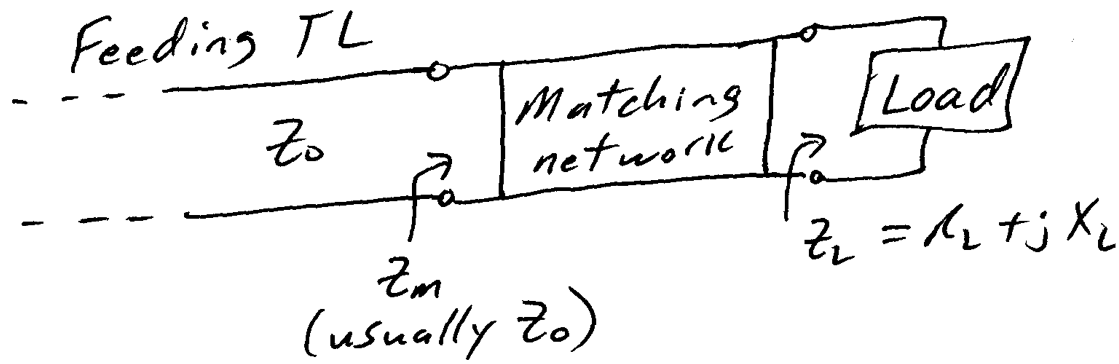
Chapter 5 Impedance Matching + Tuning

Typically, a load will not be matched to a transmission line. So, an engineer will need to design a matching or tuning network to accomplish the task.



why?

- → Minimize SWR on feeding TL
- → Keep from reflecting power back into generator
- → Deliver maximum power to load (assumes generator is matched to TL)
- → Minimize losses along TL, e.g., $|\Gamma|^2$ term in (2.94).
- → Can help improve SNR (signal-to-noise ratio)
- → Can help reduce amplitude + phase errors in distribution networks

If $R_L > 0$, we can 'always' make a matching network @ a single frequency.

Matching network considerations

→ Prefer it to be lossless

→ Prefer it to be simple ⇒ cheaper, smaller, reliable

→ Often will want bandwidth where the match is acceptable as many signals cover a range of frequencies. [Note: To match over large bandwidths is very difficult, usually have to resort to lossy/resistive matching networks.]

→ The application or type of TL/waveguide may make a particular matching network preferable or rule out others as impractical.

→ Sometimes will want the matching network to be tunable/adjustable to accomodate variations in the load(s).

We'll start by examining some matching networks made w/ lumped elements.

'Lumped elements'?

→ Need physical dimensions ≲ 1/10

→ @ microwave frequencies, parasitic capacitance and/or inductance (lead to undesired resonances) can limit usable frequency range(s) of elements

→ Fringing fields, losses, ... also a concern

Resistors → Thin film or chip (surface mount)

→ usually lossy metallic/conductive compound or doped semiconductor

→ Pretty good

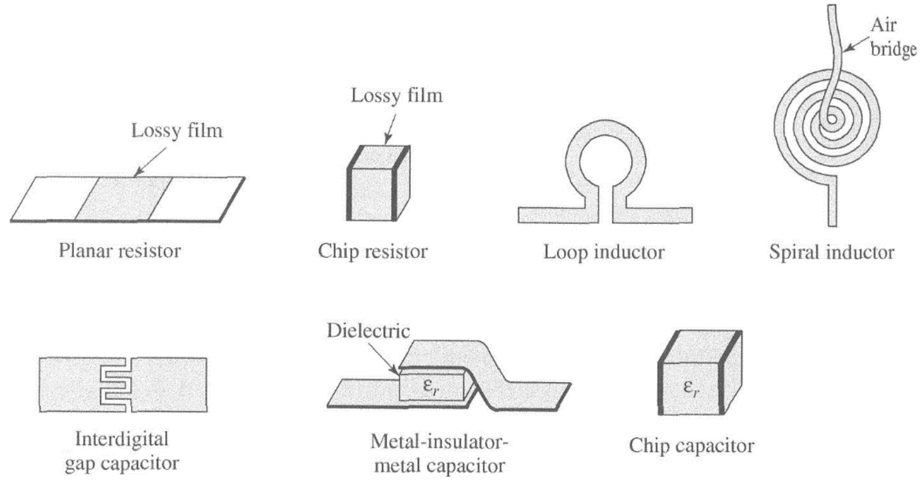
Inductors → loop or spiral (up to ~10nH)

→ Not as good due to parasitic capacitances & losses

Capacitors → narrow gap in TL or interdigitated gap

→ short stub

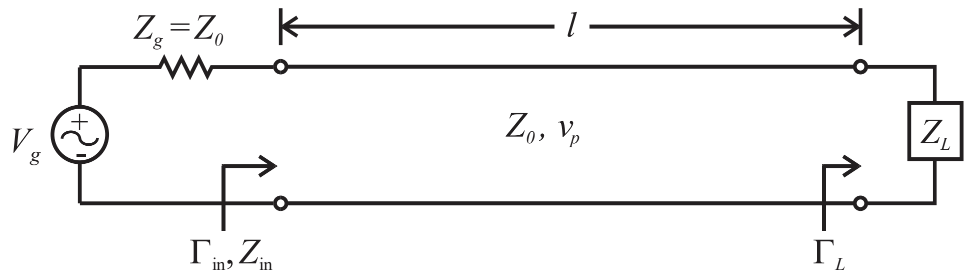
→ traditional metal-insulator-metal 'sandwich'



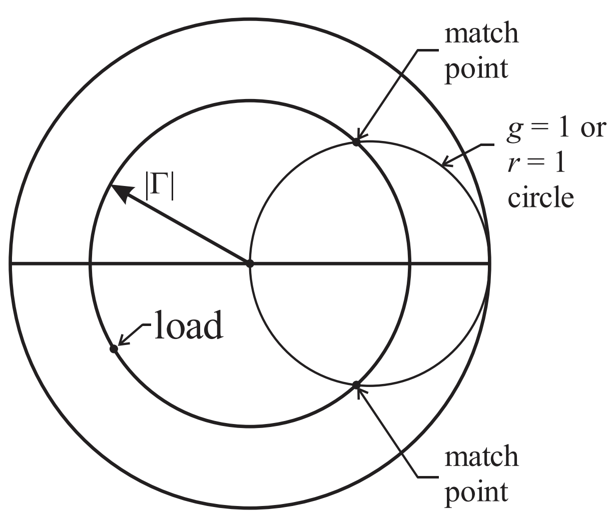
Microwave Engineering (Fourth Edition), Pozar, Wiley, 2012, ISBN 978-0-470-63155-3.

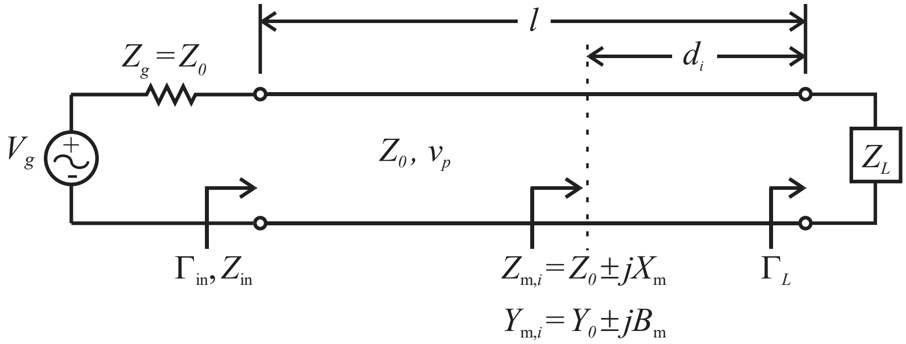
Matching load using a single lumped element

- Assume we have a source matched to the characteristic impedance Z_0 of a lossless transmission line (TL) of length l with a mismatched load Z_L attached.



- We are seeking to match the load Z_L to Z_0 as well, i.e., we want $Z_{in} = Z_0$.
- To avoid power losses, we will only use purely reactive components for matching.
- As can be seen on a Smith chart (normalized admittance or impedance), as we travel down the TL toward the generator on an arc/circle of constant $|\Gamma| < 1$, the arc/circle will intersect the $g = 1$ or $r = 1$ circle at two locations, one above & one below the horizontal axis. These locations, located a distance d_i from the load Z_L, are called the match points- normalized: $y_{m,i} = 1 \pm jb_m$ or $z_{m,i} = 1 \pm jx_m$; un-normalized: $Y_{m,i} = Y_0 \pm jB_m$ or $Z_{m,i} = Z_0 \pm jX_m$.



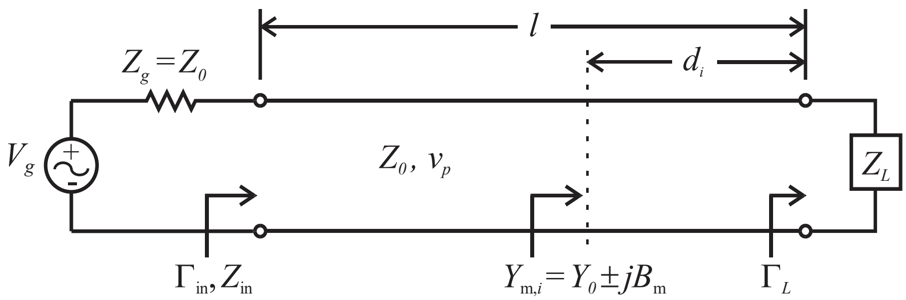


- At the match points, a lumped element with the proper susceptance or reactance can be added in parallel (to cancel $\pm jB_m$) or in series (to cancel $\pm jX_m$) resulting in a match.

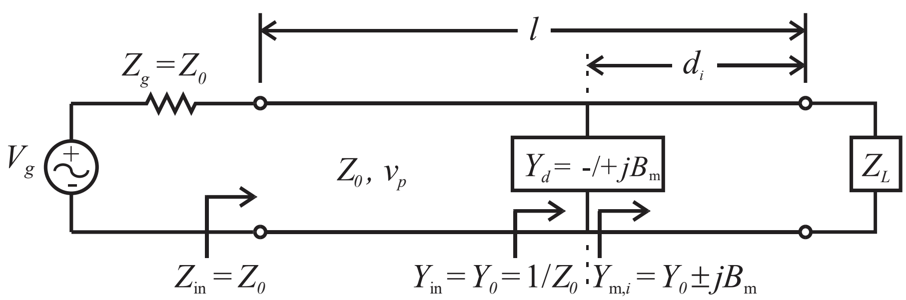
<u>Matching using a single Parallel Lumped Element</u>

- Particularly well-suited for two-wire transmission lines.

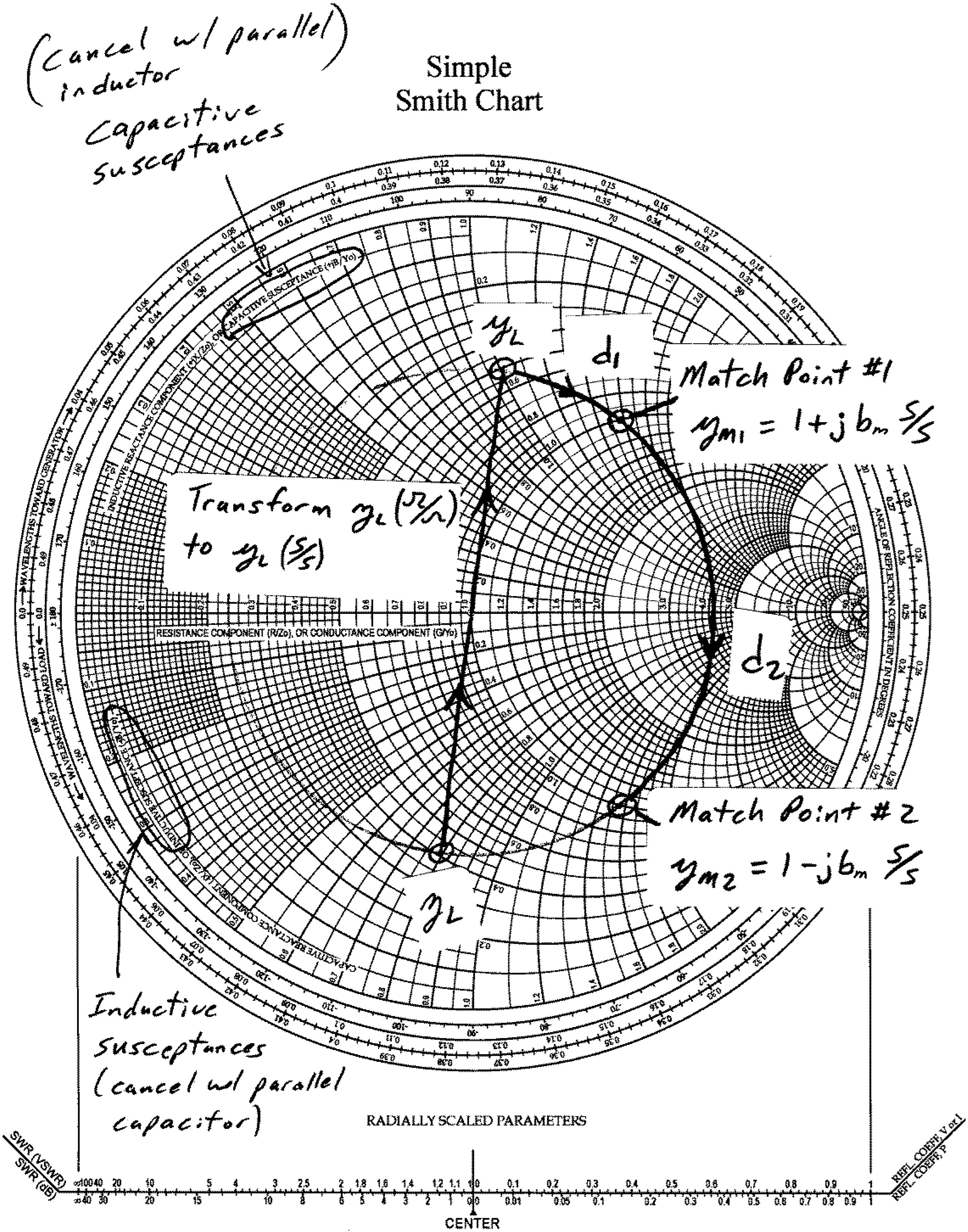
1) Calculate $z_L = Z_L/Z_0$ and plot on Smith chart.
2) Draw circle, centered on Smith chart, through z_L point. This circle of constant $|\Gamma|$ includes the locus of all possible z_{in} (and y_{in}) along the transmission line with this load.
3) Go $\lambda/4$ around the circle of constant $|\Gamma|$ from z_L point to y_L point.
4) There are two points (i.e., match point points) on the circle of constant $|\Gamma|$ that intersect the circle where the normalized conductance $g = 1$, i.e., $y_{\text{m},i} = 1 \pm jb_m$. In terms of input admittance, this is where $Y_{\text{m},i} = y_{\text{m},i}\, Y_0 = y_{\text{m},i}/Z_0 = Y_0 \pm jB_m = 1/Z_0 \pm jB_m$.
5) Find the distance d_i from y_L to the match points using the "WAVELENGTHS TOWARD GENERATOR" scale.



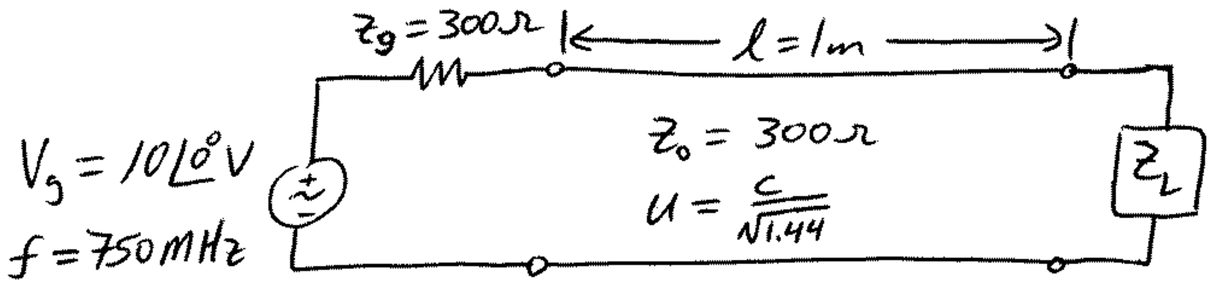
6) Select one of the match points and add a discrete component (i.e., capacitor or inductor) in parallel with a susceptance $Y_d = \mp jB_m$. To calculate the needed capacitance or inductance, remember $Y_{\text{cap}} = j\omega C$ and $Y_{\text{ind}} = -j/\omega L$.
7) Everywhere toward generator from this location will see a normalized admittance of $y_{\text{in}} = y_{\text{m},i} + y_d = (1 \pm jb_m) \mp jb_m = 1$ or normalized impedance $z_{\text{in}} = 1$, i.e., $Y_{\text{in}} = Y_0$ &/or $Z_{\text{in}} = Z_0$.



Simple Smith Chart



Example- Matching load using a parallel capacitor



Using a discrete capacitor placed in parallel w/ the transmission line, match the load $Z_L = 150 - j600\,\Omega$ to the transmission line and generator. Place the capacitor as close to the load as possible. How much power is delivered to the load?

$$z_L = \frac{Z_L}{Z_0} = \frac{150 - j600}{300} = 0.5 - j2\ \Omega/\Omega$$

→ plot z_L on Smith Chart

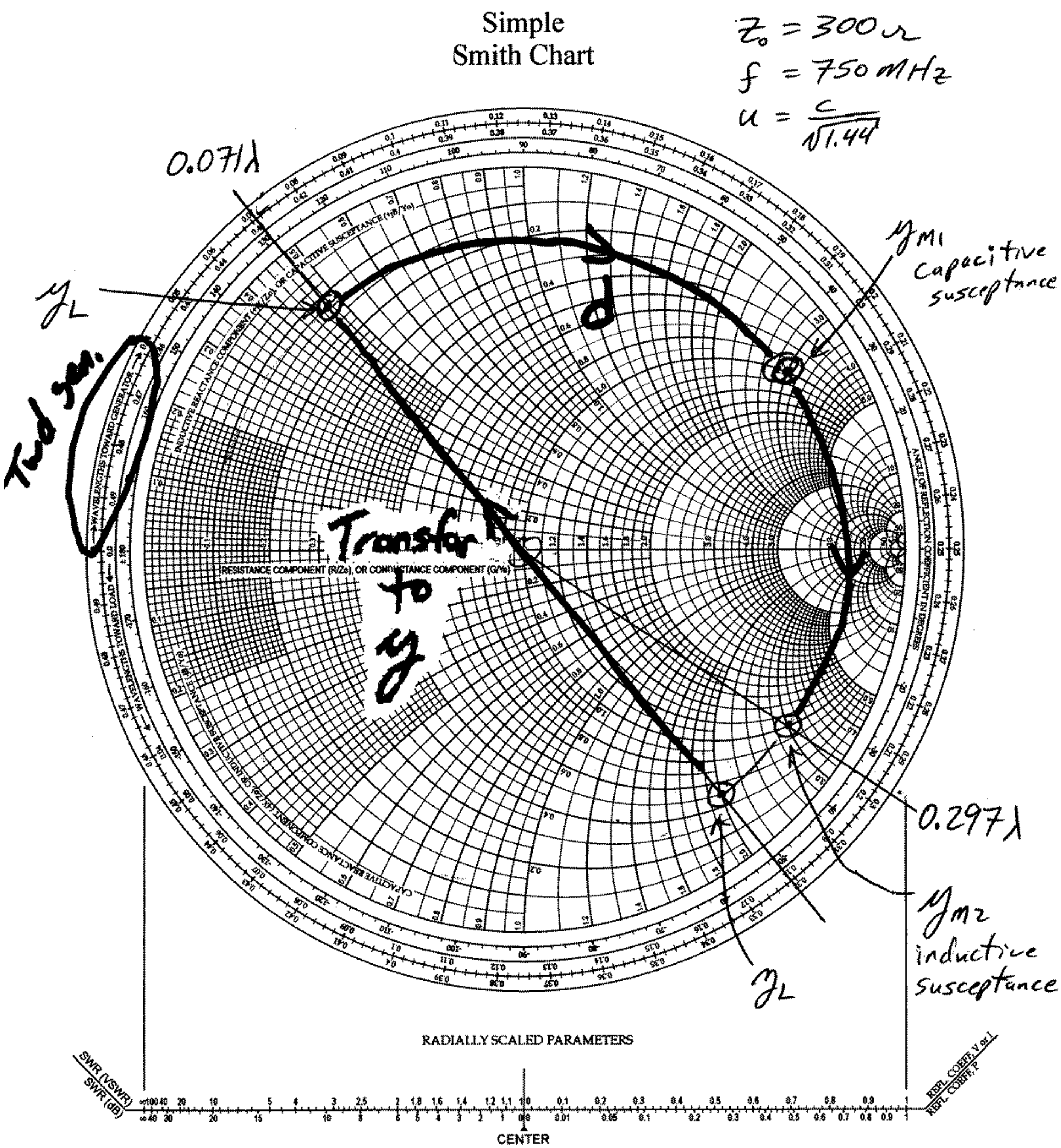
→ draw circle, centered on Smith Chart, thru z_L

→ rotate $\lambda/4$ from z_L to $y_L = 0.11765 + j0.4706$ S/S

→ Note location of match points (points where $|\Gamma|$ circle intersects $g = 1$ S/S circle).

