Chapter 4 Microwave Network Analysis

For many problems, circuits, ..., we are only interested in terminal quantities such as voltage, current, power, ...
In these situations, matrix descriptions for sub-circuits or building blocks are very useful to avoid bogging down in details of these pieces.

$\Downarrow$

Microwave Network Theory

→ History: This came out of the MIT Radiation Lab in the 1940's

4.1 Impedance and Equivalent Voltages & Currents

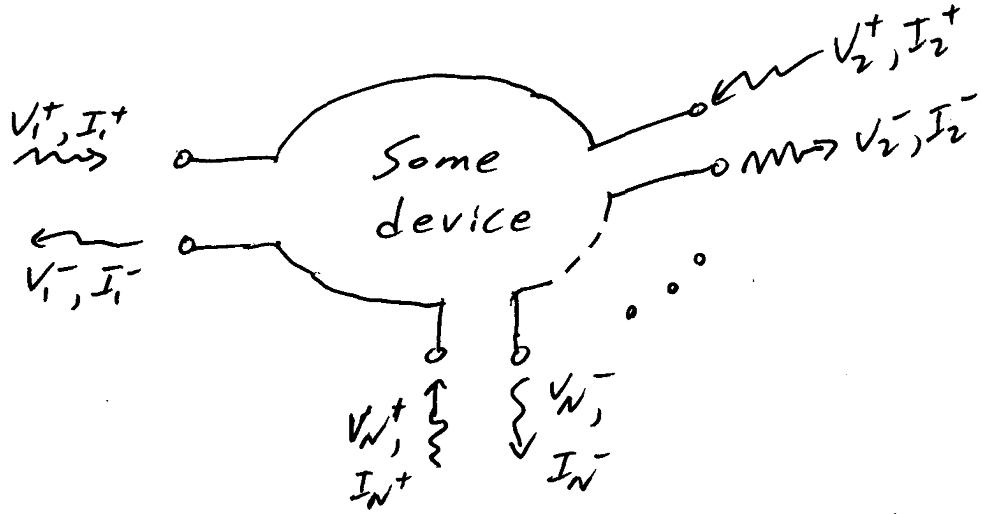
→ Not always easy or possible to measure voltages &/or currents @ microwave frequencies & devices (e.g., waveguide)

Impedance – first used by Oliver Heaviside in late 1800's for AC circuits

Types:

* Intrinsic impedance $\equiv \eta = \sqrt{\mu/\epsilon}$ only material
* Wave impedance $\equiv Z_w = E_t/H_t$ freq., material, & wave TE, TM, & TEM
* Characteristic impedance $\equiv Z_0 = \frac{1}{Y_0} = \frac{V^+}{I^+}$ ratio of voltage & current in a traveling wave

4.2 Impedance and Admittance Matrices



At each port of the device, we have both incident and reflected current & voltage waves. At the terminals (AKA: reference, phase, or terminal planes), the total current & voltage for the n^{th} terminal is

$$V_n = V_n^+ + V_n^- \qquad (4.24a)$$

$$I_n = I_n^+ + I_n^- \qquad (4.25b)$$

$\Rightarrow$ Quite similar to our phasor current & voltage equations for TLs @ $z=0$.

Define impedance (4.28) $\quad Z_{ij} = \left.\dfrac{V_i}{I_j}\right|_{I_k = 0 \text{ for all } k \neq j}$

That is, drive port or terminal j w/ a current I_j while leaving all other ports open-circuited and measure the open-circuit voltage V_i @ the port/terminal i.

4.2 cont.

Note: * Z_{ii} is the input impedance of the i^{th} port w/ all other ports open-circuited.

* Z_{ij} can be thought of as a transfer impedance between ports i & j w/ all others are open-circuited

Using Z_{ij}, we can related all port/terminal currents & voltages using the impedance matrix $[Z]$:

$$[V] = [Z][I] \qquad (4.25)$$

or

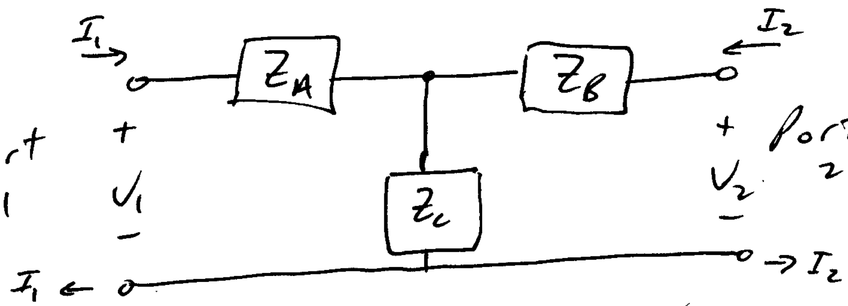
$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & & Z_{2N} \\ \vdots & & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

⇒ The voltages are phasors as well as the currents, i.e., complex.

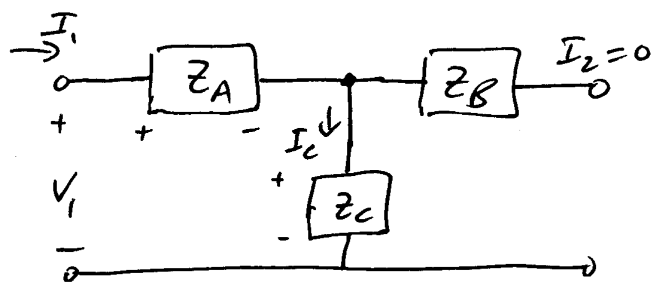
⇒ The impedances are complex numbers as well (NOT phasors).

4.2 cont.

example - Find the z parameters for the two-port T-network.



$\boxed{Z_{11}}$ → set $I_2 = 0$, $Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$



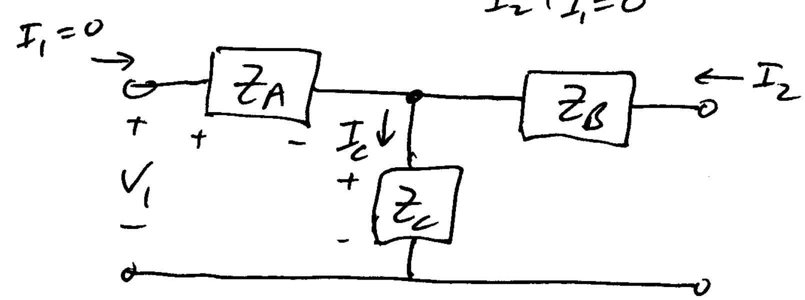
By KCL, $I_C = I_1 + \cancel{I_2}^{\,0} = I_1$

By KVL, $-V_1 + I_1 Z_A + \cancel{I_C}^{\,I_1} Z_C = 0$

$V_1 = I_1 (Z_A + Z_C)$

$Z_{11} = V_1 / I_1 = Z_A + Z_C$

$\boxed{Z_{12}}$ → set $I_1 = 0$, $Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$



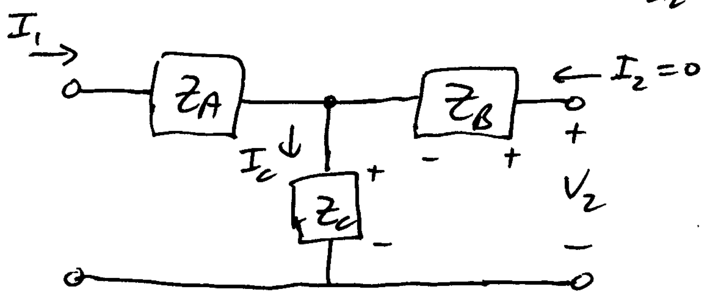
By KCL, $I_C = \cancel{I_1}^{\,0} + I_2 = I_2$

By KVL, $-V_1 + 0(Z_A) + \cancel{I_C}^{\,I_2} Z_C = 0$

$V_1 = I_2 Z_C \Longrightarrow Z_{12} = \frac{V_1}{I_2} = Z_C$

4.2 cont.

