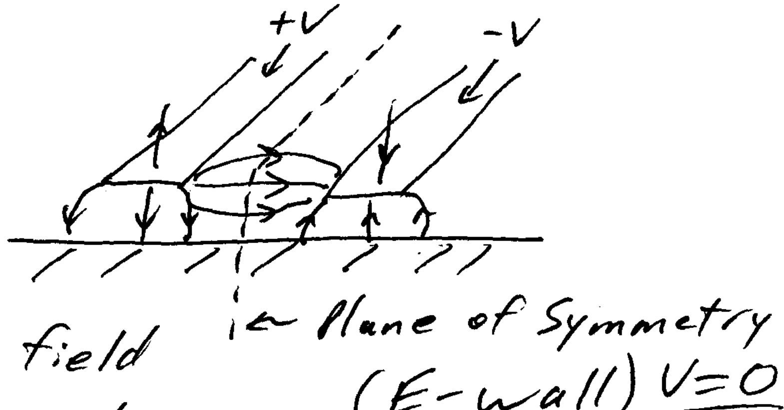
No C_{12}! (7.68)

Let $C_{11} = C_{22} = C_e$. Then, the even-mode characteristic impedance is

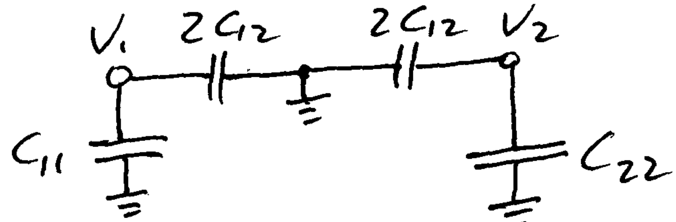
$$Z_{oe} = \frac{1}{v_{pe} C_e} \qquad (7.69)$$

7.6 cont.

Odd Mode



Here, the electric field lines are orthogonal or perpendicular to the plane of symmetry w/ lots of coupling between the TLs/traces.



(Remember $\left[\frac{1}{2C_{12}} + \frac{1}{2C_{12}}\right]^{-1} = C_{12}$, series)

The odd mode capacitance is then

$$C_o = C_{11} + 2C_{12} \qquad (7.70)$$

for each as they are in parallel.

This yields an odd mode characteristic impedance

$$Z_{0o} = \frac{1}{v_{po} C_o} \qquad (7.71)$$

⇒ For a pure TEM wave (stripline),

$$v_{pe} = v_{po} = v_p = \frac{c}{\sqrt{\epsilon_r}}.$$

⇒ For microstrip, $v_{pe} \neq v_{po}$ and $\epsilon_{re} \neq \epsilon_{ro}$ as different percentanges of the electric field lines are in air + dielectric.

7.6 cont.

How do we get C_e and C_o?

⇒ Hard problem. Can be done analytically for TLs that are pure TEM (i.e., very symmetrical). Otherwise, numerical modeling w/ CAD packages are most often used.

⇒ Alternative, use design curves/graphs

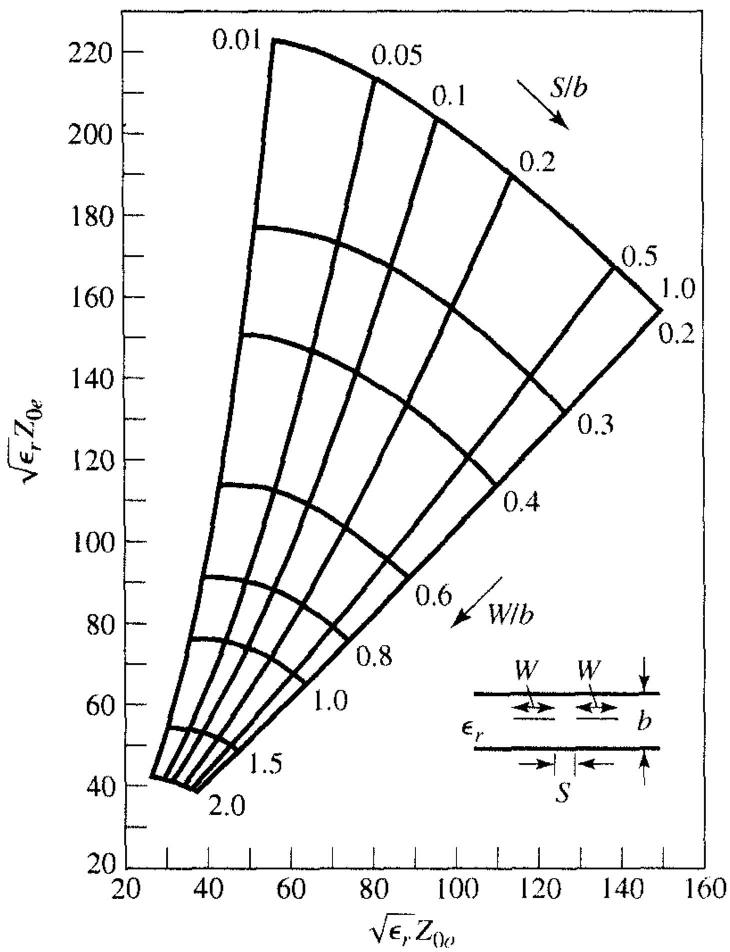


FIGURE 7.29 Normalized even- and odd-mode characteristic impedance design data for symmetric edge-coupled striplines.

Microwave Engineering (Fourth Edition), Pozar, Wiley, 2012, ISBN 978-0-470-63155-3.

⇒ Note, we can scale to any ϵ_r.

7.6 cont.