$y_{m1} = 1 + j3$ S/S ← capacitive susceptance

$y_{m2} = 1 - j3$ S/S ← inductive susceptance


Simple
Smith Chart
$Z_0 = 300\,\Omega$
$f = 750$ MHz
$u = \frac{c}{\sqrt{1.44}}$
0.071λ
y_L
Twd gen.
y_{M1}
capacitive susceptance
d
Transfor to y
0.297λ
y_{M2}
inductive susceptance
z_L
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
RADIALLY SCALED PARAMETERS
CENTER

Normalized capacitive susceptance $\quad y_{cap} = (j\omega C) Z_o$

To use a discrete capacitor for matching, select match point #2 and require

$$y_{m2} + y_{cap} = 1$$

$$(1 - j3) + j(2\pi)(750\times10^6)C(300) = 1$$

$$\hookrightarrow \quad C = \frac{3}{2\pi(750\times10^6)(300)}$$

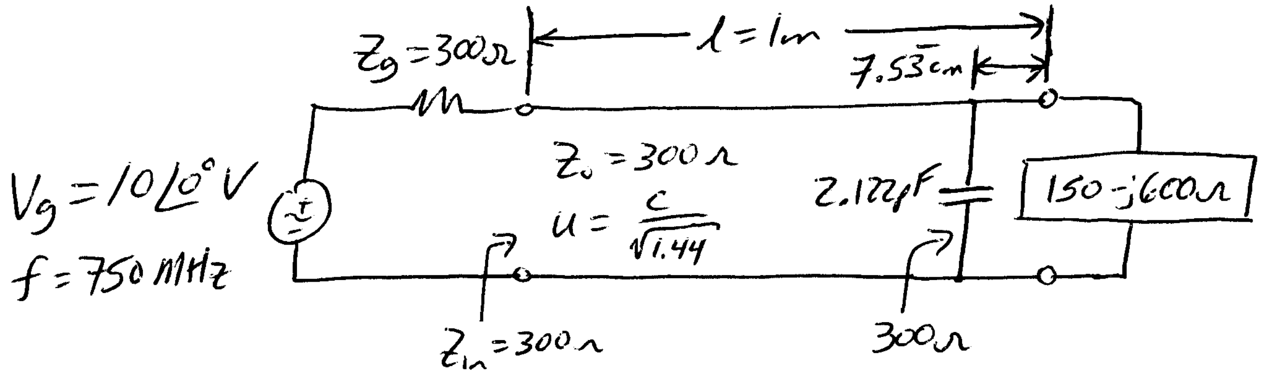
$$\underline{\underline{C = 2.122\,pF}}$$

Distance from load to y_{m2} location closest to load? ← from "Toward Generator" Scale

$$d = 0.297\lambda - 0.071\lambda = \underline{\underline{0.226\lambda}}$$

$$d = 0.226\left(\frac{3\times10^8/\sqrt{1.44}}{750\times10^6}\right) = 0.075\bar{3}\,m$$

$$\underline{\underline{d = 7.5\bar{3}\,cm}}$$



$$I(0) = \frac{V_g}{Z_g + Z_{in}} = \frac{10\angle 0°}{300+300} = \frac{1}{60}\angle 0° \text{ A}$$

$$P_{in} = \frac{1}{2}|I(0)|^2 R_{in} = \frac{1}{2}\left(\frac{1}{60}\right)^2 300 = \frac{|V_g|^2}{8R_g}$$

$$P_{in} = 41.\bar{6} \text{ mW}$$

Since capacitors do not absorb real power and the transmission line is lossless,

$$P_L = P_{in} = 41.\bar{6} \text{ mW} \quad \text{w/ matching capacitor}$$

For comparison, w/out matching capacitor,

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{(150 - j600) - 300}{(150 - j600) + 300} = 0.8246\angle -50.9°$$

$$\Gamma_{in} = \Gamma_L e^{-j2\beta\ell} = (0.8246\angle -50.9°)\, e^{-j2\frac{2\pi(750\times10^6)}{c/\sqrt{1.44}}(1)} = 0.8246\angle -52.42°$$

$$Z_{in} = Z_o \frac{1+\Gamma_{in}}{1-\Gamma_{in}} = 300\,\frac{1+0.825\angle -52.4°}{1-0.825\angle -52.4°} = 142.4 - j581.62\ \Omega$$

$$I_o = \frac{V_g}{Z_{in} + Z_g} = \frac{10\angle 0° \text{ V}}{142.4 - j581.6 + 300} = 0.01368455\angle 52.74° \text{ A}$$

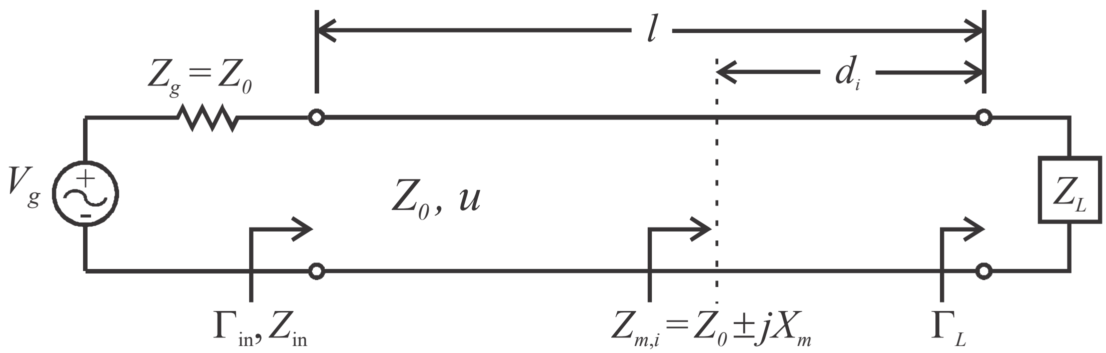
$$P_{in,noc} = P_{L,noc} = \frac{1}{2}(0.01368)^2\, 142.4 = 13.\bar{3} \text{ mW} \quad \approx 32\% \text{ of matched } P_L$$

Matching using a single Series Lumped Element

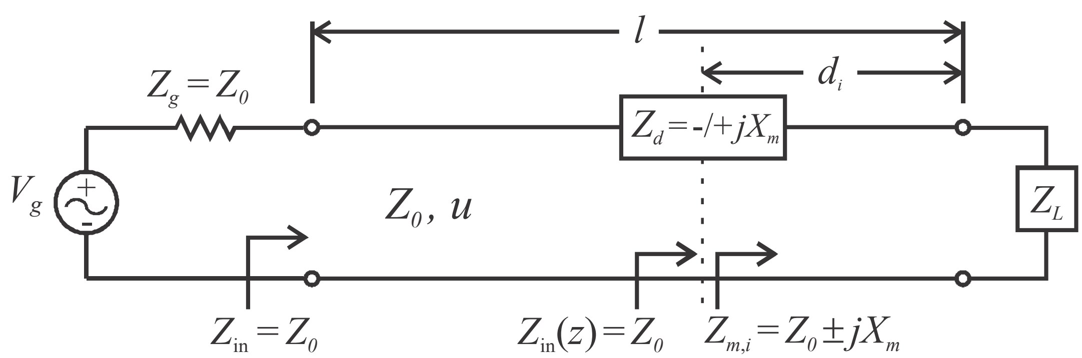
- Particularly well-suited for microstrip transmission lines.

Steps

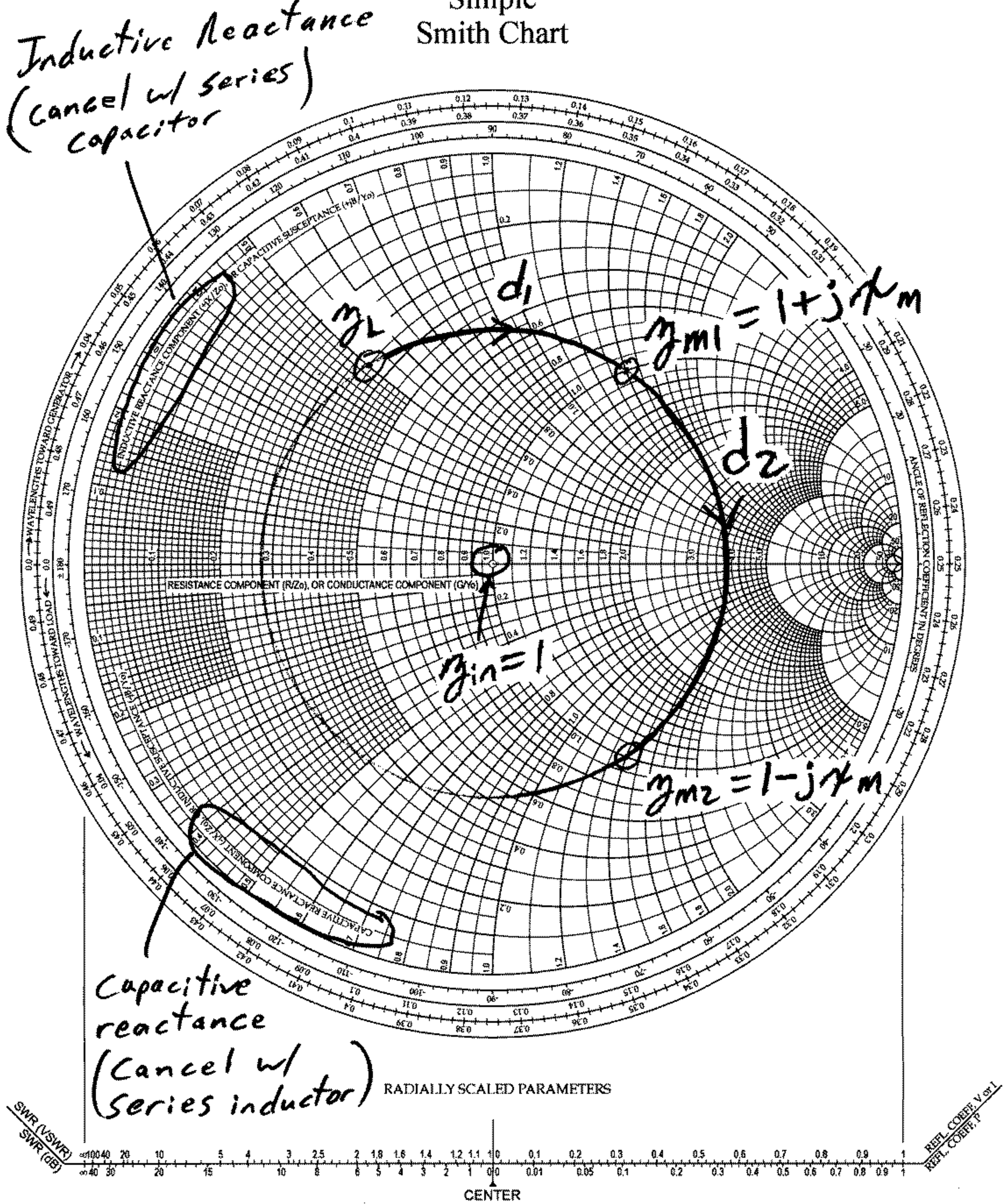
1) Calculate $z_L = Z_L/Z_0$ and plot on Smith chart.
2) Draw circle, centered on Smith chart, through z_L point. This circle of constant $|\Gamma|$ includes the locus of all possible z_{in} (and y_{in}) along the transmission line with this load.
3) There are two points (i.e., match point points) on the circle of constant $|\Gamma|$ that intersect the circle where the normalized resistance $r = 1$, i.e., $z_{m,i} = 1 \pm jx_m$. In terms of input impedance, this is where $Z_{m,i} = Z_0 \pm jX_m$.
4) Find the distance d_i from z_L to the match points using the "WAVELENGTHS TOWARD GENERATOR" scale.



5) Select one of the match points and add a discrete component (i.e., capacitor or inductor) in series with a reactance $Z_d = \mp jX_m$. To calculate the needed capacitance or inductance, remember $Z_{cap} = -j/\omega C$ and $Z_{ind} = j\omega L$.
6) Now, everywhere toward the generator from this location will see a normalized input impedance $z_{in} = (1 \pm jx_m) \mp jx_m = 1$, i.e., $Z_{in} = Z_0$.

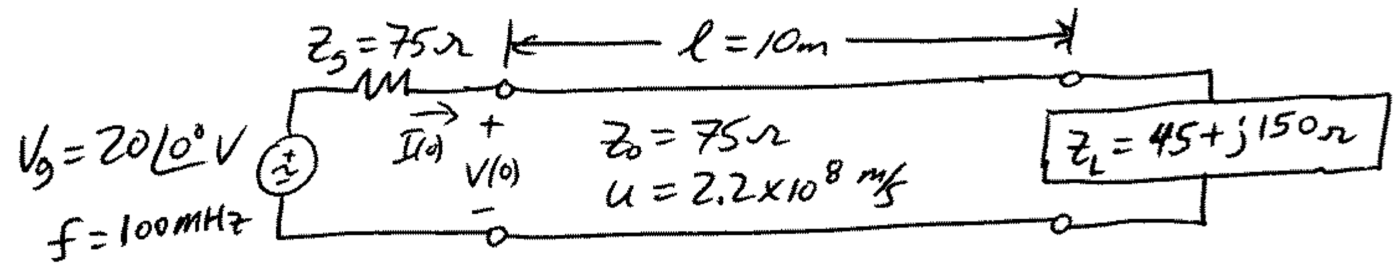


Simple
Smith Chart



<u>Example</u>- Matching load using a series inductor

Using a discrete inductor placed in series w/ the transmission line, match the load to the transmission line and generator. The inductor should be placed as close to the load as possible. How much power is delivered to the load without matching? How much w/ matching?



$$\lambda = \frac{u}{f} = \frac{2.2\times10^8}{100\times10^6} = \underline{2.2\text{m}} \qquad \frac{\ell}{\lambda} = \frac{10}{2.2} = 4.5\overline{45}$$

$$z_L = \frac{Z_L}{Z_0} = \frac{45+j150}{75} = 0.6 + j2\ \Omega/\Omega \leftarrow \text{plot on Smith Chart}$$

<u>No Match</u>

VSWR = 8.8 (from Smith Chart)

Move $\ell/\lambda = 4.5\overline{45} \xrightarrow{-4.5} 0.04545$ "Toward Generator" to

$$z_{in_{NM}} = 3.2 + j4.2\ \Omega/\Omega \quad (3.236 + j4.176\ \Omega/\Omega \text{ analytic})$$

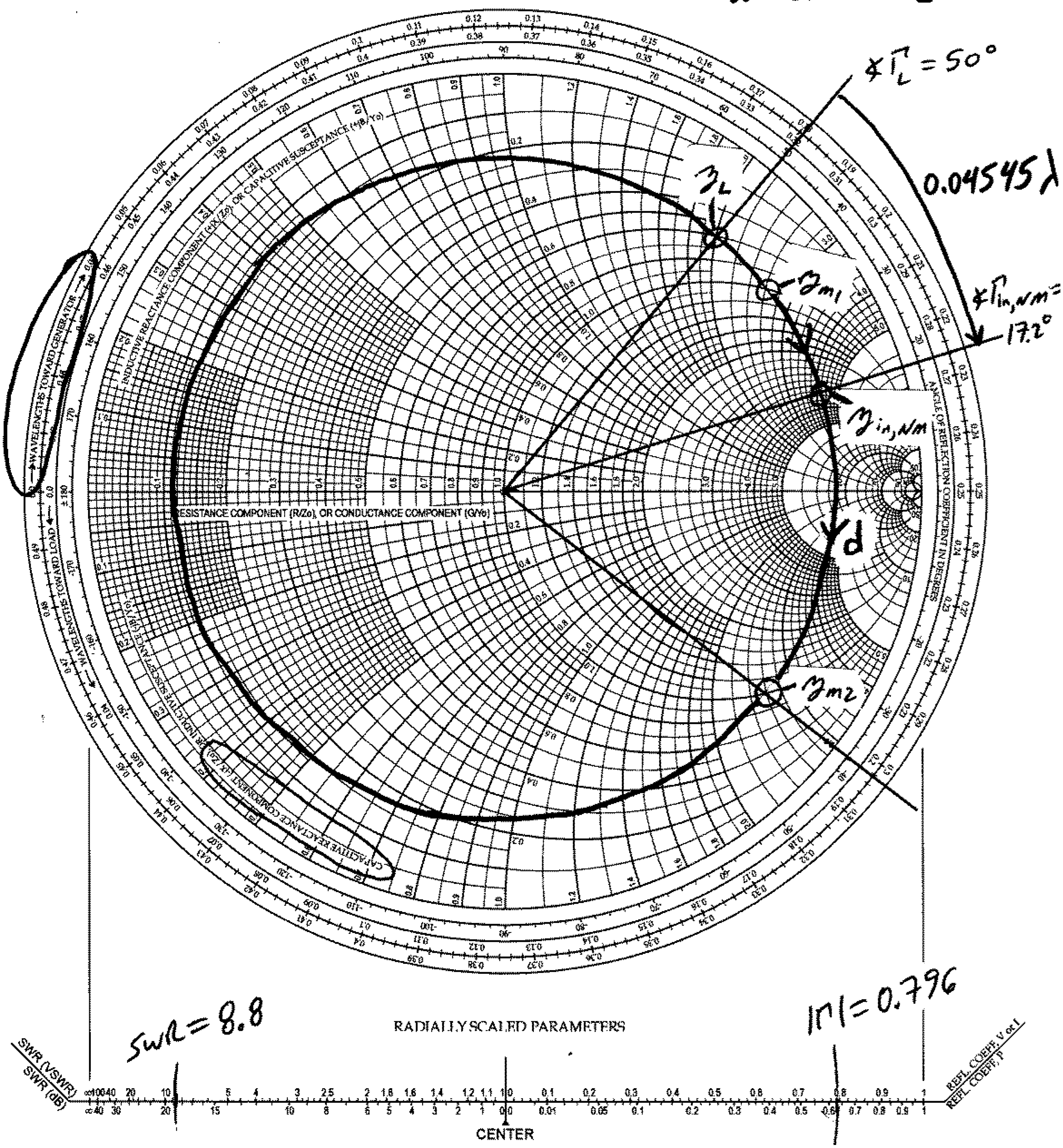
$$Z_{in,NM} = (3.2 + j4.2)75 = \underline{240 + j315\ \Omega}$$

Simple Smith Chart

$Z_o = 75\Omega$

$f = 100\ MHz$

$u = 2.2 \times 10^8\ m/s$



No Match cont.

No Match:

$$I(0) = \frac{V_g}{Z_g + Z_{in,NM}} = \frac{20\angle 0^\circ}{75 + (240 + j315)} = \underline{0.044896\angle{-45^\circ}\,A}$$

$$V(0) = V_g \frac{Z_{in,NM}}{Z_g + Z_{in,NM}} = 20\angle 0^\circ \frac{240 + j315}{75 + (240 + j315)} = 17.7792\angle 7.696^\circ\,V$$

$$P_{in,NM}(0) = P_{L,NM} = \tfrac{1}{2}\,\mathrm{Re}\{V(0)\,I^*(0)\} = \tfrac{1}{2}\,\mathrm{Re}\{17.8\angle 7.7^\circ\,(0.045\angle{+45^\circ})\}$$

$$\underline{\underline{P_{L,NM} = 0.242\ W}}$$

Matching

The match points are $y_m = 1 \pm j2.6$ ℧/℧.