$\boxed{Z_{21}}$ → set $I_2 = 0$, $Z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2=0}$



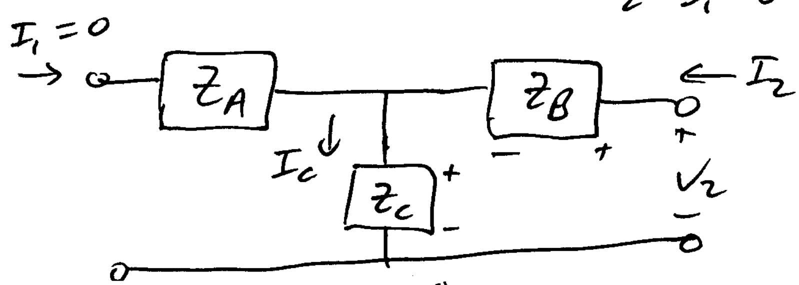
By KCL, $I_c = I_1 + \cancel{I_2}^{0} = I_1$

By KVL, $-\cancel{I_c}^{I_1} Z_c - \cancel{I_2}^{0} Z_B + V_2 = 0$

$$V_2 = I_1 Z_c$$

$$\underline{\underline{Z_{21} = V_2/I_1 = Z_c}}$$

$\boxed{Z_{22}}$ → set $I_1 = 0$, $Z_{22} = \left.\dfrac{V_2}{I_2}\right|_{I_1=0}$



By KCL, $I_c = \cancel{I_1}^{0} + I_2 = I_2$

By KVL, $-\cancel{I_c}^{I_2} Z_c - I_2 Z_B + V_2 = 0$

$$V_2 = I_2 (Z_B + Z_c)$$

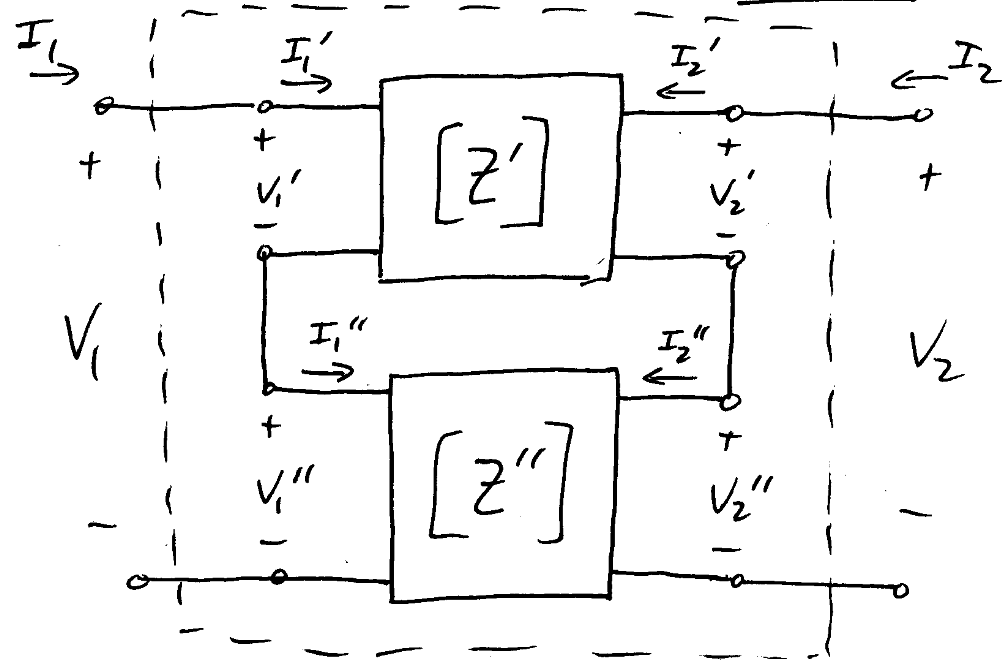
$$\underline{\underline{Z_{22} = V_2/I_2 = Z_B + Z_c}}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_A + Z_c & Z_c \\ Z_c & Z_B + Z_c \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{ for T-Network}$$

4.2 cont.

Where might we use impedance / [Z] matrices?

$\Rightarrow$ Networks connected in series.



Since network $[Z']$ is in series with network $[Z'']$, $I_1' = I_1'' = I_1$ and $I_2' = I_2'' = I_2$.

In addition, from KVL, we see that

$V_1 = V_1' + V_1''$ and $V_2 = V_2' + V_2''$.

In matrix form, we get

$$\begin{bmatrix} V_1' \\ V_2' \end{bmatrix} + \begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix} = [Z']\begin{bmatrix} I_1' \\ I_2' \end{bmatrix} + [Z'']\begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix}$$

$$\begin{bmatrix} V_1' + V_1'' \\ V_2' + V_2'' \end{bmatrix} = [Z']\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + [Z'']\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

add up

same

4.2 cont.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \left\{ [Z'] + [Z''] \right\} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Downarrow$$

For series networks, we can add up the impedance/[Z] matrices of the individual networks to get a series equivalent impedance/[Z] matrix.

I.e., $[Z] = [Z'] + [Z''] + \ldots$

In a similar fashion, define admittance

(4.29) $Y_{ij} = \left.\dfrac{I_i}{V_j}\right|_{V_k = 0 \text{ for } k \neq j}$

That is, drive port/terminal j with a voltage V_j while short circuiting all other ports ($V_k = 0$) and determine/measure the short-circuit current I_i at port i.

* Y_{ii} is the input admittance @ port i
* Y_{ij} can be thought of as a transfer admittance between ports i & j.

4.2 cont.