For microstrip, life is more complicated as we need to give a specific ϵ_r

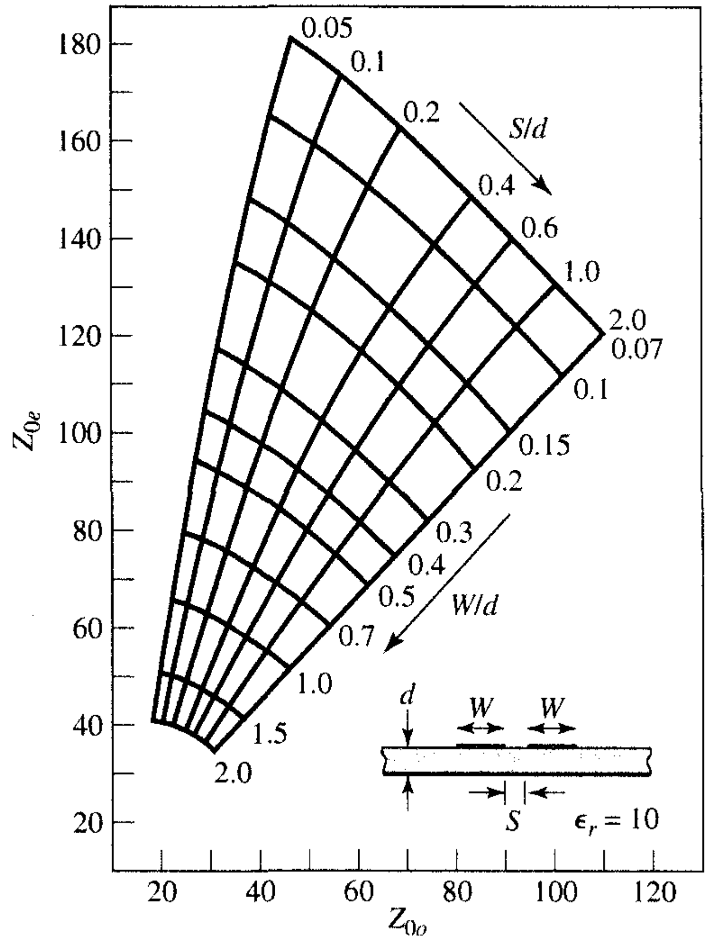


FIGURE 7.30 Even- and odd-mode characteristic impedance design data for symmetric coupled microstrip lines on a substrate with $\epsilon_r = 10$.

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Design of Coupled Line Couplers

→ A coupled line directional coupler is shown below. Port 1 is matched and driven w/ $2V_0$. Ports 2, 3, & 4 are terminated w/ Z_0

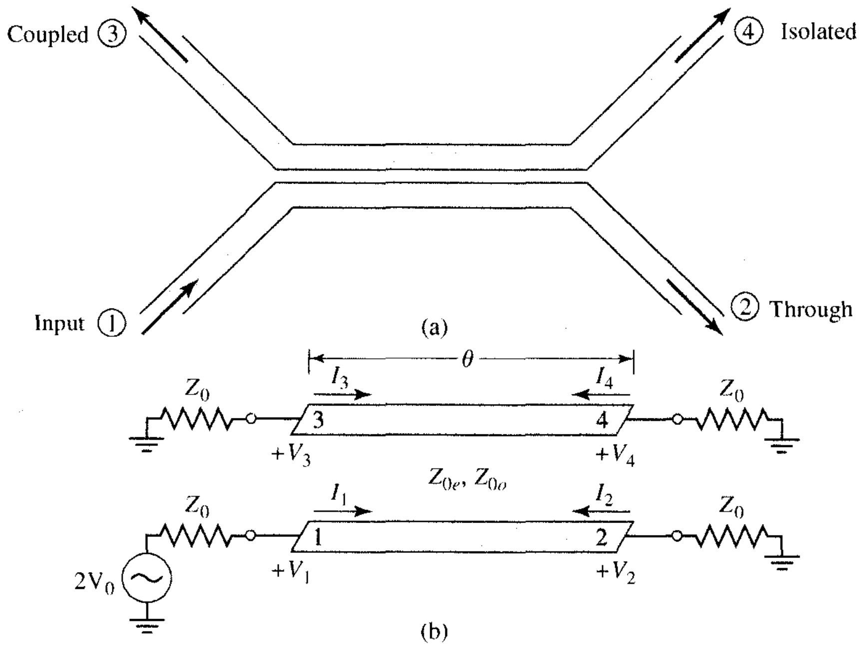

FIGURE 7.31 A single-section coupled line coupler. (a) Geometry and port designations. (b) The schematic circuit.

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Even Mode

→ Drive both ports 1 & 3 w/ matched V_0 sources.

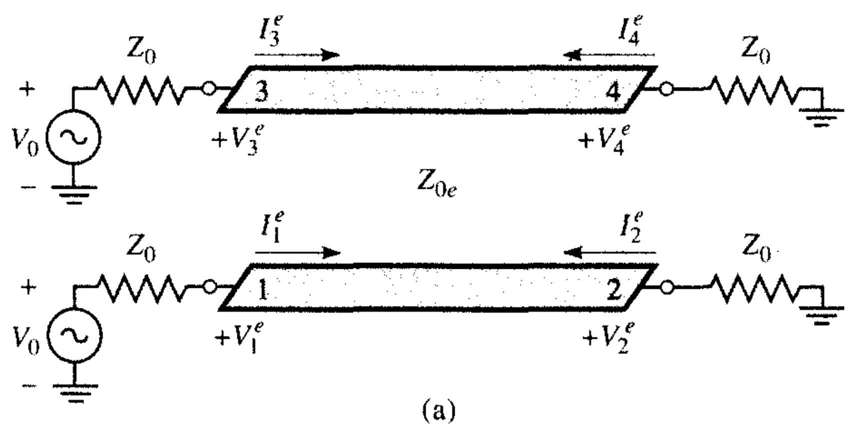

FIGURE 7.32 Decomposition of the coupled line coupler circuit of Figure 7.31 into even- and odd-mode excitations. (a) Even mode.

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* By symmetry, $I_1^e = I_3^e$ & $I_2^e = I_4^e$ for currents and $V_1^e = V_3^e$ & $V_2^e = V_4^e$ for voltages.

* Looking into port 1 and using (2.44) w/ $Z_L = Z_0$, $Z_0 = Z_{0e}$, and $\beta l = \theta$, we get

$$Z_{in}^e = Z_{0e} \frac{Z_0 + jZ_{0e} \tan\theta}{Z_{0e} + jZ_0 \tan\theta} \qquad (7.73a)$$

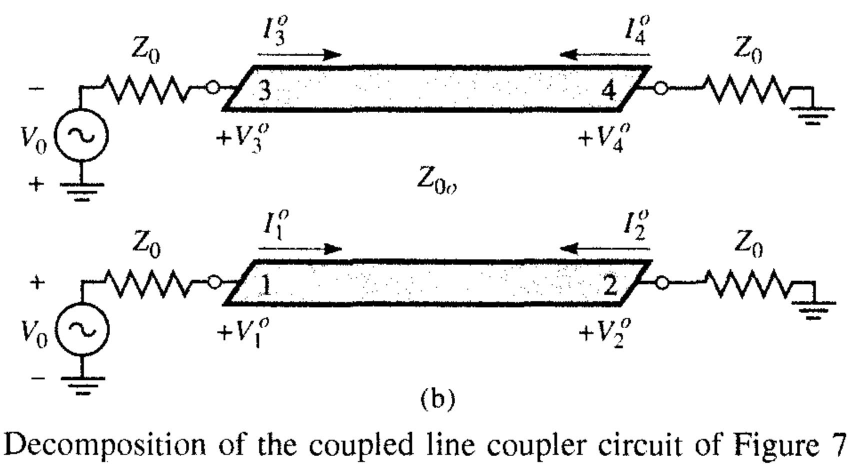
By voltage division, we can get V_1^e as

$$V_1^e = V_0 \frac{Z_{in}^e}{Z_0 + Z_{in}^e} \qquad (7.74b)$$

and current $I_1^e = \dfrac{V_0}{Z_0 + Z_{in}^e}$ (7.75b)

Odd Mode

→ Drive port 1 w/ $+V_0$ and port 3 w/ $-V_0$ matched sources.