To cancel the reactance with a series inductor $\left(+\,j\omega L / Z_0\right)$, choose $y_{m2} = 1 - j2.6$ ℧/℧ which is located $d = 0.3025\lambda - 0.1805\lambda = \underline{\underline{0.122\lambda}} = \underline{\underline{0.2684\ m}}$ from the load.

Let $y_{m2} + y_{ind} = (1 - j2.6) + \frac{j\omega L}{Z_0} = 1$

Matching cont.

$$L = \frac{2.6\,Z_0}{\omega} = \frac{2.6(75)}{2\pi(100\times10^6)}$$

$$L = 0.31035\,\mu H = 310.35\,nH$$

matched

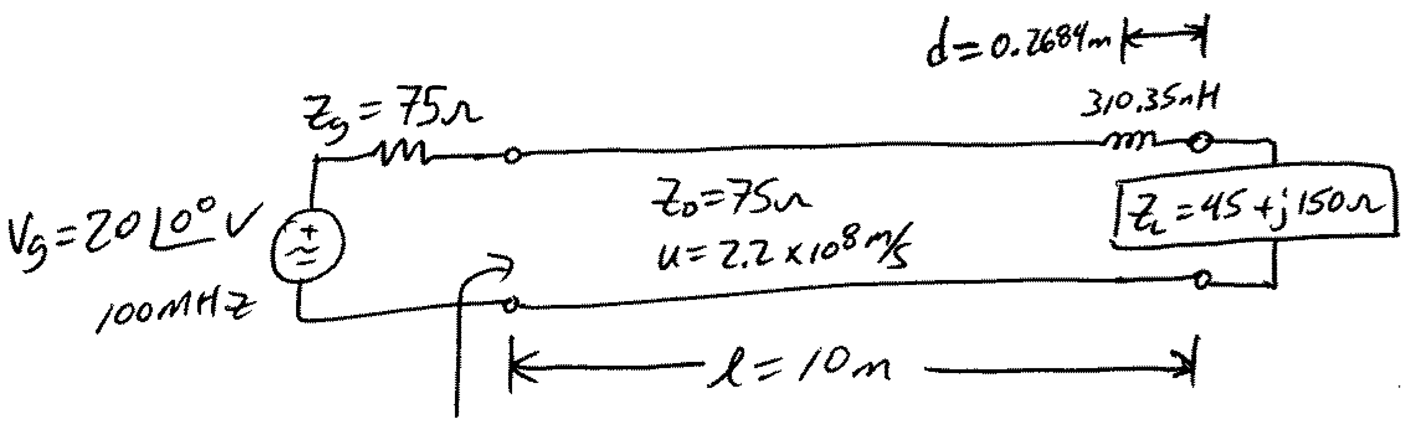
$$I(0) = \frac{V_g}{Z_g + Z_{in,m}} = \frac{20\angle 0^\circ}{75+75} = 0.1\overline{33}\angle 0^\circ\ A$$

$$V(0) = V_g\,\frac{Z_{in,m}}{Z_g + Z_{in,m}} = 20\angle 0^\circ\,\frac{75}{75+75} = 10\angle 0^\circ\ V$$

$$P_{in_m}(0) = P_{L,m} = \tfrac{1}{2}\,\mathrm{Re}\{10\angle 0^\circ\,(0.1\overline{33}\angle 0^\circ)\}$$

$$P_{L,m} = 0.66\bar{6}\ W$$

(275% of unmatched power delivered!

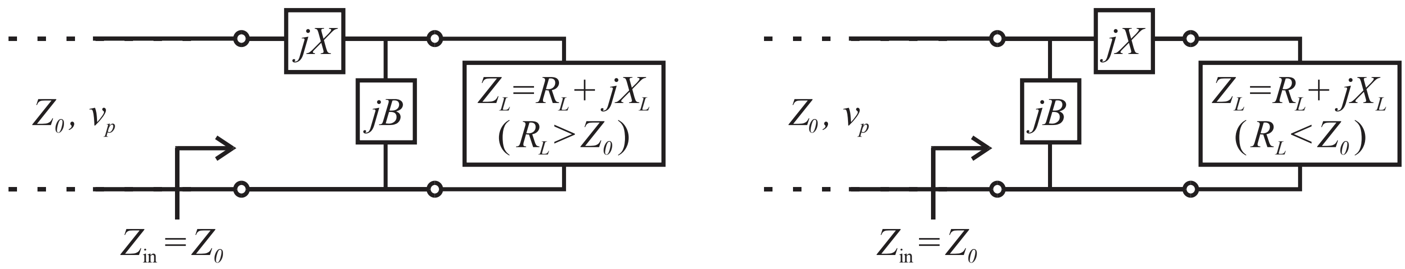


$$Z_{in} = Z_0 = 75\Omega$$

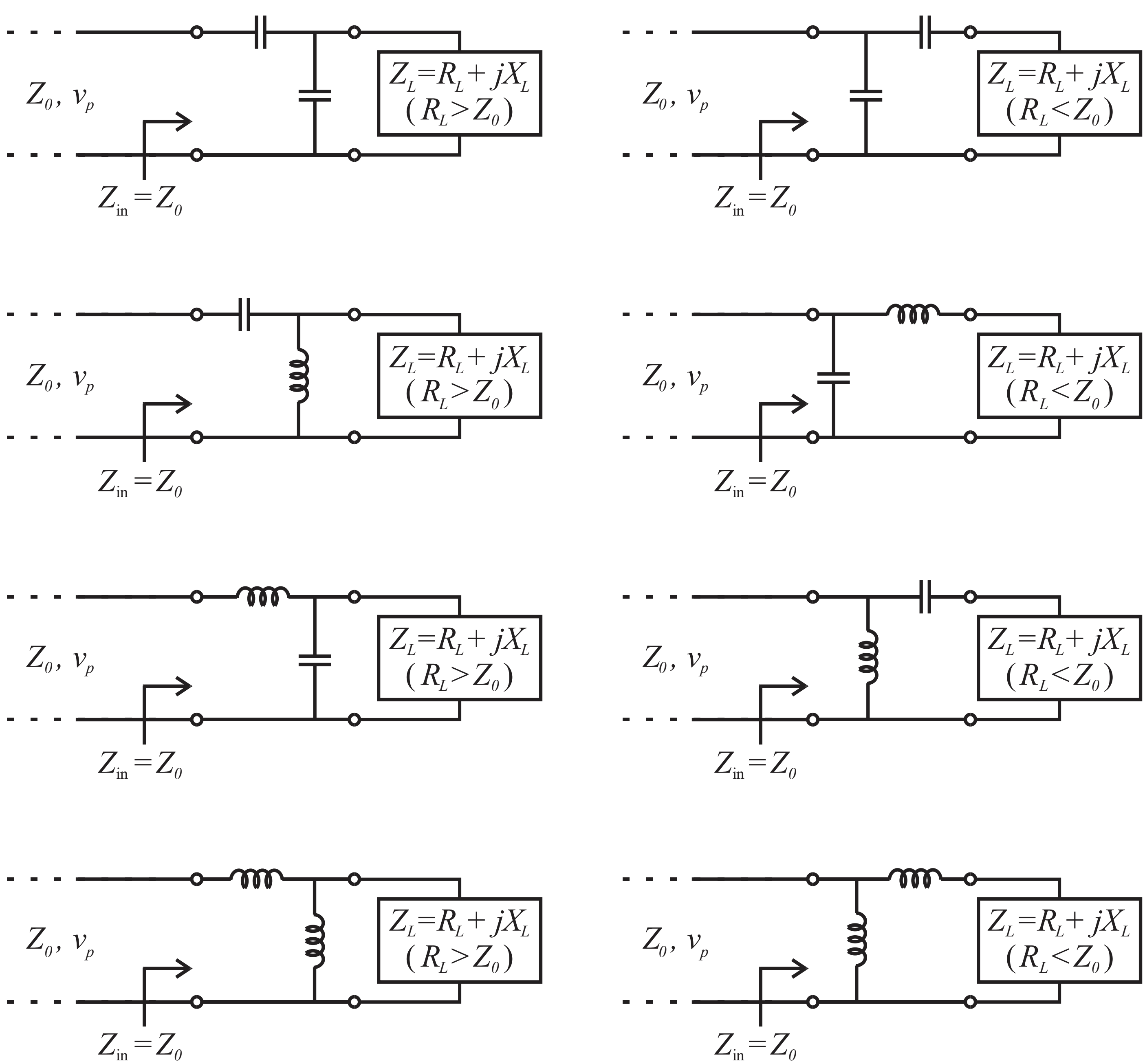
5.1 Matching w/ Lumped Elements (L Networks)

* A drawback of using a single parallel or series lumped element is that the distance d_i from the load (z_L or y_L) point to the match points ($z_{m,i} = 1 \pm j x_m$ or $y_{m,i} = 1 \pm j b_m$) may be a substantial fraction of λ. This uses valuable space on a circuit and leaves a chunk of TL w/ a standing wave.
* L-networks or L-sections get rid of the d_i length of TL at the cost of adding another lumped element.
* Note that in both matching techniques, we need two items to adjust (i.e., two degrees of freedom) in order to match/tune/transform $Z_L = R_L + jX_L$ to Z_0 at the input of the matching network.
* There are two possible L network configurations (topologies) depending on whether $R_L > Z_0$ or $R_L < Z_0$, i.e., are we inside or outside the $r = 1$ circle on the Smith chart. With L/L, L/C, C/L, & C/C possibilities for lumped elements, we have eight possible combinations.

***L*-network or *L*-section Matching w/ Lumped Elements**

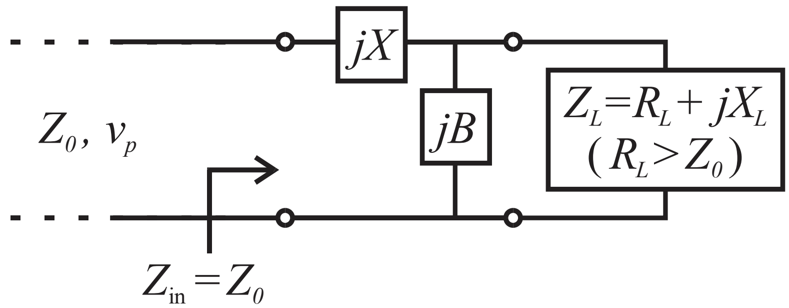


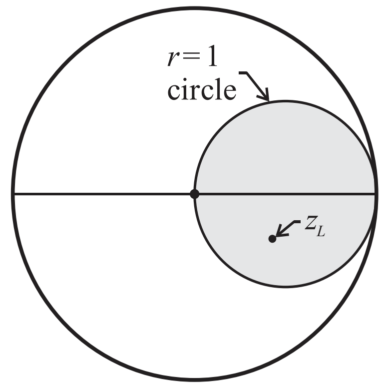
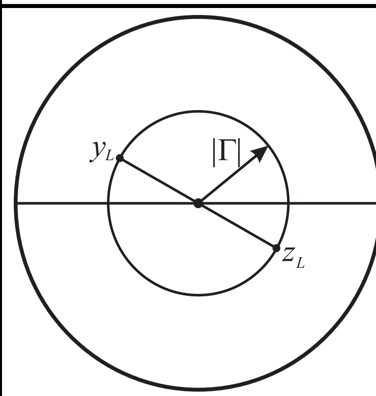
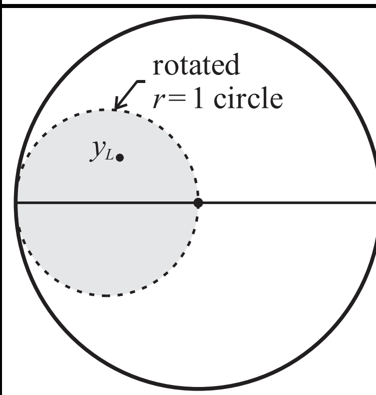
Possible combinations



***L*-network Smith Chart solution for $R_L > Z_0$ case**

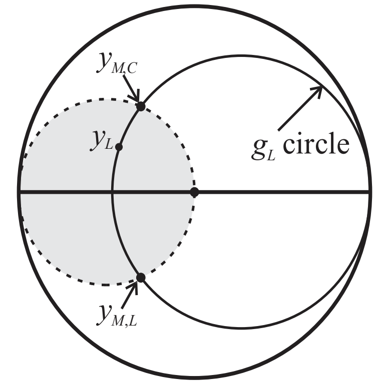
Use this *L*-network configuration-



Normalize the load impedance $z_L = Z_L/Z_0 = r_L + jx_L$ and plot it on a Smith chart. Note that z_L will be inside the r = 1 (i.e., the locus of z = 1 + jx) circle on the Smith chart as shown.	
Go λ/4 (or 180°) around Smith chart on circle of constant |Γ| to the location of the normalized load admittance $y_L = g_L + jb_L$ and plot it the Smith chart. Read and record value of $y_L = g_L + jb_L$.	
Draw a rotated (by λ/4 or 180°) r = 1 or z = 1 + jx circle on the Smith chart as shown. Note that y_L is inside this circle. [As an alternative, a combination *Z*-*Y* Smith chart will already have this circle on it.]	

Add jb (normalized jB parallel element of L-network) to y_L to move along the circle of constant g_L to where it intersects the <u>rotated</u> $r = 1$ or $z = 1 + jx$ circle on the Smith chart. As shown, there are two potential match points, $y_{M,C}$ & $y_{M,L}$. To get to the $y_{M,C}$ match point, you must add a capacitive jb (positive). To get to the $y_{M,L}$ match point, you must add an inductive jb (negative).

Why? On returning to normalized impedance z, the match points are on the $r = 1$ or $z = 1 + jx$ circle. Add a series impedance will get us to $Z_{in} = Z_0$!



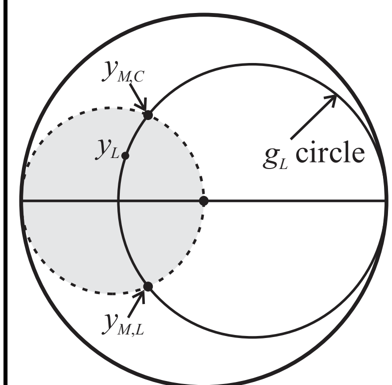
Select a match point based on whether you would prefer to use a **parallel** capacitor ($y_{M,C}$) or inductor ($y_{M,L}$). Read and record the value of the selected match point, find needed jb, and compute required **parallel** inductor L or capacitor C for the L-network.

$$\text{Capacitor: } y_{M,C} = g_L + jb_{M,C} = g_L + j(b_L + b_{\text{cap}})$$

$$\Rightarrow b_{\text{cap}} = b_{M,C} - b_L = \omega C Z_0 \Rightarrow C = b_{\text{cap}} / (\omega Z_0), \textbf{ or}$$

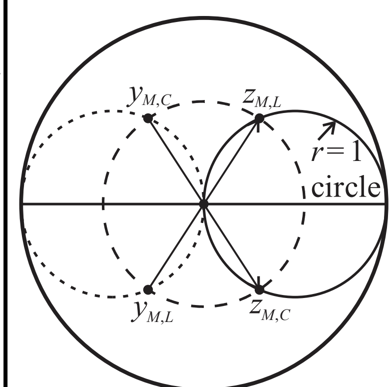
$$\text{Inductor: } y_{M,L} = g_L + jb_{M,L} = g_L + j(b_L + b_{\text{ind}})$$

$$\Rightarrow b_{\text{ind}} = b_{M,L} - b_L = -Z_0/(\omega L) \Rightarrow L = -Z_0 / (\omega b_{\text{ind}})$$



From selected match point, $y_{M,C}$ or $y_{M,L}$, go $\lambda/4$ around Smith chart on new circle of constant $|\Gamma|$ to position of corresponding normalized impedance, $z_{M,C}$ or $z_{M,L}$. Read & record value of $z_{M,C}$ or $z_{M,L}$.

- ➢ $z_{M,C} = 1 + jx_{M,C}$ where $x_{M,C} < 0$ is a normalized capacitive reactance which will require the addition of a **series** normalized <u>inductive</u> reactance for matching.
- ➢ $z_{M,L} = 1 + jx_{M,L}$ where $x_{M,L} > 0$ is a normalized inductive reactance which will require the addition of a **series** normalized <u>capacitive</u> reactance for matching.



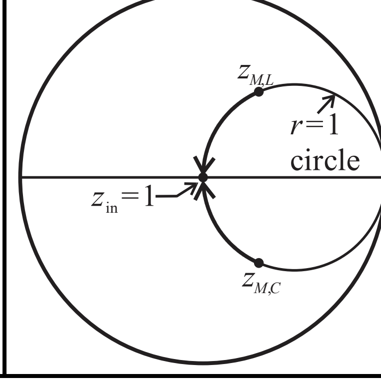
Add jx (normalized jX, **series** element on L-network) to $z_{M,C}$ or $z_{M,L}$ to move along the $r = 1$ circle to the center of the Smith chart where $z_{in} = 1$ (i.e., $Z_{in} = Z_0$).

$$\text{Inductor: } z_{in} = z_{M,C} + jx_{\text{ind}} = 1 + j(x_{M,C} + x_{\text{ind}}) = 1$$

$$\Rightarrow x_{\text{ind}} = -x_{M,C} = \omega L / Z_0 \Rightarrow L = x_{\text{ind}} Z_0 / \omega, \textbf{ or}$$

$$\text{Capacitor: } z_{in} = z_{M,L} + jx_{\text{cap}} = 1 + j(x_{M,L} + x_{\text{cap}}) = 1$$

$$\Rightarrow x_{\text{cap}} = -x_{M,L} = -1/(\omega C Z_0) \Rightarrow C = -1/(\omega Z_0 x_{\text{cap}})$$



ex. Match a $Z_L = 900 + j300\,\Omega$ load to a $300\,\Omega$ twin-wire TL using a parallel inductor in the L-network when operating at 1.2 GHz. $v_p = 0.9c$

→ $z_L = \frac{Z_L}{Z_0} = \frac{900 + j300\,\Omega}{300\,\Omega} = 3 + j1\ \Omega/\Omega$

→ Plot z_L on Smith Chart.

→ Draw circle of constant $|\Gamma|$ through z_L centered on Smith chart

→ Draw radial line through z_L and center of Smith chart to far side of circle (180° around). Plot and read $\underline{y_L = 0.3 - j0.1\ S/S}$

→ Draw a rotated r=1 circle on Smith chart and note where it intersects the $g_L = 0.3\ S/S$ circle at

$$y_{m,C} = 0.3 + j0.46\ S/S$$

and

$$y_{m,L} = 0.3 - j0.46\ S/S$$

→ Per specification to use a parallel Inductor, select the match point

$$y_{m,L} = 0.3 - j0.46\ S/S$$

→ $y_{m,L} = 0.3 - j0.46 = 0.3 + j(-0.1 + b_{ind})$

↳ $b_{ind} = -0.46 - (-0.1) = -0.36$ S/S

↳ $L = \dfrac{-Z_0}{\omega b_{ind}} = \dfrac{-300}{2\pi(1.2\times10^9)(-0.36)}$

Parallel Inductor $\underline{L = 1.10524\times10^{-7}\,H = 110.52\,nH}$

→ Draw a new circle of constant $|\Gamma|$ through $y_{m,L}$ centered on the Smith Chart.