Using the Y_{ij}, we can relate all port currents + voltages using the <u>admittance matrix</u> $[Y]$:

$$[I] = [Y][V] \qquad (4.26)$$

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & & Y_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{N1} & Y_{N2} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

By matrix math, $[V] = [Y]^{-1}[I] = [Z][I]$

$$\Rightarrow \quad [Y]^{-1} = [Z]$$

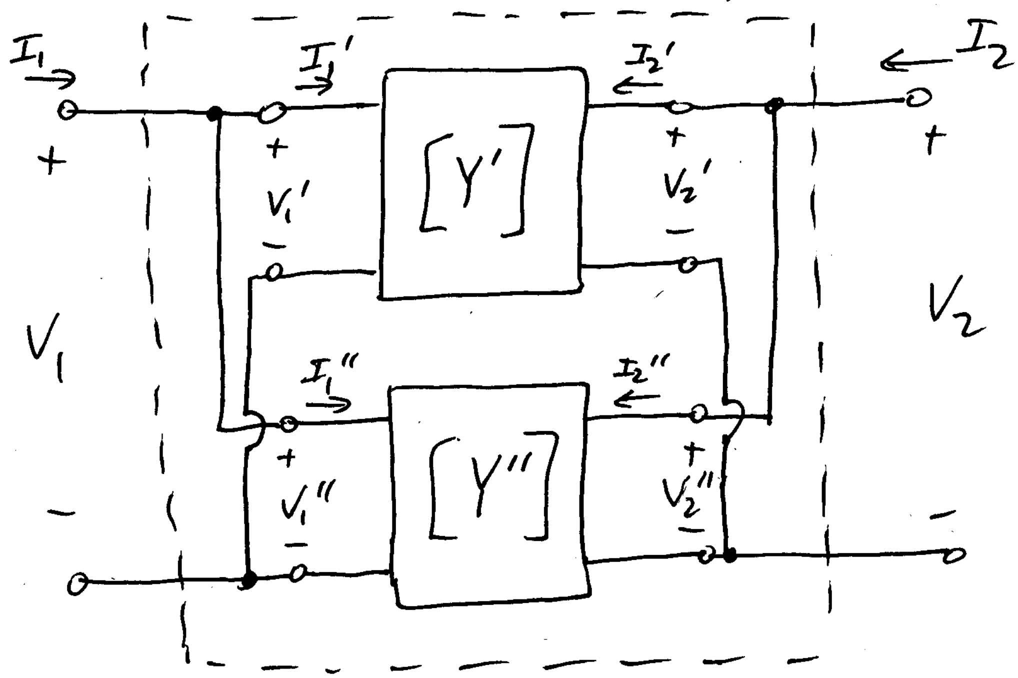
$$\text{OR} \quad [Z]^{-1} = [Y] \qquad (4.27)$$

* Again V_i + I_i are phasors while Y_{ij} are complex numbers.

4.2 cont.

Where might we use admittance/[Y] matrices?

$\Rightarrow$ Networks connected in parallel.



Since networks [Y'] and [Y''] are connected in parallel, $V_1 = V_1' = V_1''$ and $V_2 = V_2' = V_2''$.

By KCL, $I_1 = I_1' + I_1''$ and $I_2 = I_2' + I_2''$.

In matrix form, we get

$$\begin{bmatrix} I_1' \\ I_2' \end{bmatrix} + \begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix} = [Y']\begin{bmatrix} V_1' \\ V_2' \end{bmatrix} + [Y'']\begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix}$$

$$\begin{bmatrix} I_1' + I_1'' \\ I_2' + I_2'' \end{bmatrix} = [Y']\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + [Y'']\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

add-up

same

4.2 cont.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \left\{ [Y'] + [Y''] \right\} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$\Downarrow$

For parallel networks, we can add up the admittance/[Y] matrices of the individual networks to get a parallel equivalent admittance/[Y] matrix.

I.e., $[Y] = [Y'] + [Y''] + \ldots$

General notes for [Z] + [Y] matrices

1) N-port network implies that [Z] + [Y] matrices are $N \times N$ in size with N^2(real) + N^2(imaginary) = $2N^2$ quantities or degrees of freedom.

2) If the network is reciprocal (i.e., no active devices or nonreciprocal materials like ferrites or plasmas), we get symmetric or reciprocal matrices where $Z_{ij} = Z_{ji}$ and $Y_{ij} = Y_{ji}$.
$\Rightarrow$ Can swap ports + get same results.

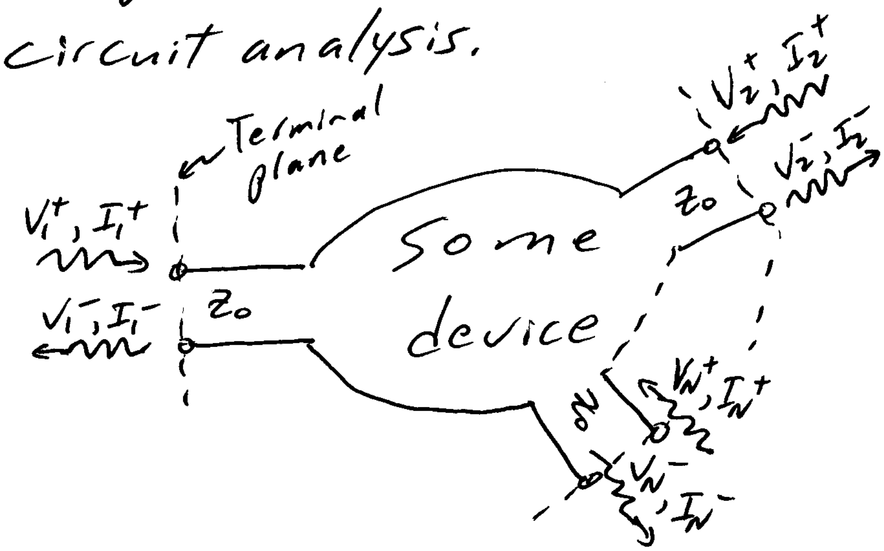
3) Lossless networks will have purely imaginary Z_{ij} and Y_{ij}, $\mathrm{Re}\{Z_{ij}\} = \mathrm{Re}\{Y_{ij}\} = 0$.

4.3 The Scattering Matrix

→ The impedance & admittance parameters find limited usage at microwave frequencies.

→ Of more use are what are known as <u>scattering parameters</u> that are based on incident, reflected/scattered/transmitted waves

→ These scattering parameters are very important for microwave engineering, comparable to currents & voltages in circuit analysis.



Here, V_n^+ are the complex amplitudes of the voltage waves incident at the ports.

Whereas, V_n^- are the complex amplitudes of the voltage waves reflected, scattered, or emerging from the ports.

4.3 cont.

Assuming the characteristic impedance of all the TLs connected to the ports is the same, e.g., Z_0, the scattering (AKA: S-) parameters are defined as

$$S_{ij} = \frac{V_i^-}{V_j^+}\bigg|_{V_k^+=0 \text{ for all } j \neq k} \quad (4.41)$$

That is, S_{ij} is the ratio of the voltage wave emerging from port i due to an incident wave @ port j when the incident waves at all other ports are zero. Practically, that means all other ports should have matched terminations/loads to avoid reflections ($\Gamma_{oc}=1$) or unwanted inputs.

$\Rightarrow S_{ii}$ is the reflection coeff. @ port i (w/ all other ports terminated), i.e., $\Gamma_{ii} = S_{ii}$

$\Rightarrow S_{ij}$ is a transmission coefficient from port j to port i (w/ all other ports terminated), i.e., $T_{ij} = S_{ij}$.