(b)

FIGURE 7.32 Decomposition of the coupled line coupler circuit of Figure 7.31 into even- and odd-mode excitations. (b) Odd mode.

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* By symmetry, $I_1^o = -I_3^o$ & $I_4^o = -I_2^o$ and $V_1^o = -V_3^o$ & $V_4^o = -V_2^o$.

* Looking into port 1 and using (2.44) w/ $Z_L = Z_0$, $Z_0 = Z_{0o}$, and $\beta \ell = \theta$, we get

$$Z_{in}^o = Z_{0o} \frac{Z_0 + jZ_{0o}\tan\theta}{Z_{0o} + jZ_0\tan\theta}, \qquad (7.73b)$$

$$V_1^o = V_0 \frac{Z_{in}^o}{Z_0 + Z_{in}^o}, \text{ and} \qquad (7.74b)$$

$$I_1^o = \frac{V_0}{Z_0 + Z_{in}^o}. \qquad (7.75a)$$

7.6 cont.

The overall input impedance at port 1 is $Z_{in} = \frac{V_1}{I_1} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o}$ (7.72)

sub in V_1^e, V_1^o, I_1^e, and I_1^o

$$Z_{in} = \frac{\frac{V_0 Z_{in}^e}{Z_0 + Z_{in}^e} + \frac{V_0 Z_{in}^o}{Z_0 + Z_{in}^o}}{\frac{V_0}{Z_0 + Z_{in}^e} + \frac{V_0}{Z_0 + Z_{in}^o}} \times \frac{(Z_0 + Z_{in}^e)(Z_0 + Z_{in}^o)}{(Z_0 + Z_{in}^e)(Z_0 + Z_{in}^o)}$$

$$= \frac{Z_{in}^e (Z_0 + Z_{in}^o) + Z_{in}^o (Z_0 + Z_{in}^e)}{(Z_0 + Z_{in}^o) + (Z_0 + Z_{in}^e)}$$

add in

$$= \frac{Z_0 (Z_{in}^e + Z_{in}^o + 2Z_0) - 2Z_0^2 + 2 Z_{in}^e Z_{in}^o}{Z_{in}^e + Z_{in}^o + 2 Z_0}$$

$$Z_{in} = Z_0 + \frac{2 (Z_{in}^e Z_{in}^o - Z_0^2)}{Z_{in}^e + Z_{in}^o + 2 Z_0} \quad (7.76)$$

<u>Choose</u> $Z_0 = \sqrt{Z_{0e} Z_{0o}}$. (7.77)

This makes (7.73a) & (7.73b)

$$Z_{in}^e = Z_{0e} \frac{\sqrt{Z_{0o}} + j\sqrt{Z_{0o}} \tan\theta}{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}} \tan\theta}$$

and

$$Z_{in}^o = Z_{0o} \frac{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}} \tan\theta}{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}} \tan\theta}$$

7.6 cont.

Now, when we multiply

$$Z_{in}^e \, Z_{in}^o = \left(Z_{oe} \frac{\sqrt{Z_{oo}} + j\sqrt{Z_{oe}} \tan\theta}{\sqrt{Z_{oe}} + j\sqrt{Z_{oo}} \tan\theta} \right) \left(Z_{oo} \frac{\sqrt{Z_{oe}} + j\sqrt{Z_{oo}} \tan\theta}{\sqrt{Z_{oo}} + j\sqrt{Z_{oe}} \tan\theta} \right)$$

$$Z_{in}^e \, Z_{in}^o = Z_{oe} Z_{oo} = Z_o^2$$

Putting this into (7.76)

$$Z_{in} = Z_o + \frac{2(Z_o^2 - Z_o^2)}{Z_{in}^e + Z_{in}^o + 2Z_o}$$

$$Z_{in} = Z_o \qquad (7.78)$$

By symmetry, so long as we have $Z_o = \sqrt{Z_{oe} Z_{oo}}$, <u>all</u> the ports are matched to Z_o!

Going back to the original circuit w/ port 1 driven by a $2V_o$ matched source, $V_1 = 2V_o \dfrac{\cancel{Z_{in}}^{Z_o}}{Z_o + \cancel{Z_{in}}^{Z_o}} = V_o$

At port 3, using our even & odd mode analysis

$$V_3 = V_3^e + V_3^o = V_1^e - V_1^o = V_o \left[\frac{Z_{in}^e}{Z_o + Z_{in}^e} - \frac{Z_{in}^o}{Z_o + Z_{in}^o} \right] \quad (7.79)$$

7.6 cont.

Substitute (7.73a) into $\dfrac{Z_{in}^e}{Z_{in}^e + Z_0}$ to get

$$\frac{Z_{in}^e}{Z_{in}^e + Z_0} = \frac{Z_{oe}\dfrac{Z_0 + jZ_{oe}\tan\theta}{Z_{oe} + jZ_0\tan\theta}}{Z_{oe}\dfrac{Z_0 + jZ_{oe}\tan\theta}{Z_{oe} + jZ_0\tan\theta} + Z_0}$$

$$= \frac{Z_{oe}Z_0 + jZ_{oe}^2\tan\theta}{Z_{oe}Z_0 + jZ_{oe}^2\tan\theta + Z_0Z_{oe} + jZ_0^2\tan\theta}$$

$$= \frac{Z_0 + jZ_{oe}\tan\theta}{2Z_0 + j\left(\dfrac{Z_{oe}^2 + \overset{Z_{oe}Z_{oo}}{Z_0^2}}{Z_{oe}}\right)\tan\theta}$$

$$= \frac{Z_0 + jZ_{oe}\tan\theta}{2Z_0 + j(Z_{oe} + Z_{oo})\tan\theta}$$

Similarly, $\dfrac{Z_{in}^o}{Z_{in}^o + Z_0} = \dfrac{Z_0 + jZ_{oo}\tan\theta}{2Z_0 + j(Z_{oe} + Z_{oo})\tan\theta}$

Using these expressions in (7.79), we get

$$V_3 = V_0\,\frac{j(Z_{oe} - Z_{oo})\tan\theta}{2Z_0 + j(Z_{oe} + Z_{oo})\tan\theta} \qquad (7.80)$$

Define coupling coefficient $\equiv C = \dfrac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}}$ (7.81)

(AKA: Voltage Coupling Coefficient)

7.6 cont.