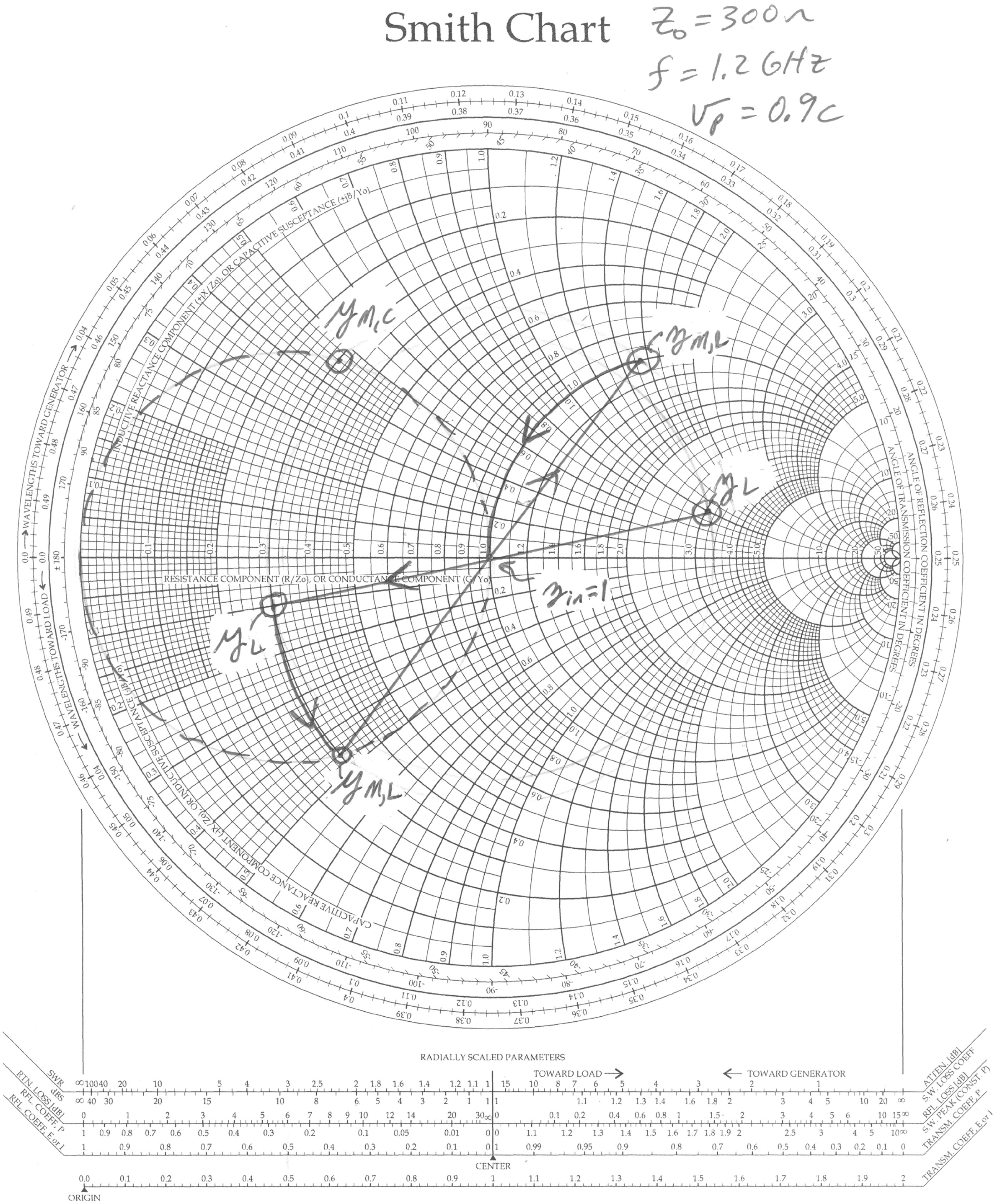
→ Draw a radial line through the center of the Smith chart from $y_{m,L}$ to far side of the $|\Gamma|$ circle (180°). Plot and read $z_{m,L} = 1 + j1.525$ Ω/Ω

→ $z_{in} = 1 = z_{m,L} + jx_{cap} = (1 + j1.525) + jx_{cap}$

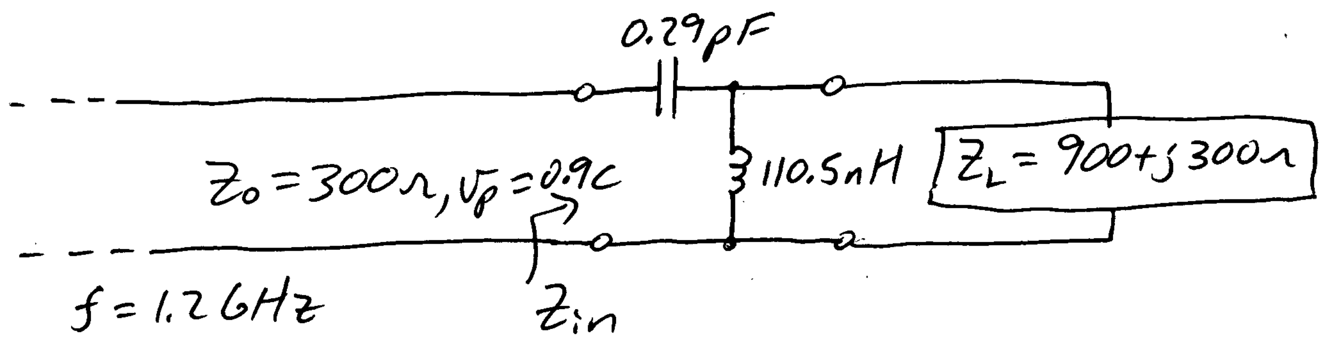
↳ $x_{cap} = -1.525$

Series Capacitor ↳ $C = \dfrac{-1}{2\pi(1.2\times10^9)300(-1.525)}$

$\underline{C = 2.899\times10^{-13}\,F = 0.29\,pF}$


Smith Chart
Zo = 300Ω
f = 1.2 GHz
vp = 0.9c
yM,C
yM,L
yL
yL
yM,L
zin = 1
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
WAVELENGTHS TOWARD GENERATOR →
← WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
RADIALLY SCALED PARAMETERS
TOWARD LOAD →
← TOWARD GENERATOR
SWR
dBS
RTN. LOSS [dB]
RFL. COEFF. P
RFL. COEFF. E or I
ATTEN. [dB]
S.W. LOSS COEFF
RFL. LOSS [dB]
S.W. PEAK (CONST. P)
TRANSM. COEFF. P
TRANSM. COEFF. E or I
CENTER
ORIGIN

Final Design (Smith Chart)



Let's use circuit theory to find Z_{in} as a check.

$$Z_{ind} = j\omega L = j2\pi(1.2 \times 10^9)(110.5 \times 10^{-9})$$
$$= j833.15\,\Omega$$

$$Z_L \,//\, Z_{ind} = \left[\frac{1}{900+j300} + \frac{1}{j833.15}\right]^{-1}$$
$$= 298.3365 + j457.528\,\Omega$$

$$Z_{cap} = \frac{-j}{\omega C} = \frac{-j}{2\pi(1.2\times10^9)\,0.29\times10^{-12}}$$
$$= -j457.34\,\Omega$$

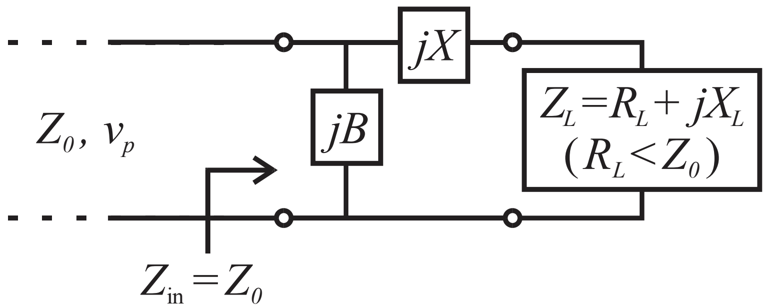
$$Z_{in} = Z_L \,//\, Z_{ind} + Z_{cap}$$
$$= (298.3365 + j457.528) - j457.34$$

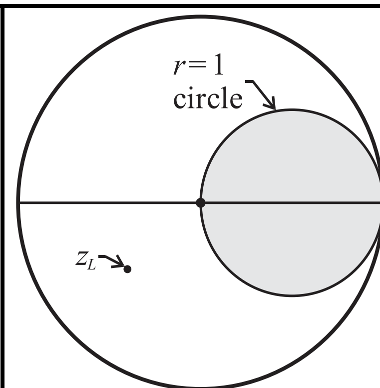
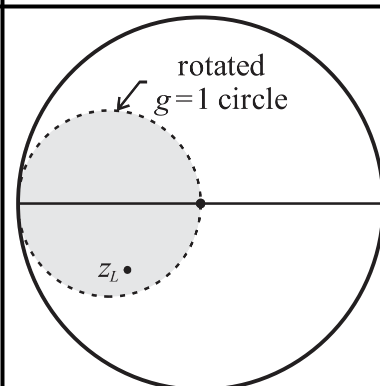
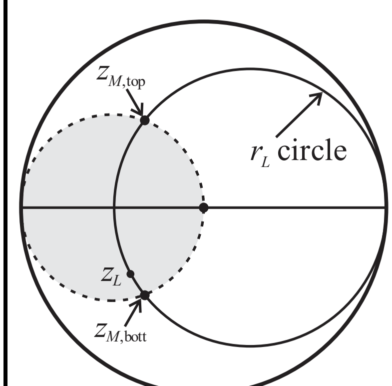
$$\underline{Z_{in} = 298.34 + j0.186\,\Omega \simeq 300\,\Omega = Z_0}$$

Not bad!

L-network Smith Chart solution for $R_L < Z_0$ case

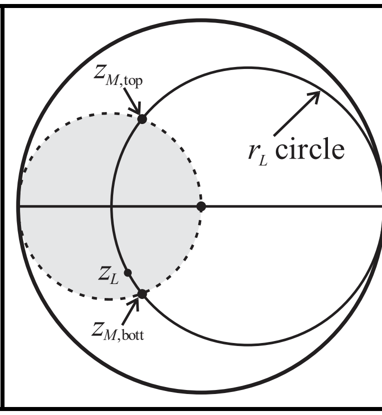
Use this *L*-network configuration-



Normalize the load impedance $z_L = Z_L/Z_0 = r_L + jx_L$ and plot it on a Smith chart. Note that z_L will be outside the $r = 1$ (i.e., the locus of $z = 1 + jx$) circle on the Smith chart as shown.	
Draw a rotated (by $\lambda/4$ or 180°) $g = 1$ or $y = 1 + jb$ circle on the Smith chart as shown. Note that z_L could be inside or outside this circle. [As an alternative, a combination *Z*-*Y* Smith chart will already have this circle on it.]	
Add jx (normalized jX, series element on the *L*-network) to z_L to move along the circle of constant r_L to where it intersects the <u>rotated</u> $g = 1$ or $y = 1 + jb$ circle on the Smith chart as shown. As shown, there are two potential match points, $z_{M,\text{top}}$ and $z_{M,\text{bott}}$. **Why?** When we convert to normalized admittance y, we will now be on the $g = 1$ or $y = 1 + jb$ circle. Adding a parallel admittance will get us to Y_0!	

Select one of the two match points, $z_{M,\text{top}}$ and $z_{M,\text{bott}}$.

- From the location of z_L, if you go CW on the circle of constant r_L to get to a match point, you must add inductive reactance (positive).
- From the location of z_L, if you go CCW on the circle of constant r_L to get to a match point, you must add capacitive reactance (negative).



Determine the necessary **series** lumped element for the *L*-network. I.e.,

$$z_{M,\text{top/bott}} = r_L + jx_\text{m} = z_L + jx = r_L + j(x_L + x) \quad \Rightarrow \quad x_\text{m} = x_L + x \quad \Rightarrow \quad x = x_\text{m} - x_L$$

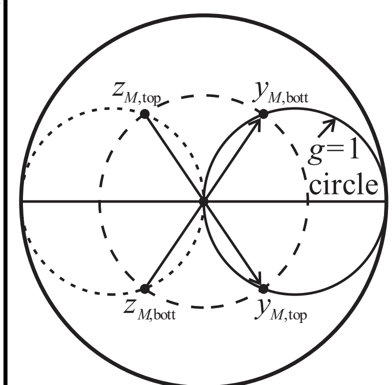
If $x > 0$, it is inductive. $x = \omega L / Z_0 \Rightarrow L = x Z_0 / \omega$

or

If $x < 0$, it is capacitive. $x = -1/(\omega C Z_0) \Rightarrow C = -1/(\omega Z_0 x)$

From selected match point, $z_{M,\text{top}}$ or $z_{M,\text{bott}}$, go $\lambda/4$ around Smith chart on new circle of constant $|\Gamma|$ to position of corresponding normalized admittance, $y_{M,\text{top}}$ or $y_{M,\text{bott}}$, and plot. Read & record value of $y_{M,\text{top}}$ or $y_{M,\text{bott}}$.

- $y_{M,\text{top}} = 1 + jb_{M,\text{top}}$ where $b_{M,\text{top}} < 0$ is a normalized inductive susceptance, requires addition of a **parallel** normalized <u>capacitive</u> susceptance for matching.
- $y_{M,\text{bott}} = 1 + jb_{M,\text{bott}}$ where $b_{M,\text{bott}} > 0$ is a normalized capacitive susceptance, requires addition of a **parallel** normalized <u>inductive</u> susceptance for matching.



Add jb (normalized jB, **parallel** element of *L*-network) to $y_{M,\text{top}}$ or $y_{M,\text{bott}}$ to move along the $g = 1$ circle to the center of the Smith chart where $y_\text{in} = 1$ (i.e., $Y_\text{in} = Y_0$).

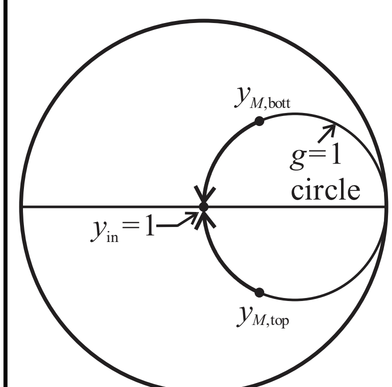
Inductor: $y_\text{in} = y_{M,\text{bott}} + jb_\text{ind} = 1 + j(b_{M,\text{bott}} + b_\text{ind}) = 1$

$\Rightarrow \quad b_\text{ind} = -b_{M,\text{bott}} = -Z_0/\omega L \quad \Rightarrow \quad L = -Z_0/(\omega b_\text{ind})$

or

Capacitor: $y_\text{in} = y_{M,\text{top}} + jb_\text{cap} = 1 + j(b_{M,\text{top}} + b_\text{cap}) = 1$

$\Rightarrow \quad b_\text{cap} = -b_{M,\text{top}} = \omega C Z_0 \quad \Rightarrow \quad C = b_\text{cap}/(\omega Z_0)$



ex. Match a $Z_L = 15 + j45\ \Omega$ load to a $75\ \Omega$ transmission line using an L-network when operating at 2.4 GHz. A C/C L-network is preferable.

$v_p = 0.7c$

$\rightarrow\ z_L = \frac{Z_L}{Z_0} = \frac{15+j45}{75} = 0.2 + j0.6\ \Omega/\Omega$

$\rightarrow$ Plot z_L on Smith chart (used combined Z & Y Smith chart).

$\rightarrow$ Follow $r_L = 0.2$ circle to intersections with rotated g=1 circle and read–

$$z_{m,top} = 0.2 + j0.4\ \Omega/\Omega$$

$$z_{m,bott} = 0.2 - j0.4\ \Omega/\Omega$$

In both cases, moved CCW along $r_L = 0.2$ circle which implies adding capacitive reactance. Looking ahead, selecting $\underline{z_{m,TOP}}$ will result in a parallel capacitor as well as the series capacitor.

$\rightarrow$ Choose $z_{m,top} = 0.2 + j0.4\ \Omega/\Omega = z_L + jx$

$\rightarrow\ x = 0.4 - 0.6 = -0.2 \Rightarrow C_{series} = \frac{-1}{\omega Z_0 x}$

$\rightarrow$ $C_{series} = \dfrac{-1}{2\pi(2.4\times10^9)(-0.2)75}$

$C_{series} = 4.421\,pF$

$\rightarrow$ Draw circle through $y_{m,top}$ and move 180° around to $y_{m,top} = 1 - j2$ S/S

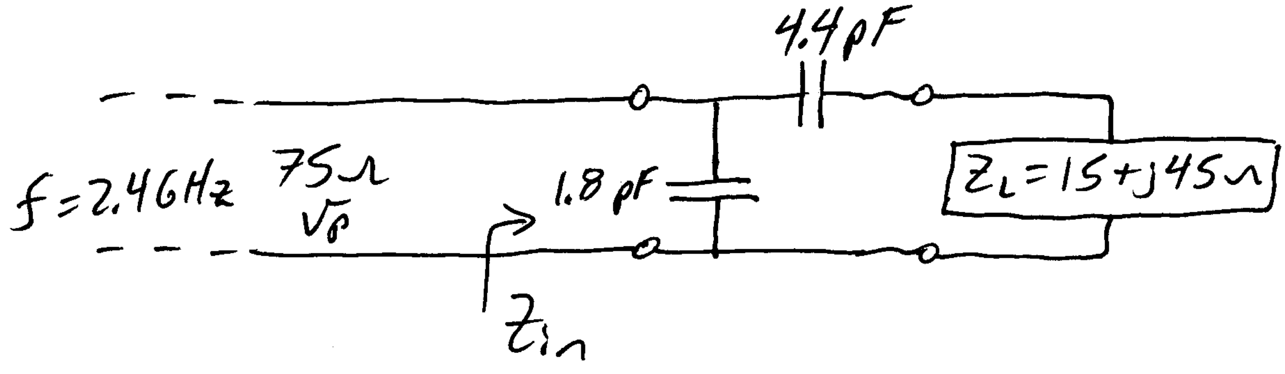
$\rightarrow$ For $y_{m,top} + jb_{cap} = y_{in} = 1$, we need

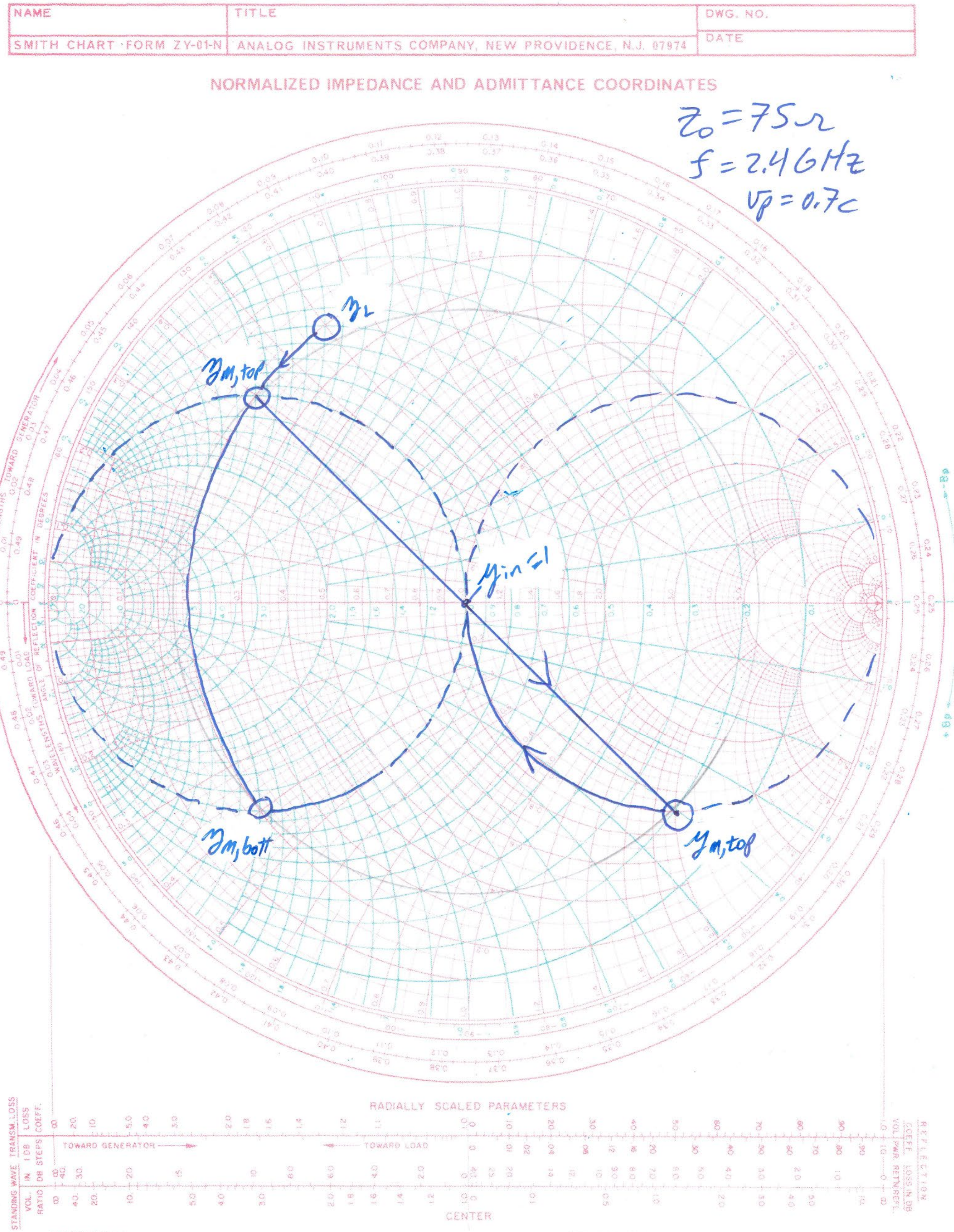
$b_{cap} = +2 = \omega C_{parallel} Z_0$

$C_{parallel} = \dfrac{2}{2\pi(2.4\times10^9)75}$

$C_{parallel} = 1.768\,pF$

Final Design




NAME
TITLE
DWG. NO.
SMITH CHART FORM ZY-01-N
ANALOG INSTRUMENTS COMPANY, NEW PROVIDENCE, N.J. 07974
DATE
NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES
Z0 = 75 Ω
f = 2.4 GHz
vp = 0.7c
zL
ym,top
yin = 1
ym,bott
ym,top
RADIALLY SCALED PARAMETERS
TOWARD GENERATOR
TOWARD LOAD
CENTER

Check Z_{in} using circuit theory

$$Z_L + Z_{C,series} = (15 + j45) + \frac{-j}{2\pi(2.4\times10^9)4.4\times10^{-12}}$$

$$= 15 + j45 - j15.0715$$

$$= 15 + j29.9285\ \Omega$$

$$Z_{in} = Z_{C,parallel} \,//\, (Z_L + Z_{C,series}$$

$$= \left[j2\pi(2.4\times10^9)(1.8\times10^{-12}) + \frac{1}{15 + j29.93} \right]^{-1}$$

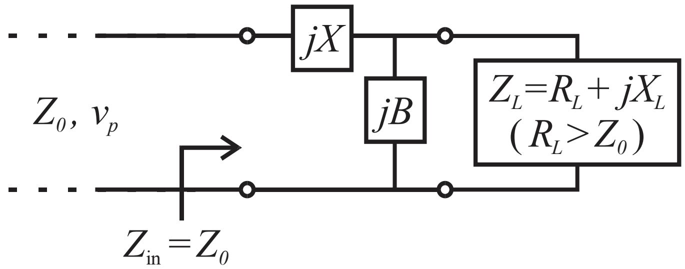
$$= \left[+j0.02714 + (0.01338 - j0.0267) \right]^{-1}$$

$$\underline{\underline{Z_{in} = 74.63 - j2.445\ \Omega}}$$

Not bad considering I rounded off capacitor values.