Note: If all other ports are NOT terminated, then the voltage scattered/emerging from the port is simply defined by S_{ii} or S_{ij}.

4.3 cont.

Once the S-parameters are known (measured or computed), we can define the scattering or [S] matrix relating incident and scattered/reflected/transmitted voltage waves

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

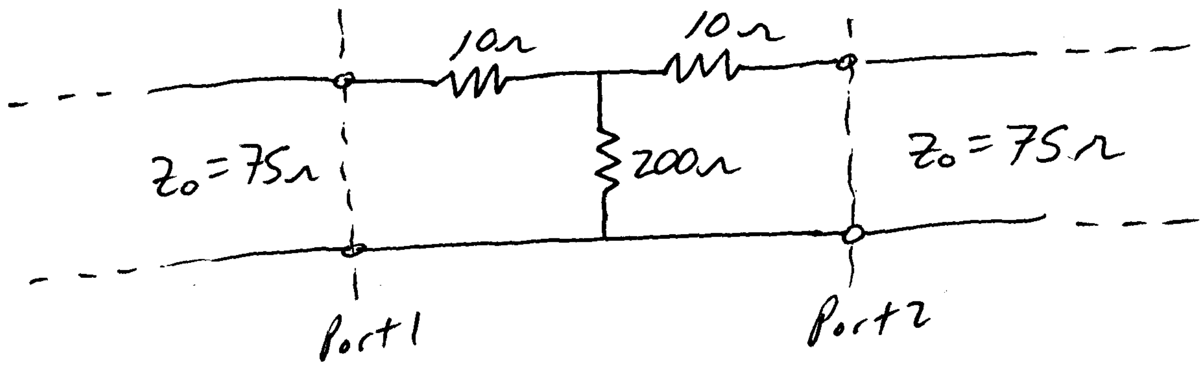
or $$[V^-] = [S][V^+] \qquad (4.40)$$

* S-parameters differ from Z- and Y-parameters where the other ports are open-circuited or short-circuited. This can be critical as some devices may not work correctly w/out load terminations, e.g., some transistor amplifiers.

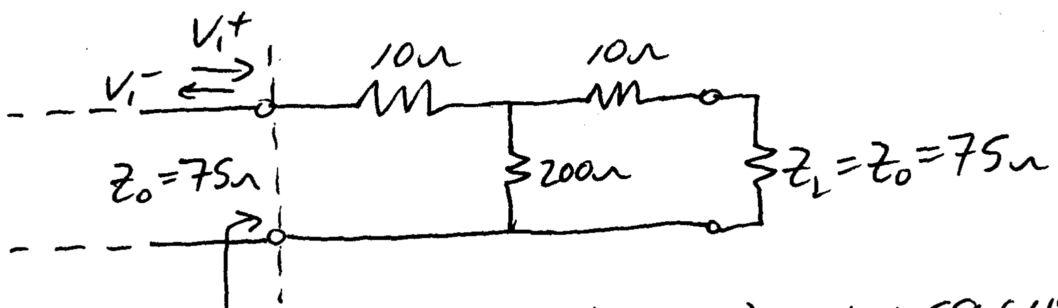
* Most often S-parameters are measured, i.e., use a vector network analyzer (VNA), or found by simulations. However, in some (simple) cases, they can be computed analytically.

4.3 cont.

example – Find the S-parameters for the resistive T-network shown.



$\boxed{S_{11}}$ $\quad S_{11} = \left.\dfrac{V_1^-}{V_1^+}\right|_{V_2^+=0}$ $\Rightarrow$ terminate port 2 w/ load $Z_L = Z_0 = 75\Omega$



$$Z_{in} = 10 + 200 \| (10 + 75) = 10 + 59.649$$
$$= 69.649\,\Omega$$

$$S_{11} = \Gamma_{11} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{69.649 - 75}{69.649 + 75}$$

$$\underline{S_{11} = -0.036992}$$

$\boxed{S_{22}}$ $\quad S_{22} = \left.\dfrac{V_2^-}{V_2^+}\right|_{V_1^+=0}$ $\Rightarrow$ terminate port 1 w/ load $Z_L = Z_0 = 75\Omega$

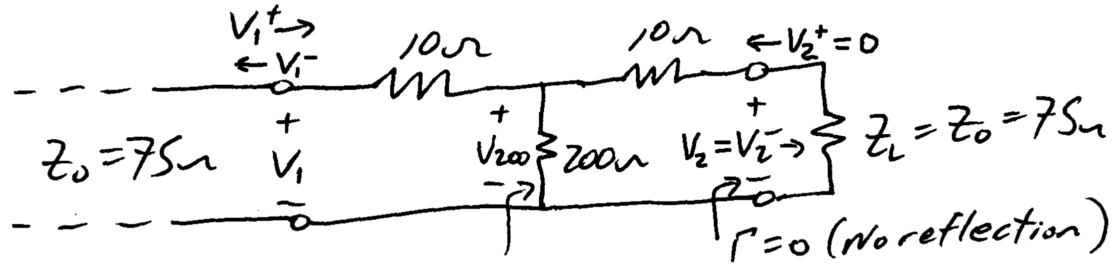
$\Rightarrow$ Since the T-network is symmetric,

$$\underline{S_{22} = S_{11} = -0.036992}$$

4.3 cont.

$\boxed{S_{21}}$ $S_{21} = \left.\dfrac{V_2^-}{V_1^+}\right|_{V_2^+=0}$ ⇒ Again terminate port 2 w/ $Z_L = Z_0 = 75\Omega$

⇒ V_2^- will be the voltage across Z_L.



$R_{eq2,p} = 200 \| (10+75) = 59.649\Omega$

By voltage division,

$$V_{200} = V_1\left(\frac{R_{eq2,p}}{R_{eq2,p}+10}\right) = V_1\left(\frac{59.649}{59.649+10}\right)$$
$$= 0.85642\, V_1$$

By voltage division again,

$$V_2^- = V_{200}\left(\frac{75}{75+10}\right) = 0.85642\, V_1\left(\frac{75}{85}\right)$$
$$= 0.7556675\, V_1 = 0.7556675(1+\underbrace{\Gamma_{11}}_{-0.037})V_1^+$$

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0} = 0.72771377$$

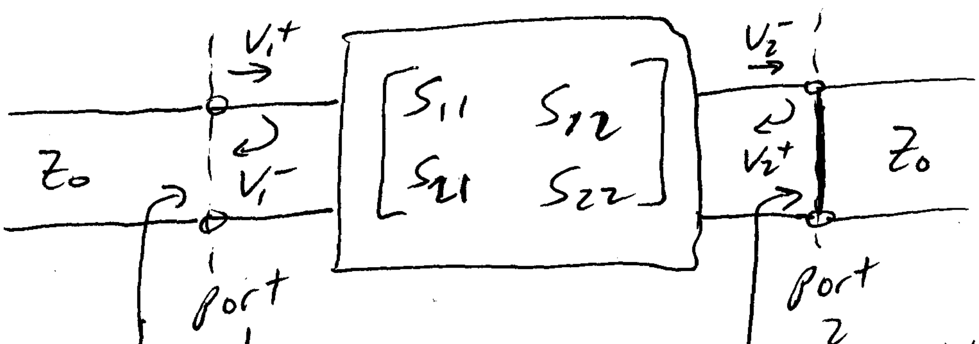
$\boxed{S_{12}}$ By symmetry, $S_{12} = S_{21} = 0.7277$

$$[S] = \begin{bmatrix} -0.0370 & 0.7277 \\ 0.7277 & -0.0370 \end{bmatrix}$$

4.3 cont.