We can easily show $\sqrt{1-C^2} = \frac{2Z_0}{Z_{oe}+Z_{oo}}$.

Re-arranging (7.80) by dividing top & bottom by $(Z_{oe}+Z_{oo})$

$$V_3 = V_0 \frac{j\frac{Z_{oe}-Z_{oo}}{Z_{oe}+Z_{oo}}\tan\theta}{\frac{2Z_0}{Z_{oe}+Z_{oo}}+j\tan\theta} = V_0 \frac{jC\tan\theta}{\sqrt{1-C^2}+j\tan\theta} \quad (7.82)$$

At port 2, from even-odd mode, we get

$$V_2 = V_2^e + V_2^o$$

$\Downarrow$ leading to

$$V_2 = V_0 \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta} \quad (7.84)$$

and $V_4 = V_4^e + V_4^o = V_2^e - V_2^o$

$\Downarrow$ leading to

$$V_4 = 0 \quad (7.83)$$

$\Rightarrow$ Port 4 is isolated!

Next, for some value of $C < 1$, plot $\left|\frac{V_2}{V_0}\right|^2$ and $\left|\frac{V_3}{V_0}\right|^2$ versus $\theta = \beta l$ as shown in Fig 7.33.

7.6 cont.

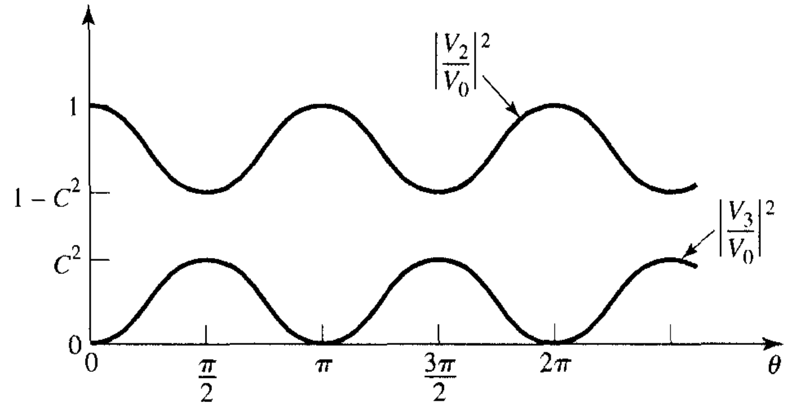


FIGURE 7.33 Coupled and through port voltages (squared) versus frequency for the coupled line coupler of Figure 7.31.

Microwave Engineering (Fourth Edition), Pozar, Wiley, 2012, ISBN 978-0-470-63155-3.

* Note, at low frequencies where $\beta l = \theta \to 0$, $\left|\frac{V_2}{V_0}\right|^2 \to 1$ and $\left|\frac{V_3}{V_0}\right|^2 \to 0$ which implies that all the power goes from port 1 to port 2 while no power goes to ports 3 & 4 (no coupling!)

* However, when $\beta l = \pi/2$ (i.e., $l = \lambda/4$) or odd multiples of $\pi/2$, we see that $\left|\frac{V_2}{V_0}\right|^2 = 1 - C^2$ and $\left|\frac{V_3}{V_0}\right|^2 = C^2$. Or, using (7.82) + (7.84), w/ $\theta = \pi/2$, we get

$$\frac{V_3}{V_0} = C \quad (7.85)$$

⇐ why we call "C" the voltage coupling factor or coefficient

and

$$\frac{V_2}{V_0} = -j\sqrt{1-C^2} \quad (7.86)$$

Note: $V_1/V_0 = 1$ and $V_4/V_0 = 0$ (unchanged).

7.6 cont.

w/ $\theta = \pi/2$ (or $\frac{3\pi}{2}, \ldots$), we see from (7.85) & (7.86) that we have a coupler w/ a $90°$ phase difference between ports 3 & 2

$\Rightarrow$ makes sense since phase delay from port 1 to 2 is ~0 while we have a $\pi/2 = 90°$ delay going down the $\lambda/4$ long TL from port 1 to port 2.

Practical note: C is typically small since large C would require very closely spaced traces/TLs $\Rightarrow$ hard to fabricate accurately.

From (7.77) $Z_0 = \sqrt{Z_{oe} Z_{oo}}$ and

(7.81) $C = \dfrac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}}$, we can

show that $$Z_{oe} = Z_0 \sqrt{\frac{1+C}{1-C}} \quad (7.87a)$$

$$Z_{oo} = Z_0 \sqrt{\frac{1-C}{1+C}} \quad (7.87b)$$

(Design equations!!)

Example- Design a nominal 16 dB (16.02 dB) coupler using symmetric 50 Ω coupled microstrip TLs on Rogers TM10i board ($\varepsilon_r \sim 10$, d = 2.54 mm = 0.1", 1 oz.) for 2.4 GHz.

$Z0 := 50\ \Omega$ $\quad d := 2.54\cdot 10^{-3}\ \text{m}$ $\quad f := 2.4\cdot 10^{9}\ \text{Hz}$

$CdB := 16.02$ $\quad C := 10^{\frac{-CdB}{20}}$ $\quad \boxed{C = 0.158125}$ $\quad$ % power $\quad C^2\cdot 100 = 2.5$

(7.87a) even-mode char. impedance $\quad Z0e := Z0\cdot\sqrt{\dfrac{1+C}{1-C}}$ $\quad \boxed{Z0e = 58.644}\ \Omega$

(7.87b) odd-mode char. impedance $\quad Z0o := Z0\cdot\sqrt{\dfrac{1-C}{1+C}}$ $\quad \boxed{Z0o = 42.63}\ \Omega$