***L*-network Analytic Solution for $R_L > Z_0$ case**

Use this *L*-network configuration-



Using circuit theory, the input impedance is found and set equal to Z_0-

$$Z_{\text{in}} = \left[jB + \frac{1}{R_L + jX_L} \right]^{-1} + jX = Z_0 .$$

Doing the algebra and complex math to solve the real and imaginary parts of the Z_{in} equation for B and X, results in

$$B = \frac{X_L \pm \sqrt{R_L / Z_0}\,\sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2} = \frac{X_L \pm \sqrt{R_L / Z_0}\,\sqrt{R_L (R_L - Z_0) + X_L^2}}{R_L^2 + X_L^2} \qquad (5.3a)$$

and

$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{B R_L} \qquad (5.3b)$$

for the susceptance of the parallel element and reactance of the series element respectively.

Notes:

- From (5.3a), there are two possible solutions for B.
- In turn, this implies that there are two possible solutions for X, depending on the value of B selected.
- Note square root terms are always positive as $R_L > Z_0 > 0$.

	Capacitor	Inductor
Impedance	$Z_{\text{cap}} = jX_{\text{cap}} = \frac{-j}{\omega C} = \frac{-j}{2\pi f C}$ $\Rightarrow \quad X_{\text{cap}} = \frac{-1}{\omega C} = \frac{-1}{2\pi f C} < 0$	$Z_{\text{ind}} = jX_{\text{ind}} = j\omega L = j2\pi f L$ $\Rightarrow \quad X_{\text{ind}} = \omega L = 2\pi f L > 0$
Admittance	$Y_{\text{cap}} = jB_{\text{cap}} = j\omega C = j2\pi f C$ $\Rightarrow \quad B_{\text{cap}} = \omega C = 2\pi f C > 0$	$Y_{\text{ind}} = jB_{\text{ind}} = \frac{-j}{\omega L} = \frac{-j}{2\pi f L}$ $\Rightarrow \quad B_{\text{ind}} = \frac{-1}{\omega L} = \frac{-1}{2\pi f L} < 0$

ex. Find analytic solution to matching a load $Z_L = 900 + j300\,\Omega$ to a $300\,\Omega$ ($v_p = 0.9c$) twin-wire TL using a parallel inductor in an L-network operating @ 1.2 GHz.

Per (5.3a), $$B = \frac{X_L \pm \sqrt{R_L/Z_0}\sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2}$$

$$= \frac{300 \pm \sqrt{900/300}\sqrt{900^2 + 300^2 - 300(900)}}{900^2 + 300^2}$$

$$= \frac{300 \pm \sqrt{3}\sqrt{630{,}000}}{900{,}000}$$

$$= 8.16379225 \times 10^{-4} \text{ or } \underline{-1.1941919 \times 10^{-3}\ S}$$

Choose negative solution for parallel inductor

$$-1.194 \times 10^{-3} = \frac{-1}{2\pi f L} \Rightarrow L_{par} = 1.11062 \times 10^{-7}\ H$$

Parallel Inductor $\underline{\underline{L_{par} = 111.06\ nH}}$

Per (5.3b), $$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{B R_L}$$

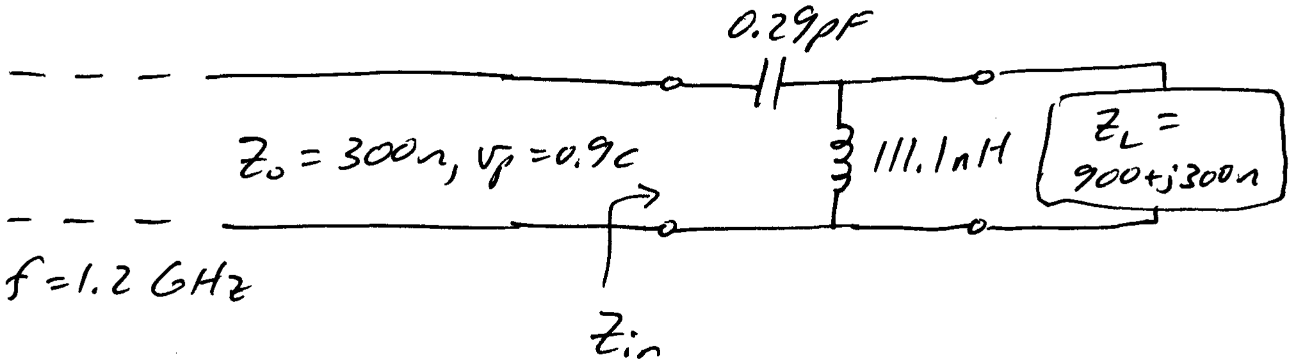
$$= \frac{1}{-1.194 \times 10^{-3}} + \frac{300(300)}{900} - \frac{300}{-1.194 \times 10^{-3}(900)}$$

$$= -458.25757\,\Omega \text{ (capacitive)}$$

$$= \frac{-1}{2\pi f C}$$

Series Capacitor ↳ $\underline{\underline{C_{series} = 2.8942 \times 10^{-13} F = 0.2894\ pF}}$

Final Design (Analytic)



⇒ Very similar to Smith Chart solution

Again, use circuit theory to check solution

$$Z_{ind} = j2\pi(1.2\times10^9)(111.1\times10^{-9}) = j837.67\,\Omega$$

$$Z_L \,\|\, Z_{ind} = \left[\frac{1}{900+j300} + \frac{1}{j837.67}\right]^{-1}$$

$$= 300.113 + j458.307\,\Omega$$

$$Z_{cap} = \frac{-j}{2\pi(1.2\times10^9)(0.29\times10^{-12})} = -j457.34\,\Omega$$

$$Z_{in} = Z_L \,\|\, Z_{ind} + Z_{cap}$$

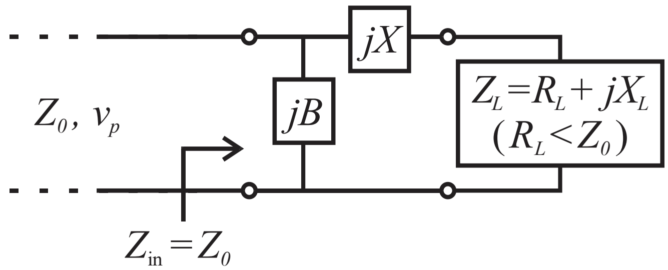
$$= (300.113 + j458.307) - j457.34$$

$$\underline{Z_{in} = 300.113 + j0.965\,\Omega}$$

Excellent match to $Z_0 = 300\Omega$

L-network Analytic solution for $R_L < Z_0$ case

Use this *L*-network configuration-



Using circuit theory, the input impedance is found and set equal to Z_0-

$$Z_{in} = \left[jB + \frac{1}{(R_L + jX_L) + jX} \right]^{-1} = Z_0 .$$

Doing the algebra and complex math to solve the real and imaginary parts of the Z_{in} equation for *B* and *X*, results in

$$X = -X_L \pm \sqrt{R_L(Z_0 - R_L)} \qquad (5.6a)$$

and

$$B = \pm \frac{\sqrt{(Z_0 - R_L)/R_L}}{Z_0} \qquad (5.6b)$$

for the reactance of the series element and susceptance of the parallel element respectively.

Notes:

- For (5.6a) and (5.6b), pair the solutions for *X* and *B* based on the sign of the '±' term.
- This also implies there are two possible solutions.
- Note square root terms are always positive as $Z_0 > R_L > 0$.

	Capacitor	Inductor
Impedance	$Z_{cap} = jX_{cap} = \frac{-j}{\omega C} = \frac{-j}{2\pi f C}$ $\Rightarrow \quad X_{cap} = \frac{-1}{\omega C} = \frac{-1}{2\pi f C} < 0$	$Z_{ind} = jX_{ind} = j\omega L = j2\pi f L$ $\Rightarrow \quad X_{ind} = \omega L = 2\pi f L > 0$
Admittance	$Y_{cap} = jB_{cap} = j\omega C = j2\pi f C$ $\Rightarrow \quad B_{cap} = \omega C = 2\pi f C > 0$	$Y_{ind} = jB_{ind} = \frac{-j}{\omega L} = \frac{-j}{2\pi f L}$ $\Rightarrow \quad B_{ind} = \frac{-1}{\omega L} = \frac{-1}{2\pi f L} < 0$

ex. Match a $Z_L = 15 + j45\ \Omega$ load to a $75\ \Omega$ TL ($v_p = 0.7c$) using an L-network operating @ 2.4 GHz. Use a C-C L-network.

Per (5.6a), $X = -X_L \pm \sqrt{R_L(Z_0 - R_L)}$

$$= -45 \pm \sqrt{15(75-15)} = -45 \pm 30$$

$$= \underline{-15} \text{ or } -75\ \Omega$$

→ Two capacitive reactance solutions. Choose the $-15\ \Omega$ solution (i.e., '+' solution) so that (5.6b) will also yield a capacitance.

$$X_{series} = -15 = \frac{-1}{2\pi(2.4\times10^9)C_{series}}$$

↳ $\underline{\underline{C_{series} = 4.421\times10^{-12}\ F = 4.421\ pF}}$

Per (5.6b), using '+' solution,

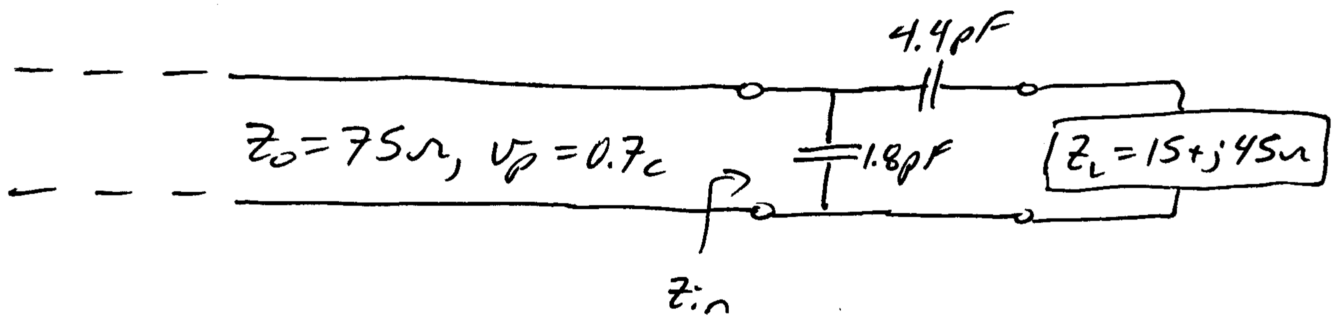
$$B = +\frac{\sqrt{(Z_0 - R_L)/R_L}}{Z_0} = \frac{\sqrt{(75-15)/15}}{75}$$

$$= 0.02\bar{6}\ S = \omega C_{parallel}$$

↳ $C_{parallel} = \dfrac{0.02\bar{6}}{2\pi(2.4\times10^9)}$

$\underline{\underline{C_{parallel} = 1.7684\times10^{-12}\ F = 1.7684\ pF}}$

Final Design (Analytic)



⇒ Identical to Smith Chart solution, after rounding capacitor values to one decimal point.

$$Z_{in} = 74.63 - j2.445\ \Omega$$

→ Good match to $Z_0 = 75\,\Omega$

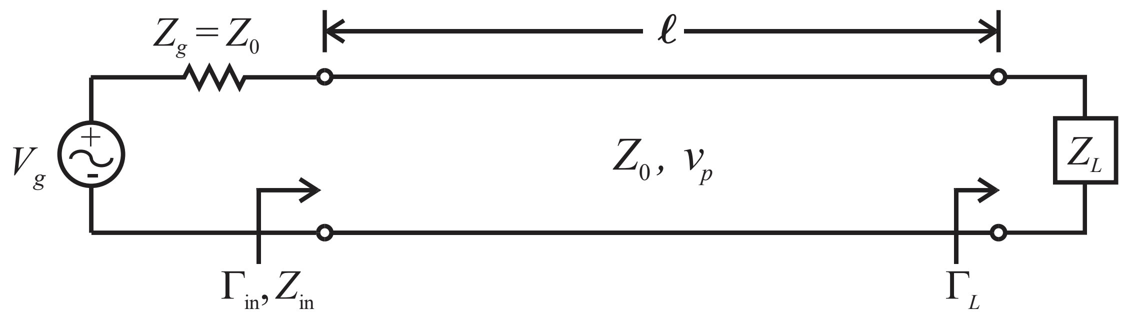
$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{(74.63 - j2.445) - 75}{(74.63 - j2.445) + 75}$$

$$= 0.016524 \angle -97.669^\circ$$

$$VSWR = \frac{1 + 0.016524}{1 - 0.016524} = \underline{1.034} \simeq 1$$

Matching load using a Single-Stub Tuning

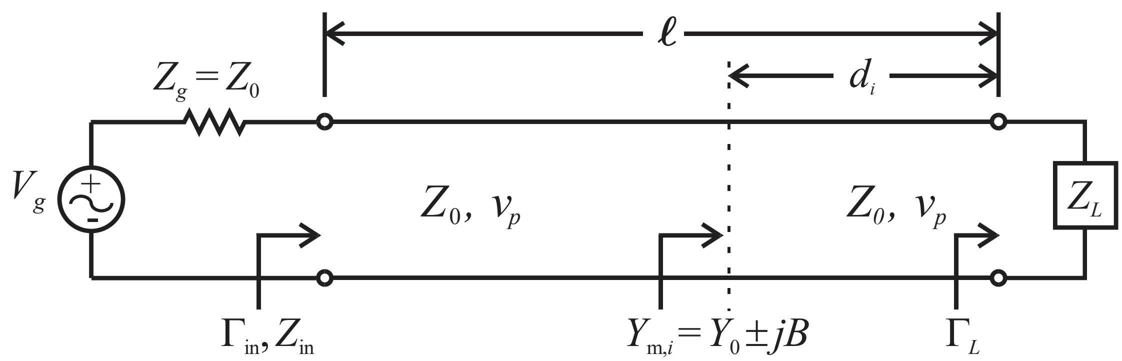
- Assume we have a source matched to the characteristic impedance Z_0 of the transmission line (TL).



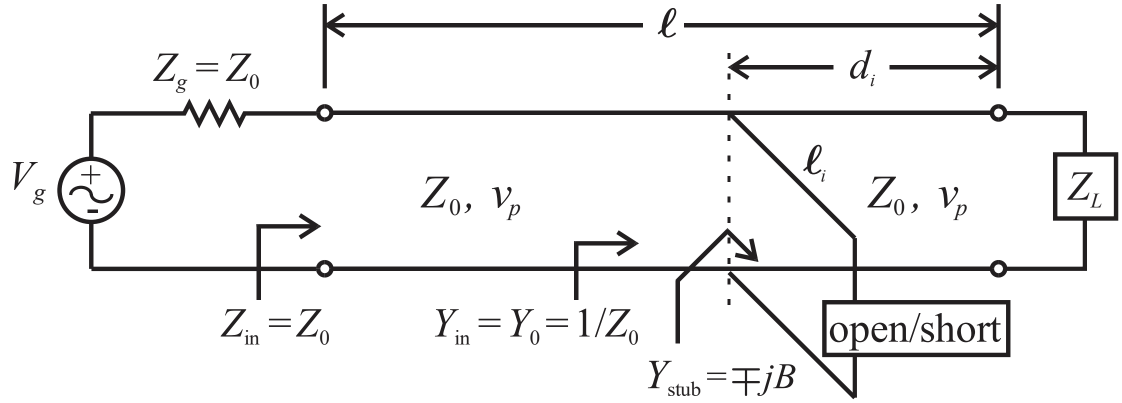
- Again, we are seeking to match the load Z_L to Z_0 as well, i.e., we want $Z_{\text{in}} = Z_0$.
- To avoid power losses, we will only use lossless (i.e., purely reactive) components for matching.
- In this case, the lossless components will be open-circuit or short-circuit terminated TL stubs.
- There are two configurations for single-stub tuning: shunt (parallel) or series.

Shunt Single-Stub

- To use a **shunt stub**, we need to place it at a location (i.e., a match point) where the input admittance is $Y_{\text{in},m} = Y_0 \pm jB$.

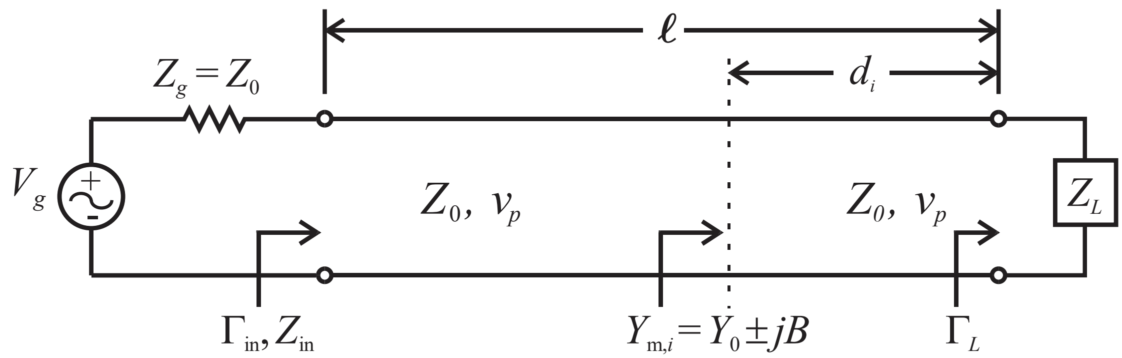


- Then, we connect a shunt (parallel) stub with an open-circuit or short-circuit termination of length selected so that the input admittance of the stub is $Y_{\text{stub}} = \mp jB$. The parallel combination results in an overall input admittance of $Y_{\text{in}} = Y_{\text{in},m} + Y_{\text{stub}} = Y_0 \pm jB \mp jB \quad \Rightarrow \quad \underline{Y_{\text{in}} = Y_0}$ (matched).