Earlier, it was mentioned that $S_{ii} = \Gamma_{ii}$ only when all other ports are terminated in matched loads. What happens if that is not true?

Let's examine a 2-port network where we short circuit port 2.



$$\Gamma_{11} = \frac{V_1^-}{V_1^+}$$

$$\Gamma_{sc} = -1 = \frac{V_2^+}{V_2^-} \begin{matrix} \leftarrow \text{reflected} \\ \leftarrow \text{incident} \end{matrix}$$

Using $\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$,

we get $V_1^- = S_{11} V_1^+ + S_{12} V_2^+$

$$V_2^- = S_{21} V_1^+ + S_{22} V_2^+$$

from $\Gamma_{sc} = -1 = \frac{V_2^+}{V_2^-} \Rightarrow -V_2^- = V_2^+$

which yields $V_1^- = S_{11} V_1^+ + S_{12}(-V_2^-)$ (A)

$$V_2^- = S_{21} V_1^+ + S_{22}(-V_2^-) \quad \text{(B)}$$

4.3 cont.

Solving (B) for V_2^- gives

$$V_2^- = \frac{S_{21}}{1+S_{22}} V_1^+$$

which we can substitute into (B)

$$V_1^- = S_{11} V_1^+ - S_{12}\left(\frac{S_{21}}{1+S_{22}}\right)V_1^+$$

So $\Gamma_{11} = \frac{V_1^-}{V_1^+} = S_{11} - \frac{S_{12}S_{21}}{1+S_{22}}$

Obviously, $\Gamma_{11} \neq S_{11}$ when port 2 is shorted!

$\Downarrow$

The S-parameters of the network or device do NOT* change w/ load(s) or source impedance(s). However, the input reflection coefficients Γ_{ii} and transmission coefficients T_{ij} can change w/ load(s) +/or source impedances.

* Assumes we have a linear network or device.

4.3 cont.

In section 4.2, it was observed that [Z] and [Y] matrices are:

1) Purely imaginary for lossless networks
2) Symmetric about the main diagonal for reciprocal networks, i.e., $Z_{ij} = Z_{ji}$.

For [S] matrices, a reciprocal network will have $[S] = [S]^t$ (4.48),

where the 't' indicates the matrix transpose operation, i.e., [S] is also symmetric about the main diagonal.

For [S] matrices, the text shows that a lossless network must have an overall average power

$$P_{avg} = \frac{1}{2} \mathrm{Re}\{[V]^t [I]^*\} = 0$$

$$\Downarrow \text{ leads to}$$

$$[S]^t [S]^* = [U] \quad \text{or} \quad [S]^* = \{[S]^t\}^{-1} \quad (4.51)$$

where $[U] = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$ is the unit/identity matrix.

4.3 cont.

Let's further examine (4.51)

$$[S]^t [S]^* = [U]$$

$$\begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & & S_{2N} \\ \vdots & & \ddots & \vdots \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix}^t \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & & S_{2N} \\ \vdots & & \ddots & \vdots \\ S_{N1} & S_{N2} & & S_{NN} \end{bmatrix}^* = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & \ddots \\ & & & 1 \end{bmatrix}$$

$$\begin{bmatrix} S_{11} & S_{21} & \cdots & S_{N1} \\ S_{12} & S_{22} & & S_{N2} \\ \vdots & & \ddots & \vdots \\ S_{1N} & S_{2N} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & \cdots & S_{1N}^* \\ S_{21}^* & S_{22}^* & & \\ \vdots & & \ddots & \\ S_{N1}^* & & & S_{NN}^* \end{bmatrix} = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & \ddots \\ & & & 1 \end{bmatrix}$$

$\Rightarrow$ Multiply row 1 with column 1 on the LHS to get: $S_{11}S_{11}^* + S_{21}S_{21}^* + \ldots S_{N1}S_{N1}^* = 1$

$\Downarrow$ In general, this implies:

$$\sum_{k=1}^{N} S_{ki} S_{ki}^* = 1 \quad \text{for } i = 1, 2, \ldots N \qquad (4.53a)$$

That is, the dot product of any column of $[S]$ with the complex conjugate of that column is equal to one (1) for a <u>lossless</u> network.

4.3 cont.

⇒ Next, multiply row 1 with column 2 on the LHS to get:

$$S_{11} S_{12}^* + S_{21} S_{22}^* + \ldots S_{N1} S_{N2}^* = 0$$

⇓ In general, this implies:

$$\sum_{k=1}^{N} S_{ki} S_{kj}^* = 0 \quad \text{for all } i \neq j \quad (4.53b)$$

That is, the dot product of any column of [S] with the complex conjugate of another column is zero (0) for a lossless network.

⇒ If we have a lossless network that is also reciprocal, i.e., $[S] = [S]^t$, then

lossless $\quad [S]^t [S]^* = [U]$

lossless + reciprocal $\quad [S]^t \{[S]^t\}^* = [U]$

⇓ This leads to the additional general result:

$$\sum_{k=1}^{N} S_{ik} S_{jk}^* = 0 \quad \text{for all } i \neq j$$

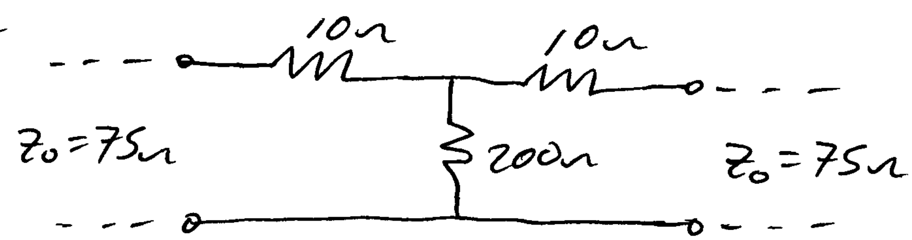
The dot product of any row of [S] with the complex conjugate of any other row is zero (0).