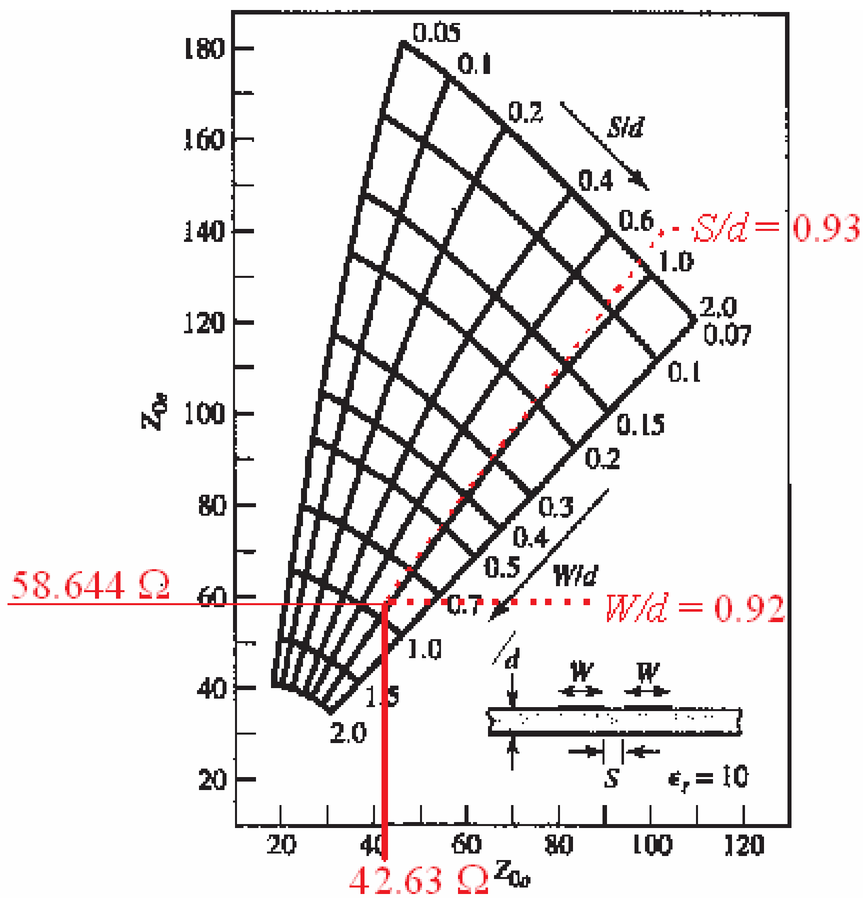


FIGURE 7.30 Even- and odd-mode characteristic impedance design data for symmetric coupled microstrip lines on a substrate with $\epsilon_r = 10$.

From Fig. 7.30 $\quad Sd := 0.93$ $\quad Wd := 0.92$

Example cont.

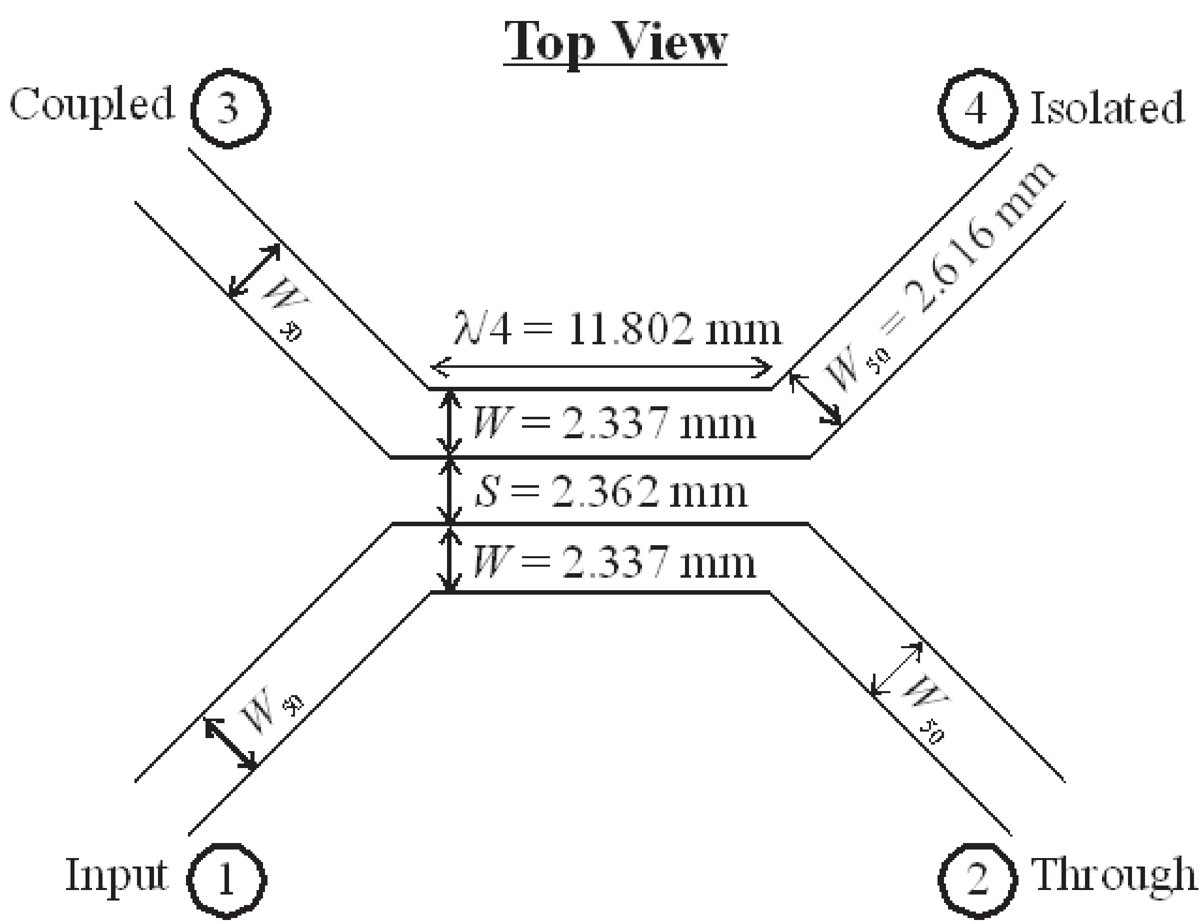
$S := Sd \cdot d$ $\boxed{S \cdot 1000 = 2.362}$ mm, trace spacing

$W := Wd \cdot d$ $\boxed{W \cdot 1000 = 2.337}$ mm, trace width

From Rogers MWI for $Z_0 = 50\ \Omega$ on a 2.54 mm thick TM10i board at 2.4 GHz-
$W_{50\Omega} = 2.616$ mm, $v_p = 1.133*10^8$ m/s, $\lambda = 0.047208$ m = 47.208 mm.