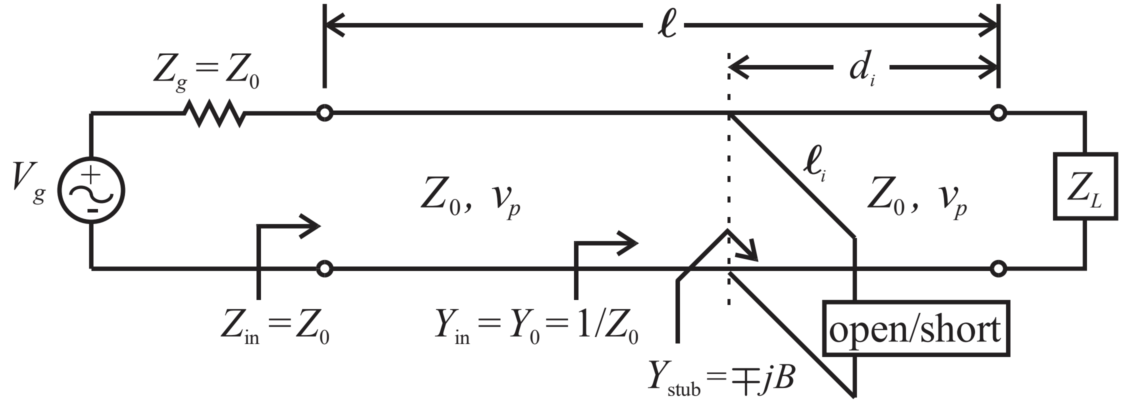
Shunt (Parallel) Single-Stub Tuning Steps

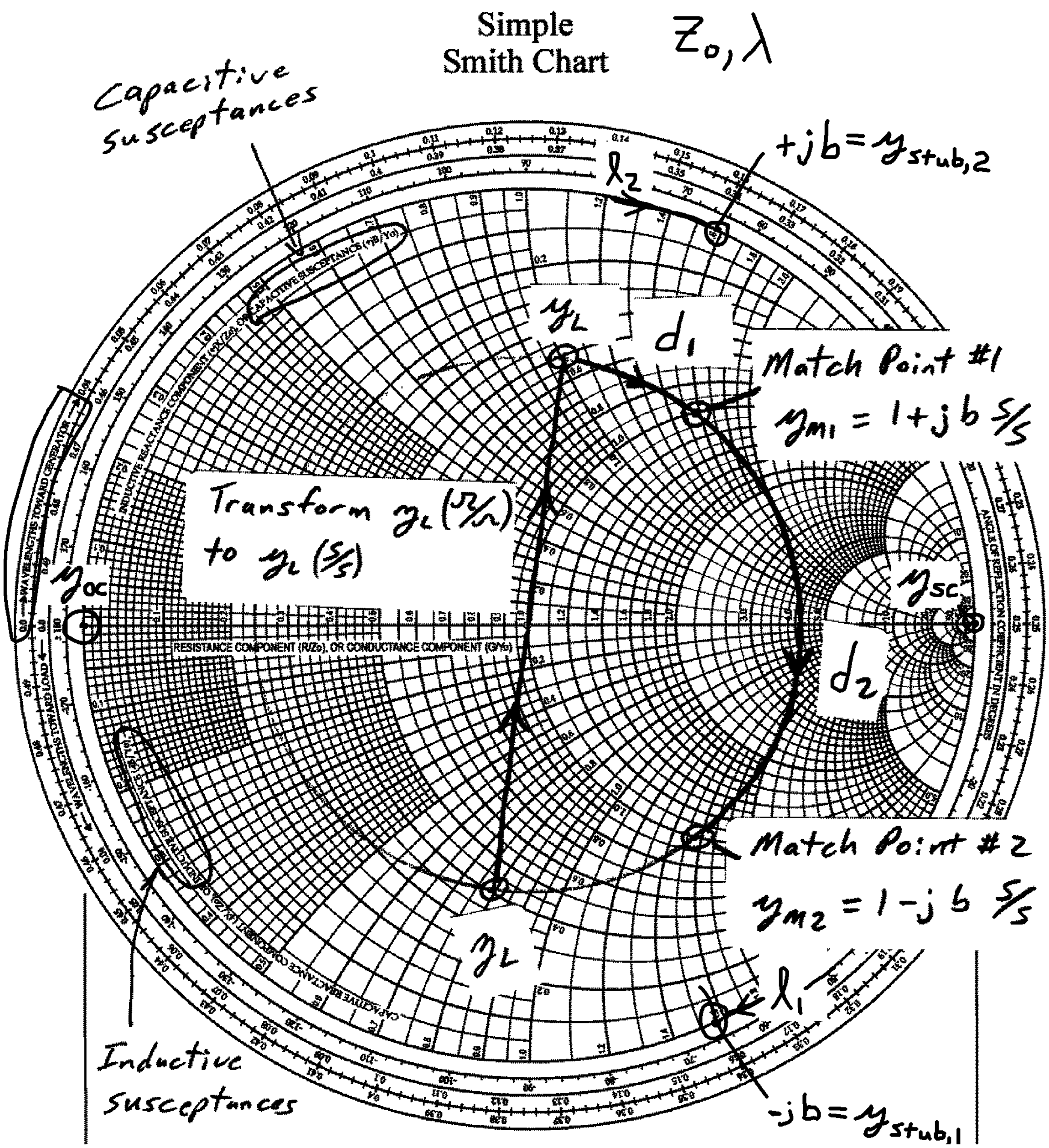
1) Calculate the normalized impedance $z_L = Z_L/Z_0$ and plot on Smith chart.
2) Draw circle, centered on Smith chart, through z_L point. This circle of constant $|\Gamma|$ includes the locus of all possible z_{in} (and y_{in}) along the TL with this load.
3) Go $\lambda/4$ around the circle of constant $|\Gamma|$ from z_L point to y_L point.
4) There are two points (i.e., match points) on the circle of constant $|\Gamma|$ that intersect the circle where the normalized conductance $g = 1$, i.e., $y_{m,i} = 1 \pm jb$. In terms of input admittance, this is where $Y_{m,i} = y_{m,i}/Z_0 = Y_0 \pm jB = 1/Z_0 \pm jB$.
5) Find the distance d_i from y_L to the match points using the "WAVELENGTHS TOWARD GENERATOR" scale.



6) Find length ℓ_i of short-circuit or open-circuit terminated stubs (use same transmission line) that will yield a normalized admittance of $y_{stub,i} = \mp jb$ by starting at either $y_{SC} \to \infty$ or $y_{OC} = 0$ and moving distance ℓ_i "WAVELENGTHS TOWARD GENERATOR" to the $\mp jb$ points.
7) Select one of the match points and the corresponding short-circuit or open-circuit terminated stub.

8) Everywhere along the TL from the stub location to the generator the normalized input admittance will be $y_{\text{in}} = y_{\text{m},i} + y_{\text{stub},i} = (1 \pm jb) \mp jb \Rightarrow y_{\text{in}} = 1$ or normalized input impedance $z_{\text{in}} = 1$, i.e., $Y_{\text{in}} = Y_0$ and/or $Z_{\text{in}} = Z_0$.





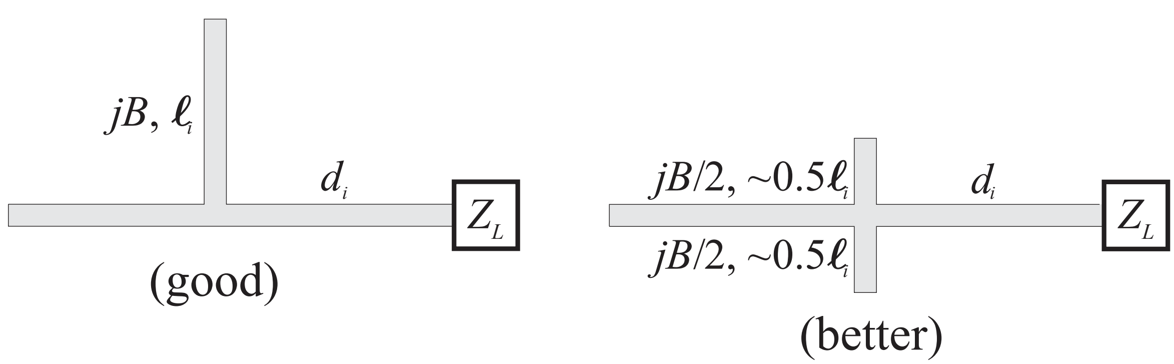
Shunt Single-stub Tuning Notes/Comments:

- Generally, shunt stubs are terminated with either open ($Y_{OC} = 0$) or short ($Y_{SC} \to \infty$) circuits for economic as well as practical reasons, i.e., can fabricate good open-circuit and short-circuit terminations with well defined locations.
- While not required, the TL used for the stubs will typically have the same characteristic impedance Z_0 as the main TL. Occasionally, a different characteristic impedance $Z_{0,\text{stub}}$ may be chosen for the stub to make it shorter/longer.
- In theory, any purely reactive load (e.g., capacitor or inductor) could be used to terminate a stub. This is seldom done as stubs are usually used to avoid the cost and difficulty of placing discrete components.
- Shunt single-stub tuners are inherently narrow-band matching solutions, both the location and length of the single-stub tuner are in terms of wavelength λ. Picking stubs as short as possible helps increase usable bandwidth, i.e., stay on relatively flat portions of the tan() and/or cot() functions.
- Often, instead of using a single parallel stub with admittance $y_{\text{stub}} = \mp jb$, a pair of stubs, each with an admittance $y_{\text{stub}} = \mp jb/2$, are used. The overall input admittance remains unchanged, i.e.,

$$y_{\text{in}} = y_{\text{m},i} + y_{\text{stub},i}/2 + y_{\text{stub},i}/2 = (1 \pm jb) \mp jb/2 \mp jb/2 = 1.$$

  This works particularly well with microstrip where the stubs can be placed on either side of the main microstrip (see below). **Why?** Better bandwidth with shorter stubs, makes electric and magnetic fields more symmetric (good), less obtrusive on circuit board, …

<u>**Top view**</u>



ex. Match a load of $Z_L = 22 - j30\,\Omega$ to a $75\,\Omega$ transmission line ($v_p = 2\times10^8\,m/s$) using an open circuit terminated shunt stub made of same TL when operating at 1.25 GHz. Make distance from load to stub as short as possible. Also, find length of stubs if a pair is used.

→ $z_L = \frac{Z_L}{Z_0} = \frac{22 - j30}{75} = 0.293 - j0.4\ \Omega/\Omega$

→ Plot z_L on Smith chart

→ Draw circle, centered on Smith chart, through z_L. Note: VSWR = 4, $|\Gamma| = 0.6$

→ Go $\lambda/4$ around Smith chart to $y_L = 1.2 + j1.63\ S/S$ by drawing radial line through z_L & center of Smith chart

→ Note where $|\Gamma| = 0.6$ circle intersects the $g = 1$ circle at match points:

$$y_{m1} = 1 + j1.5\ S/S$$

$$y_{m2} = 1 - j1.5\ S/S$$

→ Traveling along the $|\Gamma| = 0.6$ circle in the 'WAVELENGTHS TOWARD GENERATOR' (WTG) direction from y_L, the match point $y_{m2} = 1 - j1.5$ S/S is the first encountered at a distance

$$d_2 = 0.324\lambda - 0.185\lambda$$

$$\underline{\underline{d_2 = 0.139\lambda}}$$

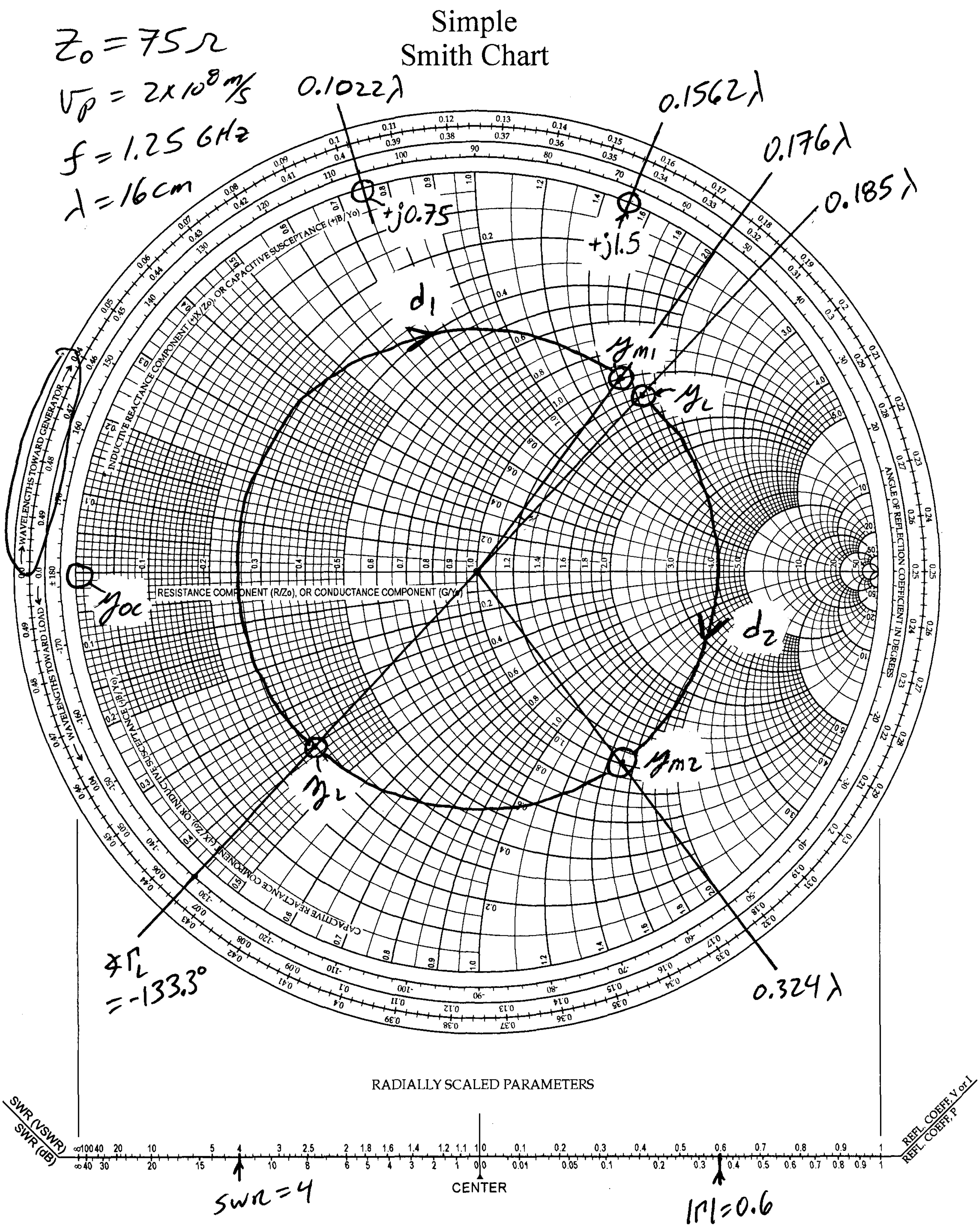
→ To cancel the $-j1.5$ S/S susceptance of y_{m2}, we will need an open circuit ($y_{oc} = 0$) stub of $+j1.5$ S/S or two open circuit stubs of $+j0.75$ S/S each.

$+j1.5$ S/S single $\underline{\underline{\ell_2 = 0.1562\lambda}}$

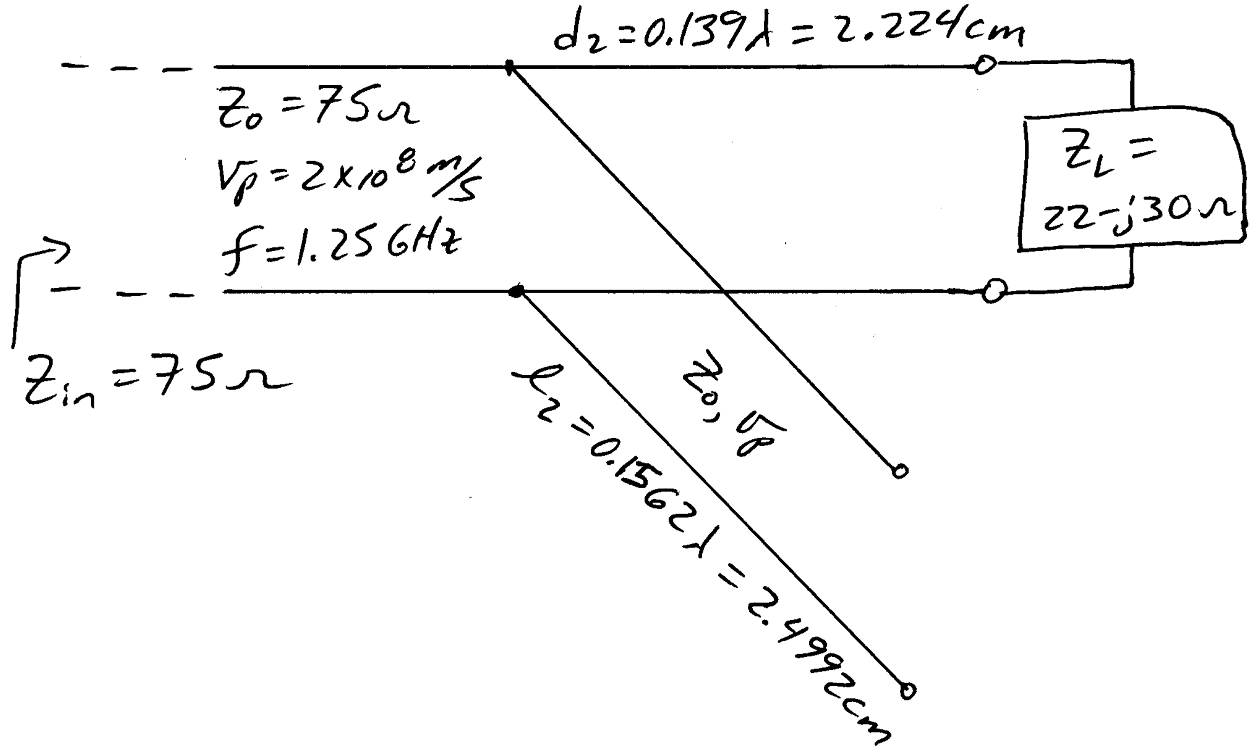
$+j0.75$ S/S pair $\underline{\underline{\ell_{2,2} = 0.1022\lambda}}$