4.3 cont.

example – Earlier it was found that

$$[S] = \begin{bmatrix} -0.037 & 0.7277 \\ 0.7277 & -0.037 \end{bmatrix}$$

for



$$[S]^t = \begin{bmatrix} -0.037 & 0.7277 \\ 0.7277 & -0.037 \end{bmatrix} = [S] \Rightarrow \underline{\text{Reciprocal!}}$$

Let's check to see if this network is lossless by computing (4.51)

$$[S]^t [S]^* \stackrel{?}{=} [U]$$

$$\begin{bmatrix} -0.037 & 0.7277 \\ 0.7277 & -0.037 \end{bmatrix} \begin{bmatrix} -0.037 & 0.7277 \\ 0.7277 & -0.037 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} (-0.037)^2 + 0.7277^2 & (-0.037)0.7277 + 0.7277(-0.037) \\ 0.7277(-0.037) + (-0.037)0.7277 & 0.7277^2 + (-0.037)^2 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.5309 & -0.05385 \\ -0.05385 & 0.5309 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

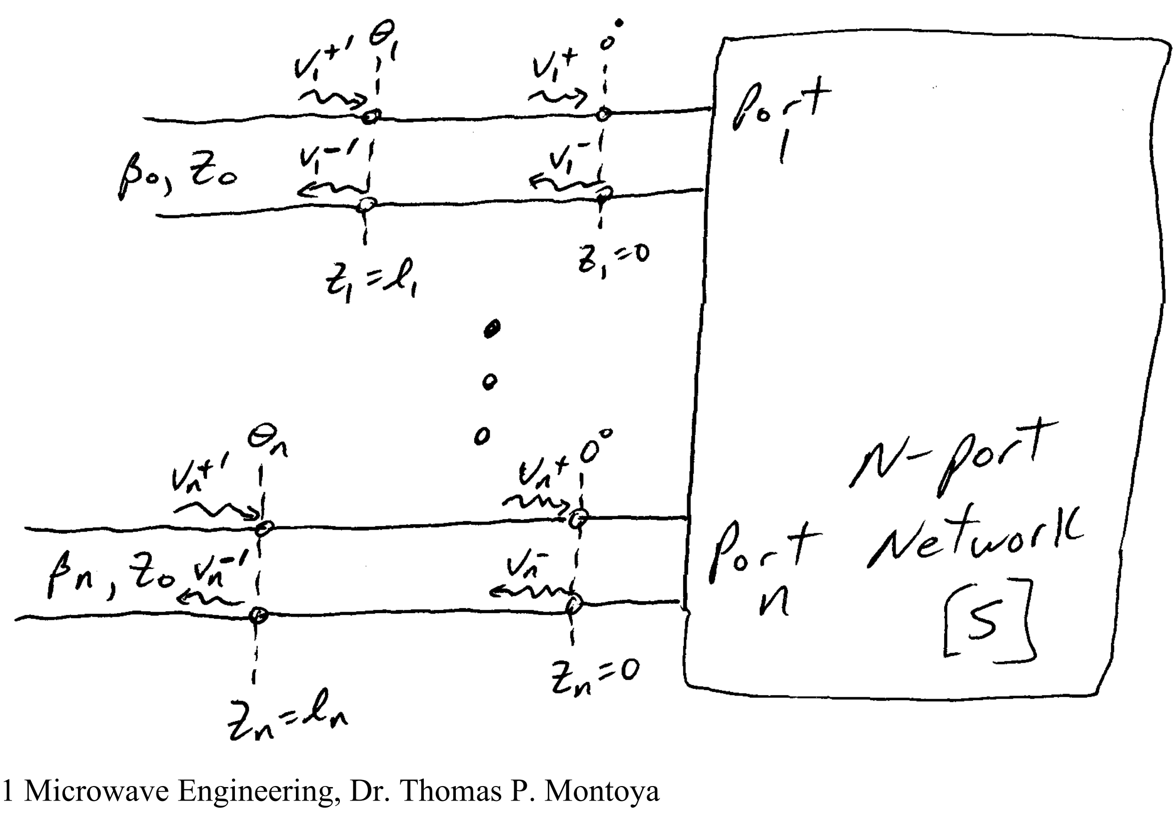
<u>NOT lossless!</u>

4.3 cont.

Shifting Reference Planes

* When we defined S-parameters for a device or network, reference/phase/terminal planes were selected with a phase of 0° (arbitrary selection) or location of 0. Have [S]-matrix.

* It is very common for the reference planes to move away from the initial selection where the S-parameters were measured/defined. Why? Adding connections and/or adaptors! Need [S']-matrix.



4.3 cont.

Per the definition of [S] matrices (4.40)

$$[V^-] = [S][V^+]. \qquad (4.54a)$$

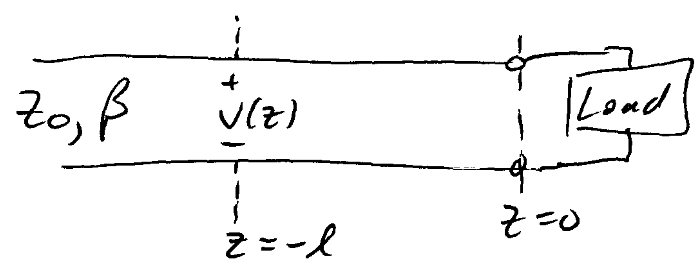
we want to determine

$$[V^{-\prime}] = [S'][V^{+\prime}], \qquad (4.54b)$$

preferably with out lots of extra work.

* Assuming the reference planes have moved outward along lossless TLs, we can turn to TL theory where the phasor voltage V(z) is given

$$V(z) = \underset{\text{forward}}{V_0^+ e^{-j\beta z}} + \underset{\text{backward}}{V_0^- e^{j\beta z}} \qquad (2.34)$$



* Comparing this with our shifted reference planes geometry, we get

$$V_n^{+\prime} = V_n^+ e^{+j\beta_n \ell_n} = V_n^+ e^{j\theta_n} \qquad (4.55a)$$

$$V_n^{-\prime} = V_n^- e^{-j\beta_n \ell_n} = V_n^- e^{-j\theta_n} \qquad (4.55b)$$

4.3 cont.

Solving (4.55a) & (4.55b) for V_n^+ & V_n^-, yields

$$V_n^+ = e^{-j\theta_n} V_n^{+\prime}$$

$$V_n^- = e^{+j\theta_n} V_n^{-\prime}$$

$\Downarrow$ put in matrix form

$$[V^+] = \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & e^{-j\theta_2} & \\ 0 & & \ddots & \\ & & & e^{-j\theta_N} \end{bmatrix} [V^{+\prime}]$$

$$[V^-] = \begin{bmatrix} e^{j\theta_1} & & 0 \\ & e^{j\theta_2} & \\ 0 & & \ddots & \\ & & & e^{j\theta_N} \end{bmatrix} [V^{-\prime}]$$

Now, substitute these into (4.54a)

$$\begin{bmatrix} e^{j\theta_1} & & 0 \\ & \ddots & \\ 0 & & e^{j\theta_N} \end{bmatrix} [V^{-\prime}] = [S] \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & \ddots & \\ 0 & & e^{-j\theta_N} \end{bmatrix} [V^{+\prime}]$$

Multiply through by the inverse of the first matrix* to get

$$[V^{-\prime}] = \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & \ddots & \\ 0 & & e^{-j\theta_N} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & & \\ & \ddots & \\ 0 & & e^{-j\theta_N} \end{bmatrix} [V^{+\prime}]$$

* Inverse of a diagonal matrix is a diagonal matrix with each diagonal element inverted.