$vp50 := 1.133 \cdot 10^8$ m/s $\lambda := \dfrac{vp50}{f}$ $\lambda = 0.047208$ m

$\boxed{\dfrac{\lambda}{4} \cdot 1000 = 11.802}$ mm



7.7 The Lange Coupler

Problem - A single pair of coupled TLs
can not achieve high coupling,
e.g., 3 dB or 6 dB.

Solution approaches - Use multiple TLs
and couple on both sides of traces.

An example is the Lange Coupler that
is capable of 3 dB coupling ratios w/
an octave (or more) bandwidth. It is
shown in Fig 7.38a

→ 90° phase shift between outputs
(like quad hybrid)

→ drawbacks are narrow lines & need
for jumper wires.

A second example is the unfolded Lange
Coupler (see Fig 7.38b).

→ easier to model

→ fewer jumper wires

→ easier to model

→ All TLs/traces have same width
and spacing

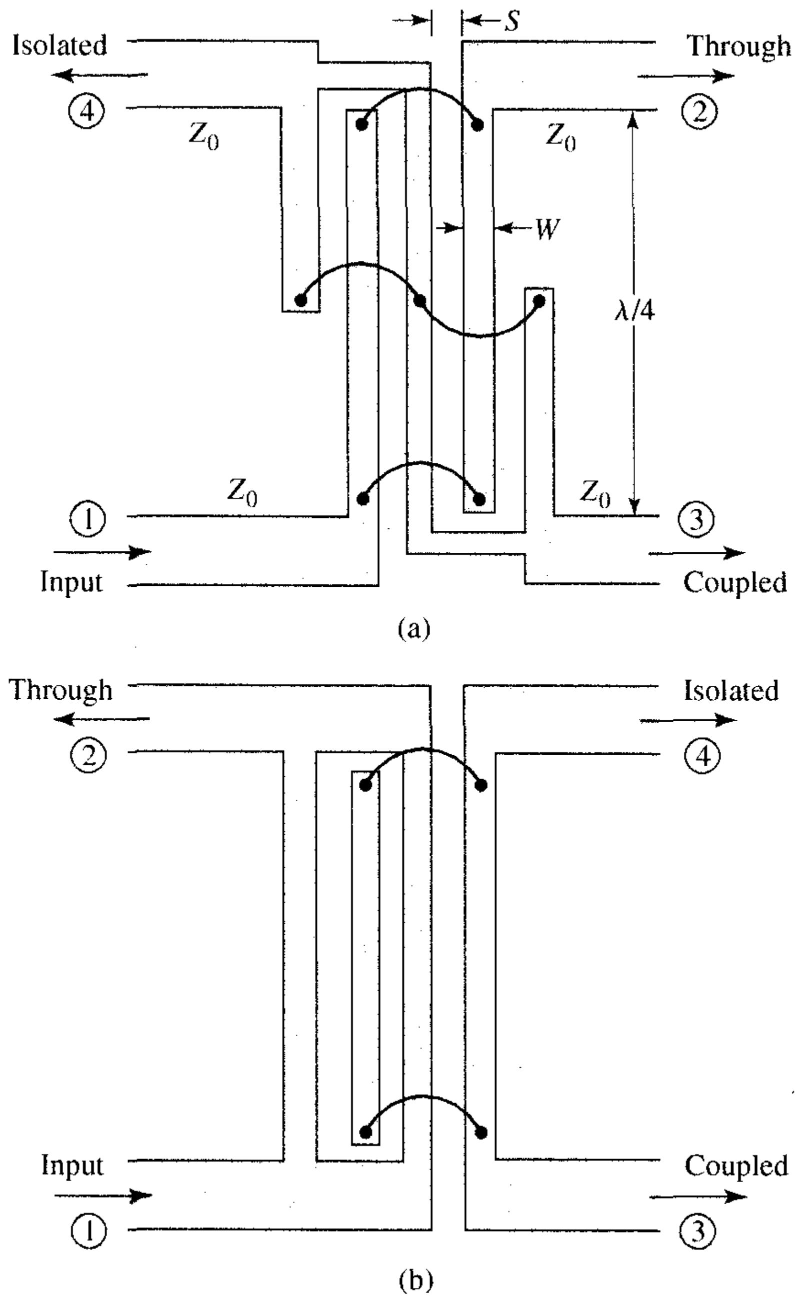


FIGURE 7.38 The Lange coupler. (a) Layout in microstrip form. (b) The unfolded Lange coupler.

7.7 cont.

For the unfolded Lange coupler, we can model it with a four-wire coupled TL model (Fig 7.39a). This can be reduced to an approximate two-wire model (See Fig 7.39b) if we assume that only immediately adjacent TLs couple strongly, i.e., ignore weaker couplings w/ non-adjacent TLs.

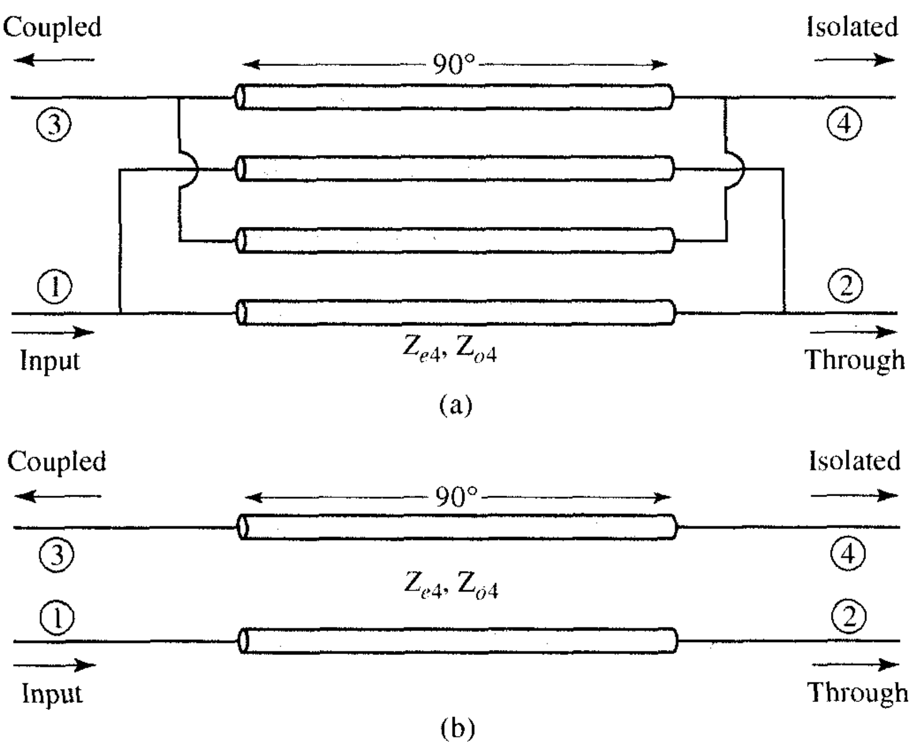


FIGURE 7.39 Equivalent circuits for the unfolded Lange coupler. (a) Four-wire coupled line model. (b) Approximate two-wire coupled line model.