→ To calculate actual dimensions, find wavelength

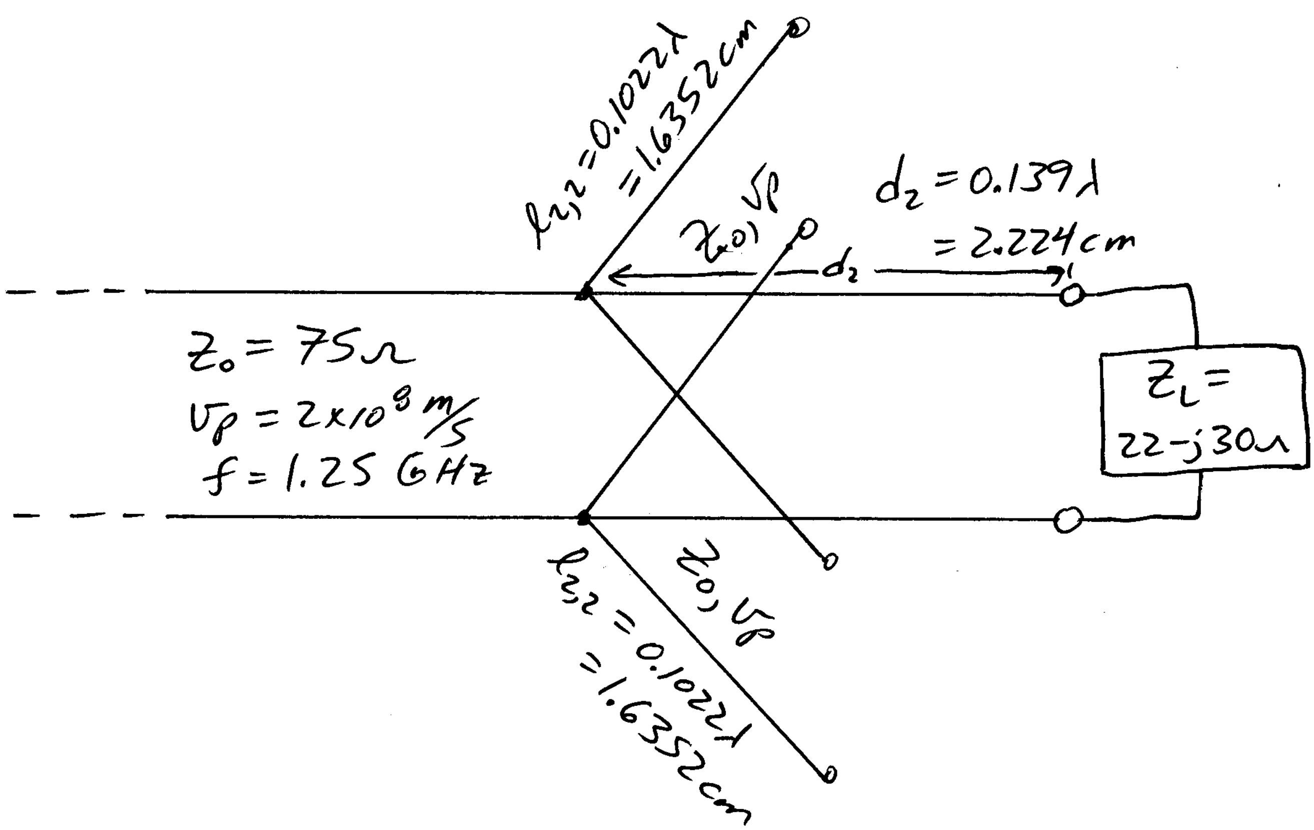
$$\lambda = v_p/f = \frac{2 \times 10^8}{1.25 \times 10^9} = 0.16\,\text{m} = 16\,\text{cm}$$


Simple
Smith Chart
Zo = 75Ω
vp = 2×10^8 m/s
f = 1.25 GHz
λ = 16cm
0.1022λ
0.1562λ
0.176λ
0.185λ
+j0.75
+j1.5
d1
d2
ym1
yL
yoc
yL
ym2
∡ΓL = -133.3°
0.324λ
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
RADIALLY SCALED PARAMETERS
SWR (VSWR)
SWR (dB)
REFL. COEFF. V or I
REFL. COEFF. P
CENTER
SWR = 4
|Γ| = 0.6

Single Open Circuit Shunt Stub Design

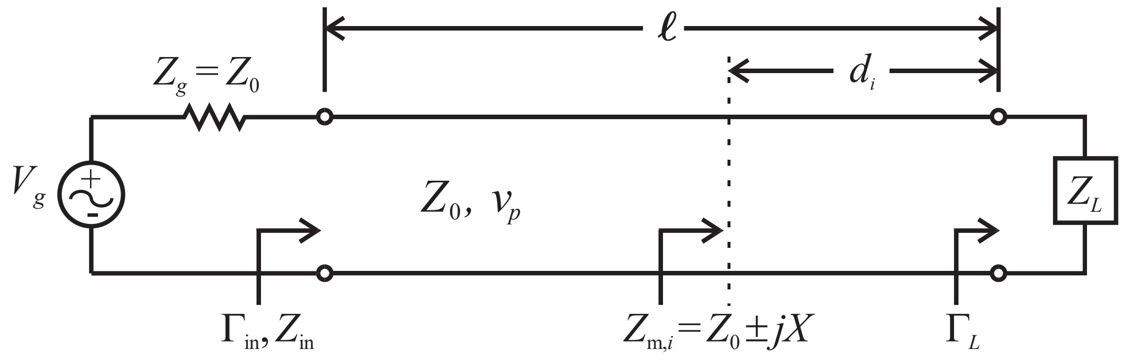


Pair of Open Circuit Shunt Stubs Design

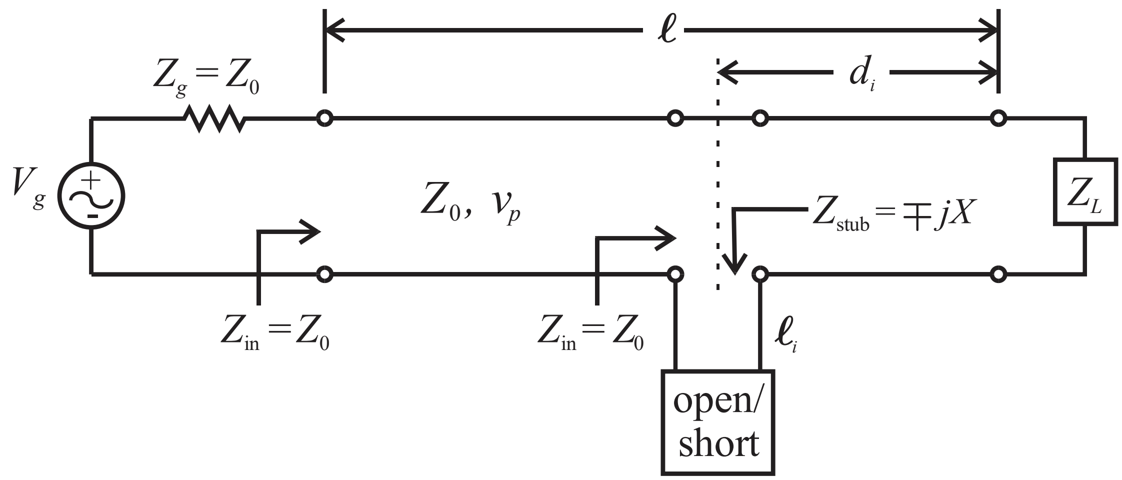


Series Single-Stub

- To use a **series stub**, we need to place it at a location (i.e., a match point) where the input impedance is $Z_{\text{in},m} = Z_0 \pm jX$.



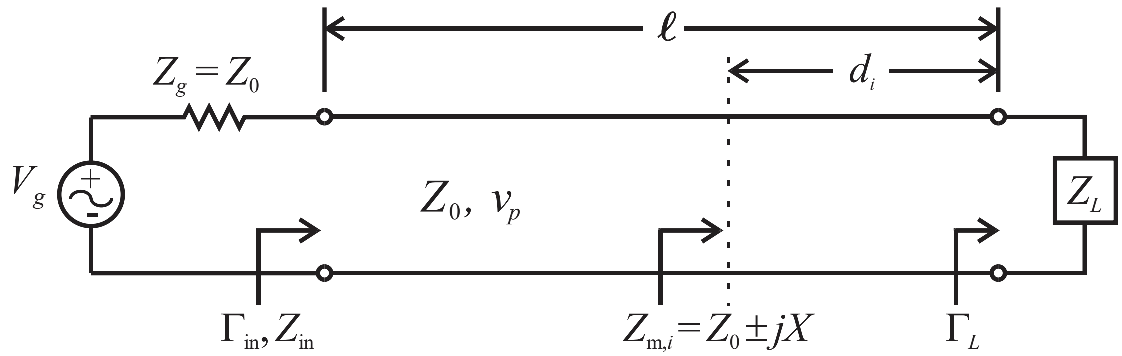
- Then, we connect a series stub with an open-circuit or short-circuit termination of length selected so that the input impedance of the stub is $Z_{\text{stub}} = \mp jX$. The series combination results in an overall input impedance of $Z_{\text{in}} = Z_{\text{in},m} + Z_{\text{stub}} = Z_0 \pm jX \mp jX \quad \Rightarrow \quad \underline{Z_{\text{in}} = Z_0}$ (matched).



Series Single-Stub Tuning Steps

1) Calculate the normalized impedance $z_L = Z_L/Z_0$ and plot on Smith chart.
2) Draw circle, centered on Smith chart, through z_L point. This circle of constant $|\Gamma|$ includes the locus of all possible z_{in} (and y_{in}) along the TL with this load.
3) There are two points (i.e., match points) on the circle of constant $|\Gamma|$ that intersect the circle where the normalized resistance $r = 1$, i.e., $z_{\text{m},i} = 1 \pm jx$. In terms of input impedance, this is where $Z_{\text{m},i} = z_{\text{m},i} Z_0 = Z_0 \pm jX$.

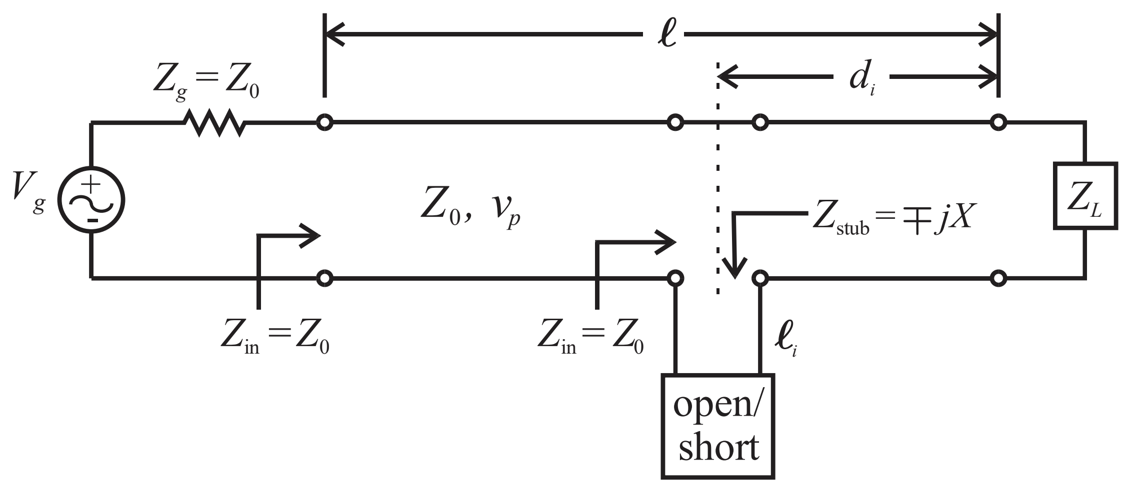
4) Find the distance d_i from z_L to the match points using the "WAVELENGTHS TOWARD GENERATOR" scale.



5) Find length ℓ_i of the short-circuit or open-circuit terminated stubs (use same transmission line) that will yield a normalized impedance of $z_{\text{stub},i} = \mp jx$ by starting at either $z_{SC} = 0$ or $z_{OC} \to \infty$ and moving a distance ℓ_i "WAVELENGTHS TOWARD GENERATOR" to the $\mp jx$ points.

6) Select one of the match points and the corresponding short or open circuit terminated stub.

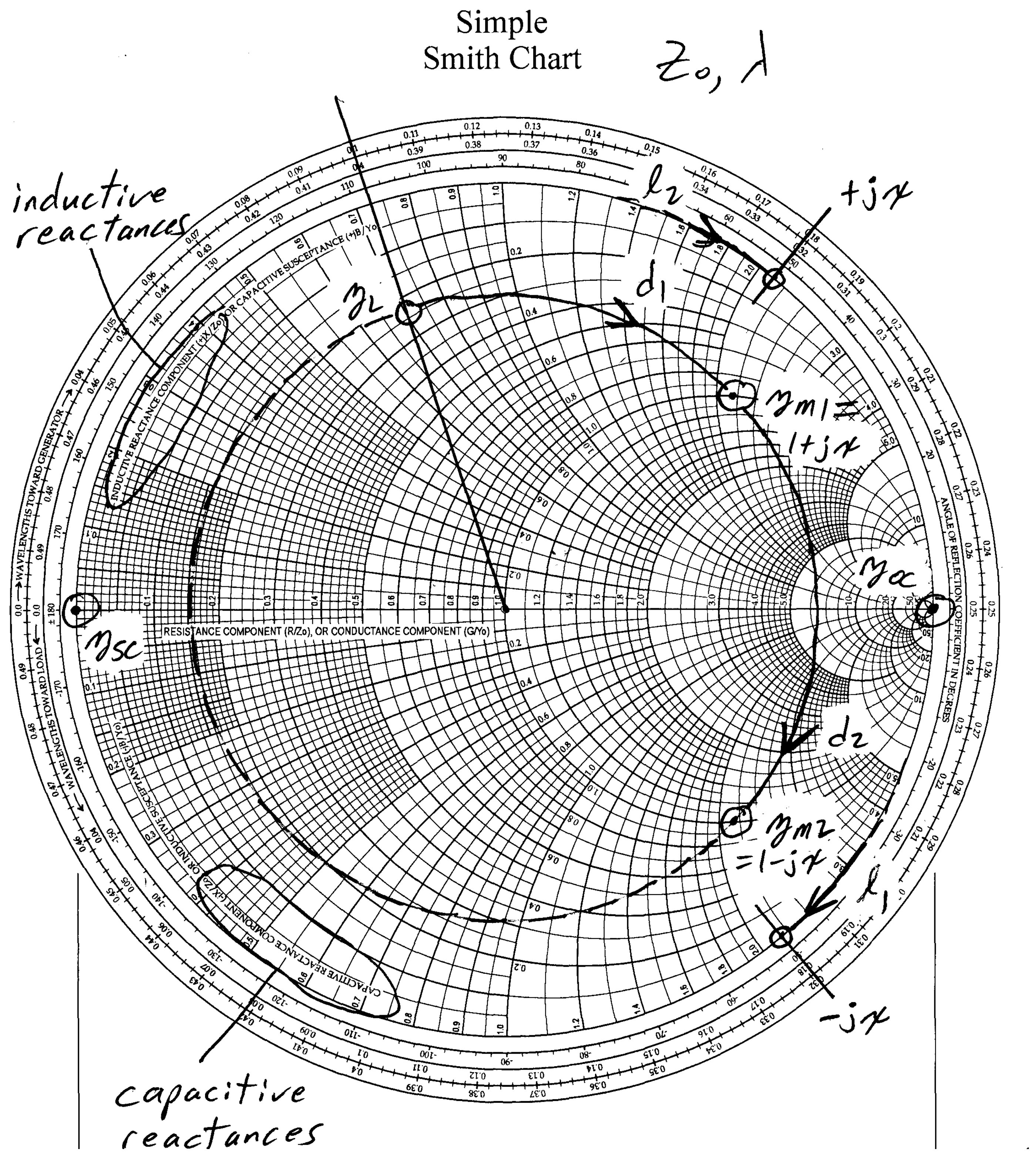
7) Everywhere along the transmission line toward the generator from this location will see a normalized input impedance of $z_{\text{in}} = z_{\text{m},i} + z_{\text{stub},i} = (1 \pm jx) \mp jx = 1$, i.e., $Z_{\text{in}} = Z_0$.



Series Single-stub Tuning Notes/Comments:

- Generally, series stubs are terminated with either open ($Z_{OC} \to \infty$) or short ($Z_{SC} = 0$) circuits for economic as well as practical reasons, i.e., can fabricate good open-circuit and short-circuit terminations with well defined locations.
- While not required, the TL used for the stubs will typically be Z_0.

- In theory, any purely reactive load could be used to terminate a stub, but seldom is done.
- Single-stub tuners are inherently narrow-band matching. Picking stubs as short as possible helps increase bandwidth.



ex. At 4 GHz, match a load $Z_L = 60 + j180\ \Omega$ to a twin-wire transmission line ($300\ \Omega$, $v_p = 2.8 \times 10^8$ m/s) using an open circuit terminated series stub. Make the stub as close as possible to the load and as short as possible.

→ Calculate & plot $z_L = \frac{Z_L}{Z_0} = \frac{60 + j180}{300}$

$$z_L = 0.2 + j0.6\ \Omega/\Omega$$

→ Draw circle of constant $|\Gamma|$ through z_L (Note: $|\Gamma| \simeq 0.74$ and $SWR \simeq 6.8$)

→ Note where $|\Gamma| = 0.74$ circle intersects $r = 1$ circle at

$$\underline{z_{m1} = 1 + j2.2\ \Omega/\Omega}$$

$$z_{m2} = 1 - j2.2\ \Omega/\Omega$$

→ Traveling in the WTG direction along $|\Gamma| \simeq 0.74$ circle, the match point z_{m1} is the first encountered at a distance

$$d_1 = 0.191\lambda - 0.088\lambda = \underline{\underline{0.103\lambda}}$$

→ To counteract the $+j2.2\ \Omega/\Omega$ of inductive reactance, the open circuit terminated stub will need a normalized capacitive reactance of $-j2.2\ \Omega/\Omega$

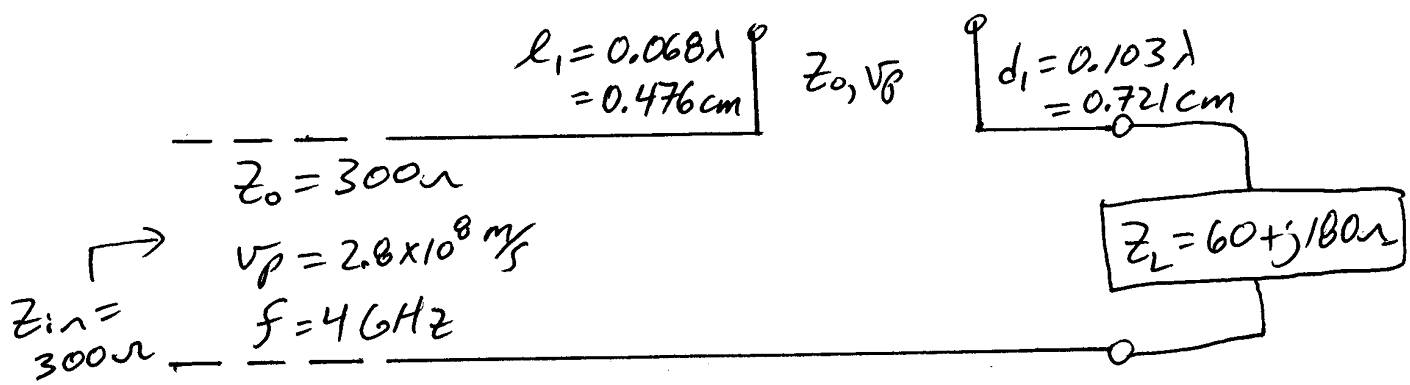
→ starting @ $y_{oc} \to \infty$ @ 0.25λ, the required stub length is