4.3 cont.

Comparing this equation w/ (4.54b), we see

$$[S'] = \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & \ddots & \\ 0 & & e^{-j\theta_N} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & \ddots & \\ 0 & & e^{-j\theta_N} \end{bmatrix} \quad (4.56)$$

Multiplying out this matrix equation yields the elements of $[S']$ to be

$$S_{mn}' = S_{mn}\, e^{-j(\theta_m + \theta_n)} \quad \text{(anywhere).}$$

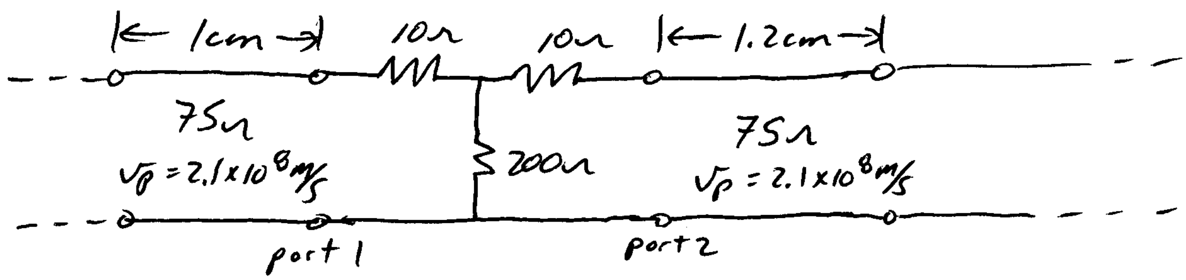
In the case that $m = n$ (along diagonal), we get $S_{nn}' = S_{nn}\, e^{-j2\theta_n}$

which tells us the <u>phase</u> of S_{nn} is shifted by <u>twice</u> the electrical length $\beta_n l_n = \theta_n$ when the terminal plane is shifted, i.e., round-trip distance for waves.

Note: This is very similar to $\Gamma(l) = \Gamma(0)\, e^{-j2\beta l}$ for reflection coefficients along lossless TLs!

4.3 cont.

example – Let's consider the earlier example of a resistive T-network implemented so that port 1 is connected to the rest of the circuit by a 1cm long 75Ω TL w/ $v_p = 2.1 \times 10^8$ m/s and port 2 is connected by a 1.2cm long 75Ω TL w/ $v_p = 2.1 \times 10^8$ m/s. Assume the frequency of operation is 1.4GHz. Find $[S']$.



We know $[S] = \begin{bmatrix} -0.037 & 0.7277 \\ 0.7277 & -0.037 \end{bmatrix}$

$$\beta = \frac{\omega}{v_p} = \frac{2\pi(1.4 \times 10^9)}{2.1 \times 10^8} = \underline{41.8879 \ \tfrac{rad}{m} = \beta_1 = \beta_2}$$

$$\theta_1 = \beta_1 \ell_1 = 41.8879(1 \times 10^{-2}) = 0.418879 \text{ rad} = 24^\circ$$

$$\theta_2 = \beta_2 \ell_2 = 41.8879(1.2 \times 10^{-2}) = 0.502655 \text{ rad} = 28.8^\circ$$

$$[S'] = \begin{bmatrix} e^{-j24^\circ} & 0 \\ 0 & e^{-j28.8^\circ} \end{bmatrix} \begin{bmatrix} -0.037 & 0.7277 \\ 0.7277 & -0.037 \end{bmatrix} \begin{bmatrix} e^{-j24^\circ} & 0 \\ 0 & e^{-j28.8^\circ} \end{bmatrix}$$

$$= \begin{bmatrix} -0.037\,e^{-j24^\circ} & 0.7277e^{-j24^\circ} \\ 0.7277\,e^{-j28.8^\circ} & -0.037\,e^{-j28.8^\circ} \end{bmatrix} \begin{bmatrix} e^{-j24^\circ} & 0 \\ 0 & e^{-j28.8^\circ} \end{bmatrix}$$

4.3 cont.

example cont. –

$$[S'] = \begin{bmatrix} -0.037 e^{-j24^\circ} e^{-j24^\circ} & 0.7277 e^{-j24^\circ} e^{-j28.8^\circ} \\ 0.7277 e^{-j28.8^\circ} e^{-j24^\circ} & -0.037 e^{-j28.8^\circ} e^{-j28.8^\circ} \end{bmatrix}$$

$$= \begin{bmatrix} -0.037 e^{-j48^\circ} & 0.7277 e^{-j52.8^\circ} \\ 0.7277 e^{-j52.8^\circ} & -0.037 e^{-j57.6^\circ} \end{bmatrix}$$

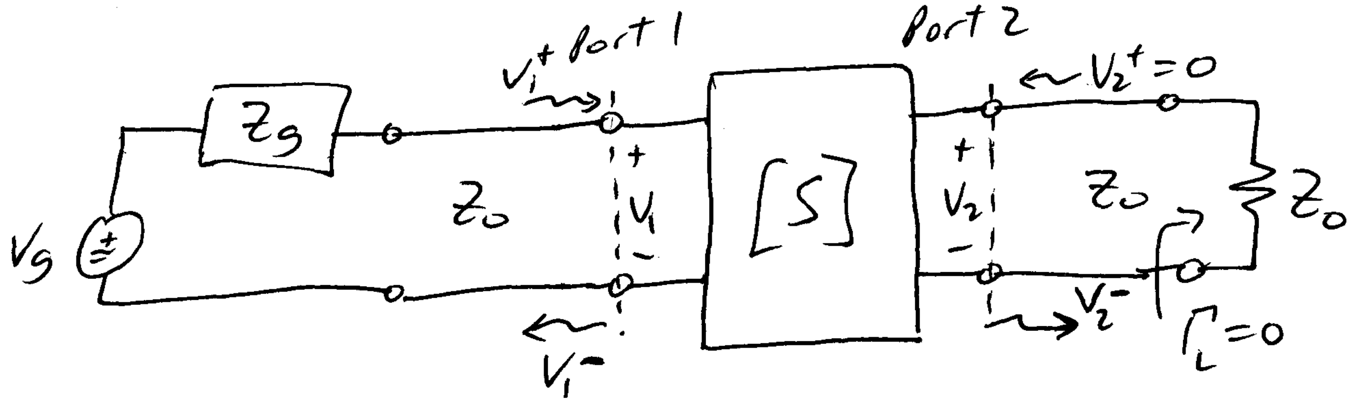
$$= \begin{bmatrix} -0.037 \angle -48^\circ & 0.7277 \angle -52.8^\circ \\ 0.7277 \angle -52.8^\circ & -0.037 \angle -57.6^\circ \end{bmatrix}$$

$$[S'] = \begin{bmatrix} 0.037 \angle 132^\circ & 0.7277 \angle -52.8^\circ \\ 0.7277 \angle -52.8^\circ & 0.037 \angle 122.4^\circ \end{bmatrix}$$

4.3 cont.