This leads to the capacitance models shown in Fig 7.40.

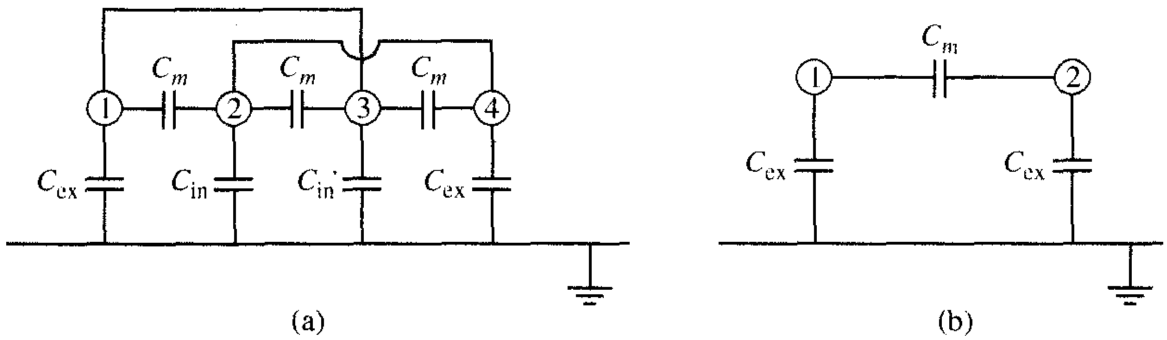


FIGURE 7.40 Effective capacitance networks for the unfolded Lange coupler equivalent circuits of Figure 7.39. (a) Effective capacitance network for the four-wire model. (b) Effective capacitance network for the two-wire model.

Unlike the two-wire coupled TLs of section 7.6, the capacitance to ground varies depending on whether the TL is on the inside C_{in} or outside/exterior C_{ex}. There is also a mutual capacitance C_m. These capacitances are approximately related by

$$C_{in} = C_{ex} - \frac{C_{ex} C_m}{C_{ex} + C_m} \quad (7.92)$$

For the four-wire model, the even mode capacitance (no C_m since all have same excitation) to ground is

$$C_{e4} = C_{ex} + C_{in} \quad (7.93a)$$

7.7 cont.

For the odd mode, the capacitance to ground is $C_{o4} = C_{ex} + C_{in} + 6C_m$ (7.93b)

Using these capacitances, the even and odd mode characteristic impedances are

$$Z_{e4} = \frac{1}{v_p C_{e4}} \qquad (7.94a)$$

$$Z_{o4} = \frac{1}{v_p C_{o4}} \qquad (7.94b)$$

For the two-wire model, we get even and odd mode capacitances of

$$C_e = C_{ex} \qquad (7.95a)$$

$$C_o = C_{ex} + 2C_m \qquad (7.95b)$$

which can be used to simplify the four-wire capacitances to

$$C_{e4} = \frac{C_e(3C_e + C_o)}{C_e + C_o} \qquad (7.96a)$$

$$C_{o4} = \frac{C_o(3C_o + C_e)}{C_e + C_o} \qquad (7.96b)$$

Now, using these and (7.94a) + (7.94b), we can find the even + odd mode four-wire model impedances in terms of two-wire

7.7 cont.

even and odd mode impedances like those done in section 7.6. Use the equations

$$Z_{e4} = \frac{Z_{oo} + Z_{oe}}{3Z_{oo} + Z_{oe}} Z_{oe} \qquad (7.97a)$$

$$Z_{o4} = \frac{Z_{oo} + Z_{oe}}{3Z_{oe} + Z_{oo}} Z_{oo} \qquad (7.97b)$$

Using the results of section 7.6, let the characteristic impedance desired (Z_o) be

$$Z_o = \sqrt{Z_{e4} Z_{o4}} = \sqrt{\frac{Z_{oe} Z_{oo} (Z_{oo} + Z_{oe})^2}{(3Z_{oo} + Z_{oe})(3Z_{oe} + Z_{oo})}} \qquad (7.98)$$

and define the voltage coupling coefficient

$$C = \frac{Z_{e4} - Z_{o4}}{Z_{e4} + Z_{o4}} = \frac{3(Z_{oe}^2 - Z_{oo}^2)}{3(Z_{oe}^2 + Z_{oo}^2) + 2Z_{oe}Z_{oo}} \qquad (7.99)$$

Lastly, we get <u>design equations</u> for a given desired C and Z_o to be

$$Z_{oe} = \frac{4C - 3 + \sqrt{9 - 8C^2}}{2C\sqrt{(1-C)/(1+C)}} Z_o \qquad (7.100a)$$

$$Z_{oo} = \frac{4C + 3 - \sqrt{9 - 8C^2}}{2C\sqrt{(1+C)/(1-C)}} Z_o \qquad (7.100b)$$

$\Rightarrow$ Approximate, for unfolded Lange coupler!

7.7 cont.

Example – Design a 4dB unfolded Lange coupler on $\epsilon_r = 10$ substrate of thickness 1mm using microstrip for a 50Ω circuit @ 1GHz.

$$C\,(dB) = -20\log_{10} C$$

$$\hookrightarrow \quad C = 10^{-4/20} = \underline{0.63096}$$

Per (7.100a) + (7.100b),

$$Z_{oe} = \frac{4(0.631) - 3 + \sqrt{9 - 8(0.631)^2}}{2(0.631)\sqrt{(1-0.631)/(1+0.631)}}\,(50)$$

$$\underline{Z_{oe} = 161.2015\,\Omega}$$

$$Z_{oo} = \frac{4(0.631) + 3 - \sqrt{9 - 8(0.631)^2}}{2(0.631)\sqrt{(1+0.631)/(1-0.631)}}\,(50)$$

$$\underline{Z_{oo} = 58.6608\,\Omega}$$

From Fig 7.30 (next page), we get

$$\underline{S/d = 0.1468} \quad \text{and} \quad \underline{W/d = 0.08313}$$

which, using d = 1mm, yields

$$S = \underline{0.147\,mm} \quad \text{and} \quad W = \underline{0.083\,mm}$$

The length $\lambda/4$ is trickier, and will usually require a simulation to pin down.