$$\ell_1 = 0.318\lambda - 0.25\lambda = \underline{\underline{0.068\lambda}}$$

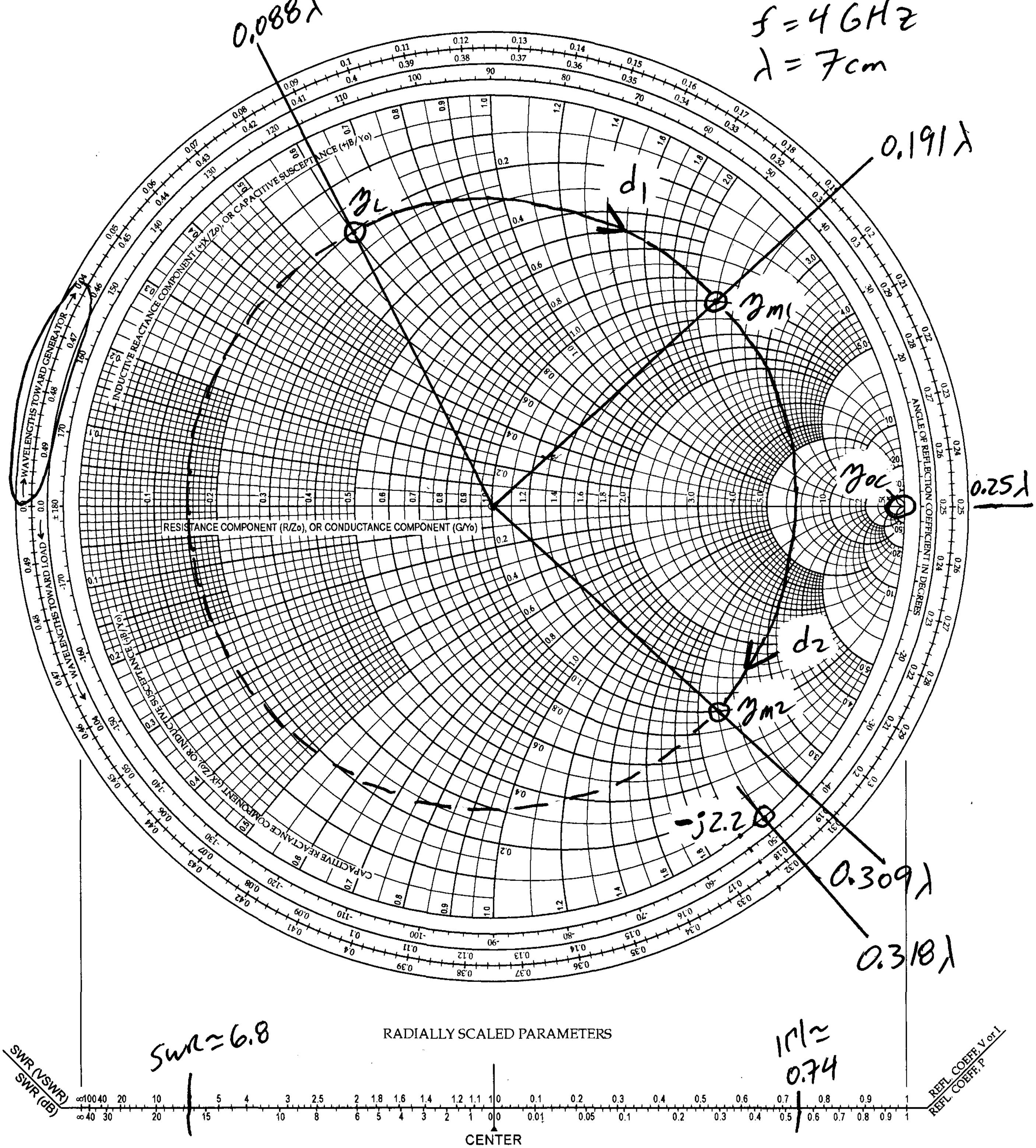
→ Find wavelength in order to calculate actual dimensions

$$\lambda = v_p/f = \frac{2.8\times10^8}{4\times10^9} = 0.07\,m = 7\,cm$$

Series Open Circuit Stub Tuning

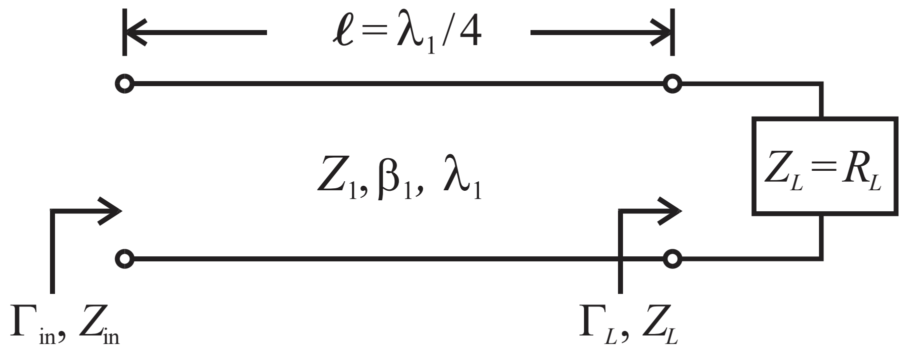


Simple
Smith Chart



2.5/5.4 The Quarter-Wave Transformer

Consider the following lossless transmission line (TL) circuit.



We can determine the input impedance in two ways. First, using (2.61),

$$Z_{\text{in}} = Z_1\left[\frac{R_L + jZ_1\tan(\beta_1\ell)}{Z_1 + jR_L\tan(\beta_1\ell)}\right] = Z_1\left[\frac{R_L/\tan(\beta_1\ell) + jZ_1}{Z_1/\tan(\beta_1\ell) + jR_L}\right]$$

where in this case

$$\beta_1\,\ell = \frac{2\pi}{\lambda_1}\frac{\lambda_1}{4} = \frac{\pi}{2}$$

which implies $\tan(\beta_1\ell) = \tan(\pi/2) \to \infty$. Therefore, the input impedance becomes

$$Z_{\text{in}} = Z_1\left[\frac{R_L/\infty + jZ_1}{Z_1/\infty + jR_L}\right] \quad \Rightarrow \quad \underline{Z_{\text{in}} = \frac{Z_1^{\,2}}{R_L}}.$$

An alternate approach is to use reflection coefficients

$$\Gamma_L = \frac{Z_L - Z_1}{Z_L + Z_1} = \frac{R_L - Z_1}{R_L + Z_1}$$

and

$$\Gamma_{\text{in}} = \Gamma_L e^{-j2\beta_1\ell} = \Gamma_L e^{-j2\frac{2\pi}{\lambda_1}\frac{\lambda_1}{4}} = \Gamma_L e^{-j\pi} \quad \Rightarrow \quad \Gamma_{\text{in}} = -\Gamma_L.$$

Now, the input impedance is

$$Z_{\text{in}} = Z_1\left[\frac{1+\Gamma_{\text{in}}}{1-\Gamma_{\text{in}}}\right] = Z_1\left[\frac{1-\Gamma_L}{1+\Gamma_L}\right] = Z_1\left[\frac{1-\dfrac{R_L - Z_1}{R_L + Z_1}}{1+\dfrac{R_L - Z_1}{R_L + Z_1}}\right] \quad \Rightarrow \quad \underline{Z_{\text{in}} = \frac{Z_1^{\,2}}{R_L}}.$$

How can we use this result to match a load to a TL? That is, we desire to make $Z_{in} = Z_0$.

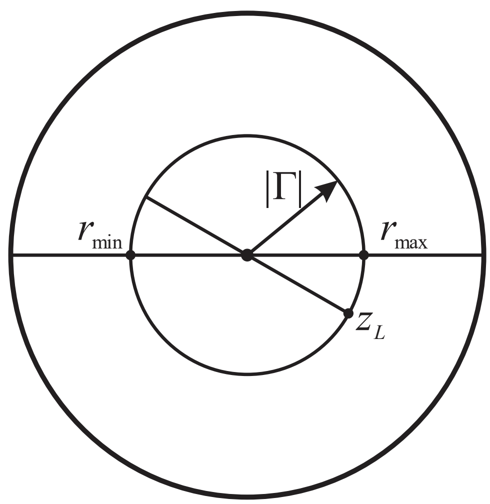
$$Z_{in} = Z_0 = \frac{Z_1^2}{R_L} \quad \Rightarrow \quad \underline{Z_1 = \sqrt{Z_0 R_L}}$$

By properly selecting the impedance Z_1 of a quarter-wave section ($\lambda_1/4$) of TL, we can transform a load **resistance** of R_L to an input impedance equal to Z_0, i.e., a **Quarter-Wave Transformer (QWT)**.

Concerns

1) What if $Z_L \neq R_L$?

   If we plot the normalized load impedance z_L on a Smith chart and draw a circle, centered on Smith chart, through z_L, the circle represents the locus of all possible normalized input impedances z_{in} along a lossless Z_0 TL connected to the load. Where the circle crosses the real axis, we have r_{max} and r_{min} (i.e., R_{max} and R_{min}). Therefore, we can create a resistive load $Z_L' = R_{max}$ or $Z_L' = R_{min}$ by introducing a length/section of Z_0 TL before the QWT.



2) We need a customized characteristic impedance Z_1 for the QWT.

   With coaxial, twin-wire, and similar TLs, this is NOT a practical solution. It is difficult to manufacture a custom TL with a Z_1 characteristic impedance as well as attaching/connecting it to the desired TL with characteristic impedance Z_0. This typically limits QWTs to microstrip, planar, & stripline TL applications where changing the characteristic impedance is simply a matter of varying the width of the TLs.

3) When we change characteristic impedance in a TL, it is not unusual for the phase velocity and wavelength to change as well, i.e., $\lambda_1 \neq \lambda_0$ for the Z_1 and Z_0 TLs respectively.

4) From the very name, **Quarter-Wave Transformer**, we can expect the QWT work perfectly only at the frequency where $\ell = \lambda_1/4$. This typically limits the usable bandwidth of a QWT to a narrow band of frequencies around the design frequency f_0.

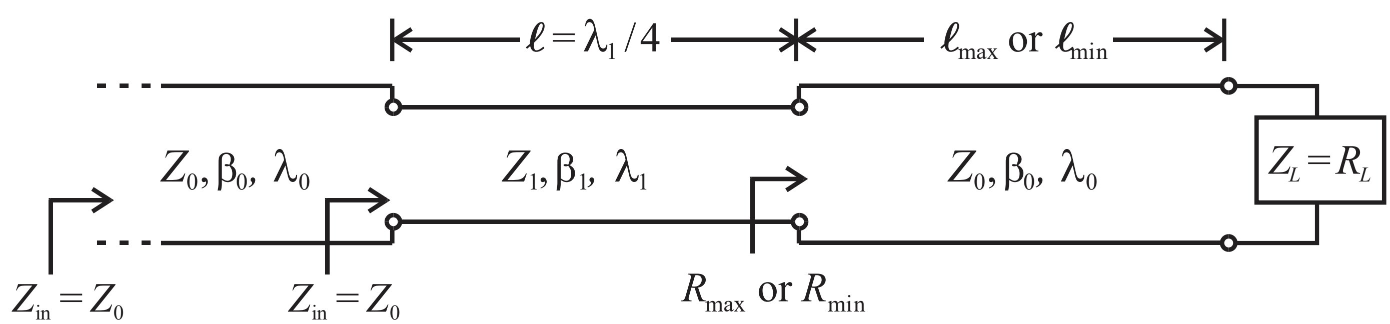
   In section 5.4 of the text, the author derives an approximate expression for the fractional bandwidth that can be expected for a QWT to be

$$\frac{\Delta f}{f_0} \approx 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_0 R_L}}{\left|R_L - Z_0\right|}\right]$$

   where Γ_m is the maximum acceptable reflection coefficient magnitude and f_0 is your center/design operating frequency.

QWT Design Steps

1) Calculate the normalized load impedance $z_L = Z_L/Z_0$ and plot on Smith chart.
2) Draw circle, centered on Smith chart, through z_L point. This circle of constant $|\Gamma|$ includes the locus of all possible z_{in} along the transmission line with this load.
3) Note the locations and values of r_{min} and r_{max}, i.e., our match points.
4) Select one of the two match points. Compute $R_{\text{min}} = r_{\text{min}} Z_0$ or $R_{\text{max}} = r_{\text{max}} Z_0$. [Note: you can look ahead to step 6 to see if one choice is 'better'.]
5) Determine, using the 'WAVELENGTHS TOWARD GENERATOR' scale, the distance ℓ_{min} or ℓ_{max} from the load z_L to the selected match point along a section of Z_0 transmission line.
6) Compute the characteristic impedance $Z_1 = \sqrt{Z_0 R_{\text{max}}}$ or $Z_1 = \sqrt{Z_0 R_{\text{min}}}$ of the QWT. By definition, the length of the QWT is $\ell = \lambda_1/4$.
7) After the QWT, attach any length of the desired Z_0 transmission line to get to the generator. The load is matched!



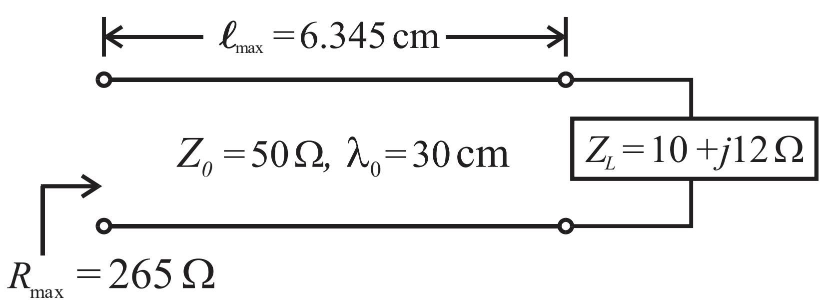
Example- Match a load of $Z_L = 10 + j\,12\ \Omega$ to a 50 Ω microstrip transmission line (λ = 30 cm) using a quarter-wave transformer (QWT) and 50 Ω microstrip. Restriction- the match should be as short as possible.

1) Normalize Z_L and plot on Smith chart

- Normalize $z_L = Z_L / Z_0 = (10 + j\,12)/50 \;\Rightarrow\;$ **$\underline{z_L = 0.2 + j0.24\ \Omega/\Omega}$**.
- Plot z_L on Smith chart by finding intersection of $r = 0.2$ circle & $x = 0.24$ arc.

2) Find first point along 50 Ω microstrip where the impedance is real

- Use compass to draw arc of constant |Γ| from z_L point on Smith chart in the "WAVELENGTHS TOWARD GENERATOR" direction until reaching the horizontal/real axis to right of origin.
- Read $\underline{r_{max} = 5.3}$ on Smith chart. This corresponds to $R_{max} = R_{max}\, Z_0 = (5.3)\,50$ $\Rightarrow$ **$\underline{R_{max} = 265\ \Omega}$**.
- Find distance from z_L to r_{max} by drawing radial line from the center of Smith chart through z_L and the "WAVELENGTHS TOWARD GENERATOR" scale, reading 0.0385 and noting r_{max} is at 0.25 on the scale. The distance $\ell_{max} = (0.25\text{-}0.0385)\lambda = 0.2115\lambda \;\Rightarrow\;$ **$\underline{\ell_{max} = 6.345\ \text{cm}}$**.



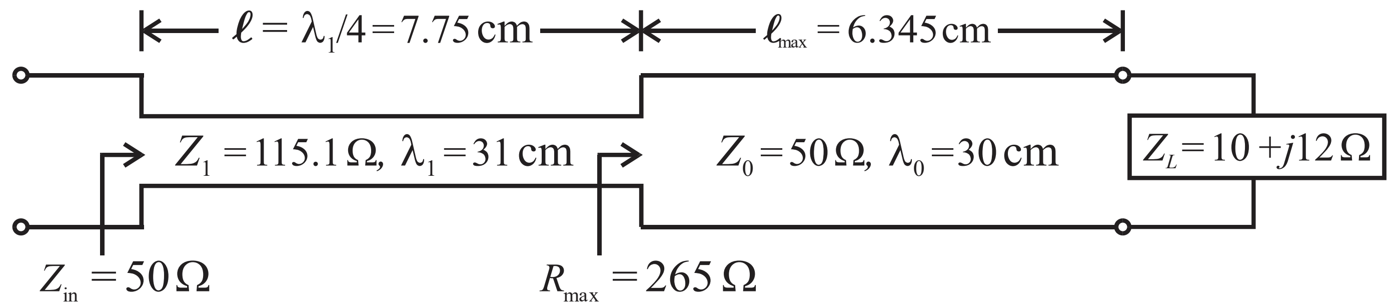
- Now that we have a real impedance, use a QWT to match to 50 Ω (next step).

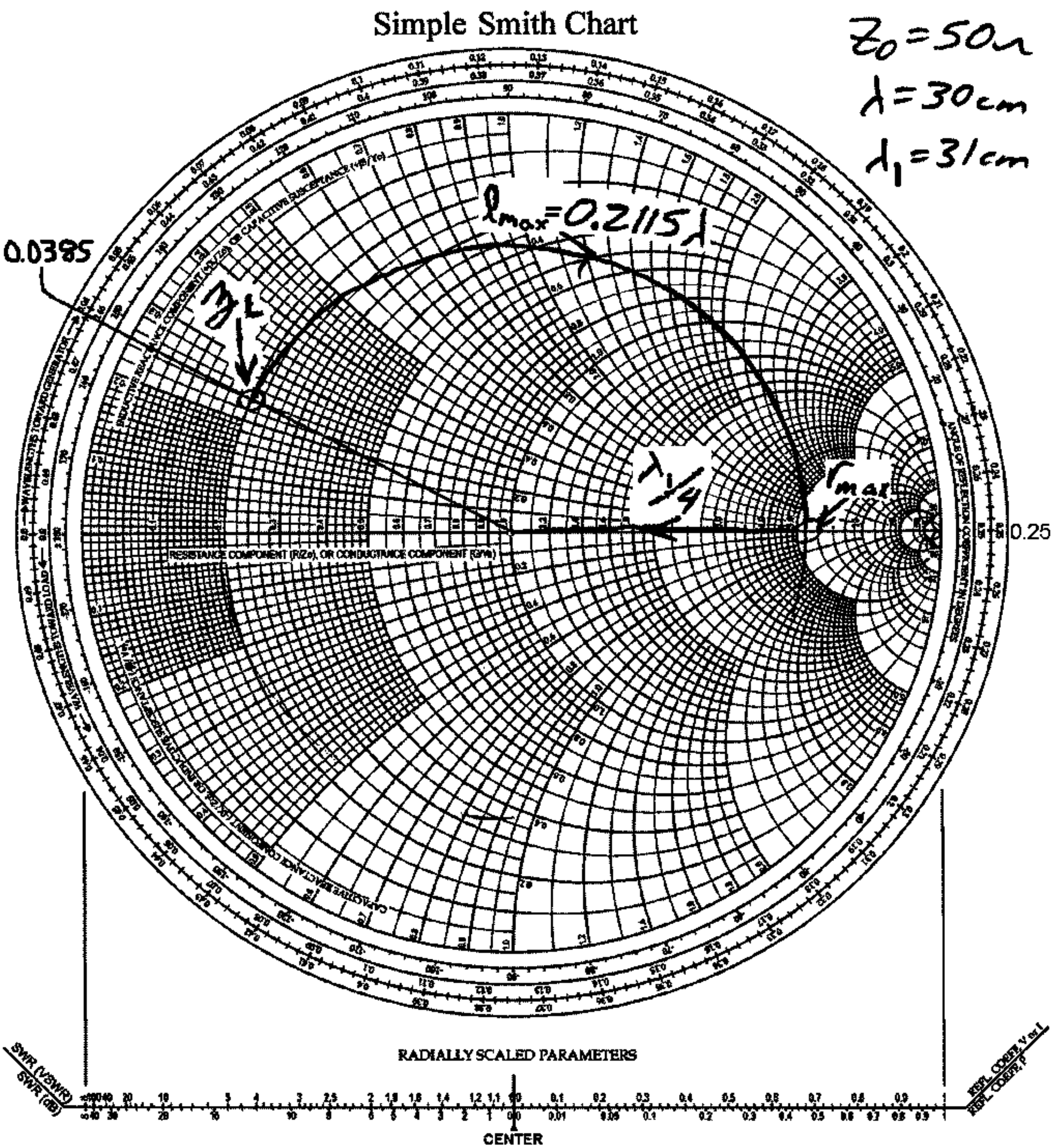
3) Design QWT to match R_{max} to 50 Ω

- Use equation to find characteristic impedance of QWT

$$Z_1 = \sqrt{Z_0 R_{max}} = \sqrt{50(265)} \;\Rightarrow\; \underline{\mathbf{Z_1 = 115.109\ \Omega}}.$$

- By definition, a QWT has a length $\ell = \lambda_1/4$. The wavelength λ_1 on 115.1 Ω microstrip will NOT be the same as λ_0 = 30 cm for the 50 Ω microstrip (Note: wavelength for microstrip depends on circuit board material & thickness as well as the microstrip width). For the sake of this example, assume λ_1 = 31 cm. Hence, **$\underline{\ell = \lambda_1/4 = 7.75\ \text{cm}}$**.


ℓ = λ1/4 = 7.75 cm
ℓmax = 6.345 cm
Z1 = 115.1 Ω, λ1 = 31 cm
Z0 = 50 Ω, λ0 = 30 cm
ZL = 10 + j12 Ω
Zin = 50 Ω
Rmax = 265 Ω


Simple Smith Chart
Z0 = 50Ω
λ = 30cm
λ1 = 31cm
ℓmax = 0.2115λ
0.0385
zL
λ1/4
rmax
0.25
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
RADIALLY SCALED PARAMETERS
CENTER