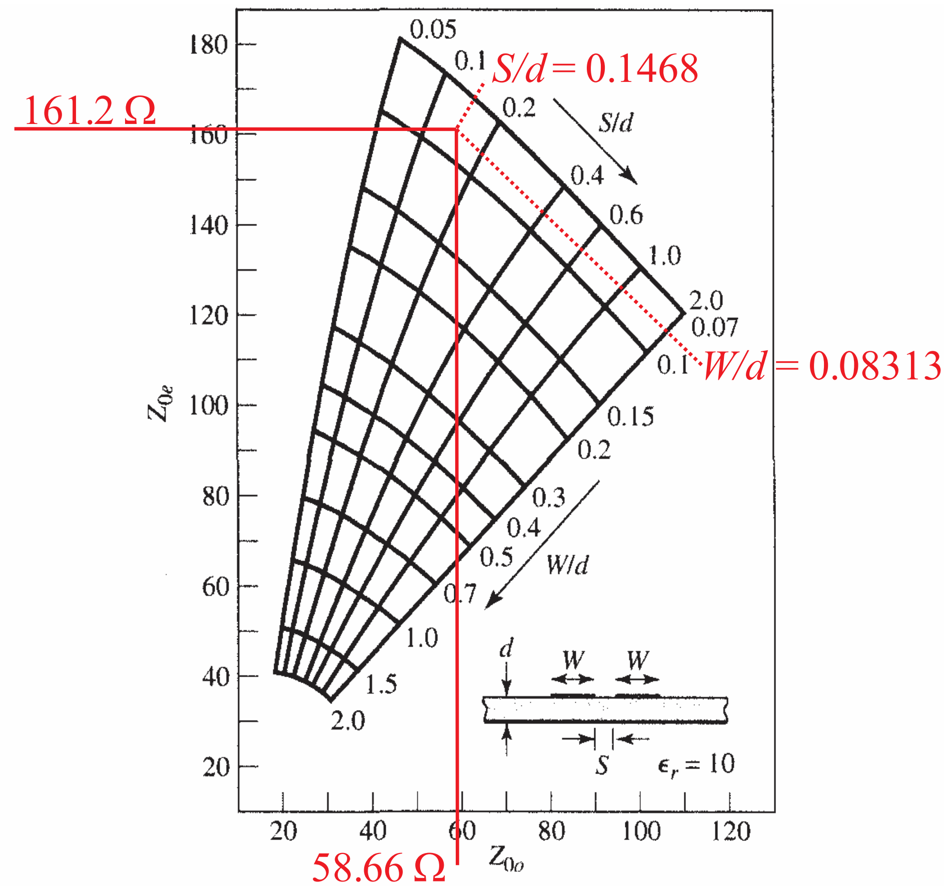


FIGURE 7.30 Even- and odd-mode characteristic impedance design data for symmetric coupled microstrip lines on a substrate with $\epsilon_r = 10$.

Using MWI, at 1GHz, a 50Ω microstrip trace has a width of 0.9245mm.

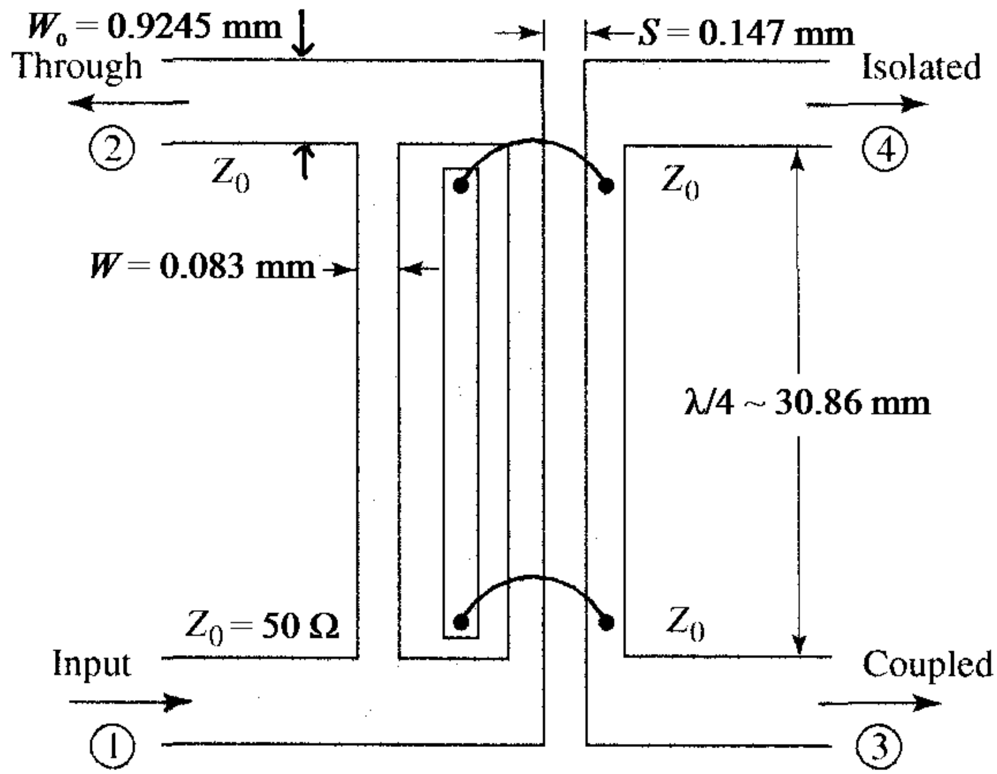
Using MWI, at 1GHz, a pair of edge-coupled microstrip traces with S = 0.147mm and W = 0.083mm has $\epsilon_{re} = 6.3016$ and $\epsilon_{ro} = 5.5166$.

$$\text{Average } \epsilon_r = \frac{6.3016 + 5.5166}{2} = 5.909$$

$$\text{Geom. mean } \epsilon_r = \sqrt{6.3016(5.5166)} = 5.896$$

$\Rightarrow$ Use $\epsilon_r = 5.9$ $\Rightarrow v_p = c/\sqrt{\epsilon_r} = 1.2342 \times 10^8$ m/s

$$\lambda = v_p/f = \frac{1.2342 \times 10^8}{1 \times 10^9} = 0.12342\text{ m} \Rightarrow \lambda/4 = 30.86\text{ mm}$$



W_0 = 0.9245 mm
Through
S = 0.147 mm
Isolated
2
Z_0
4
Z_0
W = 0.083 mm
λ/4 ~ 30.86 mm
Z_0 = 50 Ω
Z_0
Input
Coupled
1
3