S-parameters and Time-Average Power

To illustrate the relationship between S-parameters and time-average power flow, consider the two-port network inserted between a generator, connected to port 1, and a matched load, connected to port 2.



From $[V^-] = [S][V^+]$, we get

$$V_1^- = S_{11} V_1^+ + S_{12} V_2^+$$

$$V_2^- = S_{21} V_1^+ + S_{22} V_2^+$$

Note, at Port 1, the total voltage is

$$V_1 = V_1^+ + V_1^-$$

and the time-average power is

$$P_{1,tot} = P_{1,inc} - P_{1,ref} = \frac{|V_1^+|^2}{2Z_0} - \frac{|V_1^-|^2}{2Z_0}$$

4.3 cont.

However, at port 2, the total voltage is

$$V_2 = \cancel{V_2^+}^{\,0} + V_2^- = V_2^-$$

as we have a matched load.

$$P_{2,tot} = \underbrace{P_{2,ref}}_{\to 0} + P_{2,trans} = \frac{|V_2^-|^2}{2Z_0}.$$

* Consider the power ratio of reflected to incident at port 1 w/ port 2 matched

$$\frac{P_{1,ref}}{P_{1,inc}} = \frac{\frac{|V_1^-|^2}{2Z_0}}{\frac{|V_1^+|^2}{2Z_0}} = \frac{|V_1^-|^2}{|V_1^+|^2} = \left|\frac{V_1^-}{V_1^+}\right|^2$$

Then, note that, w/ port 2 matched, we define $S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0}$

$$\Downarrow$$

$$\left.\frac{P_{1,ref}}{P_{1,inc}}\right|_{V_2^+=0} = |S_{11}|^2$$

That is, the relative/fraction time-average power at port 1 equals $|S_{11}|^2$ when port 2 is matched.

4.3 cont.

* Consider the ratio of the power incident at port 1 to the power transmitted/delivered to port 2

$$\frac{P_{2,trans}}{P_{1,inc}} = \frac{\frac{|V_2^-|^2}{2Z_0}}{\frac{|V_1^+|^2}{2Z_0}} = \frac{|V_2^-|^2}{|V_1^+|^2} = \left|\frac{V_2^-}{V_1^+}\right|^2$$

By definition, $S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$

$$\Downarrow$$

$$\underline{\frac{P_{2,trans}}{P_{1,inc}} = |S_{21}|^2}$$

That is, the fraction/relative time-average power transmitted/transfered from port 1 to port 2 equals $|S_{21}|^2$ when port 2 is matched.

⇒ If we swap the ports to which the load and generator are connected, we would get $\left.\frac{P_{2,ref}}{P_{2,inc}}\right|_{V_1^+=0} = |S_{22}|^2$ and $\frac{P_{1,trans}}{P_{2,inc}} = |S_{12}|^2$.

4.3 cont.

⇒ Now, we have an idea of how to use $|S_{ii}|^2$ and $|S_{ij}|^2$ to interpret or predict power flows under matched conditions.

Next, what relationships might hold true if our two-port network is <u>lossless</u>?

⇒ From (4.51), we expect [S] to be unitary,

$$[S]^t [S]^* = [U]$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^t \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^* = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} S_{11} & S_{21} \\ S_{12} & S_{22} \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* \\ S_{21}^* & S_{22}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Downarrow$$

$$S_{11} S_{11}^* + S_{21} S_{21}^* = |S_{11}|^2 + |S_{21}|^2 = 1 \qquad \text{(A)}$$

$$S_{12} S_{12}^* + S_{22} S_{22}^* = |S_{22}|^2 + |S_{12}|^2 = 1 \qquad \text{(B)}$$

and

$$S_{11} S_{12}^* + S_{21} S_{22}^* = 0$$

$$S_{12} S_{11}^* + S_{22} S_{21}^* = 0$$

4.3 cont.

* (A) + (B) are always true for lossless two-port networks. However, for our circuit, we can see that (A) is a statement of Conservation of Power, i.e., power reflected @ port 1 + power transmitted from port 1 to 2 = 100%

Note, if port 2 were not matched, (A) + (B) would still be true, but not a statement on conservation of power.

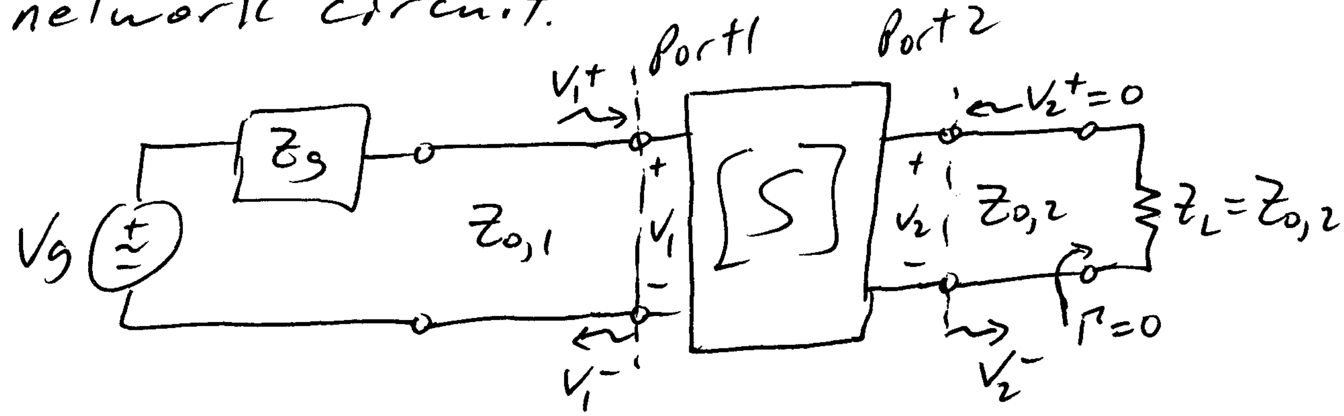
4.3 cont.

Generalized Scattering Parameters

What if the characteristic impedance(s) $Z_{0,i}$ at the various ports are not necessarily the same?

⇒ Some new definitions are needed if we wish to keep our power flow relationships.

* Let's consider the following two-port network circuit.



Now,

$$P_{1,inc} = \frac{|V_1^+|^2}{2Z_{0,1}} \qquad P_{2,trans} = \frac{|V_2^-|^2}{2Z_{0,2}}$$

$$P_{1,ref} = \frac{|V_1^-|^2}{2Z_{0,1}}$$

4.3 cont.

Looking at power ratios again

$$\left.\frac{P_{1,ref}}{P_{1,inc}}\right|_{V_2^+=0} = \frac{\dfrac{|V_1^-|^2}{2Z_{0,1}}}{\dfrac{|V_1^+|^2}{2Z_{0,1}}} = \left|\frac{V_1^-}{V_1^+}\right|^2 = |S_{11}|^2$$

as before. However,

$$\left.\frac{P_{2,trans}}{P_{1,inc}}\right|_{V_2^+=0} = \frac{\dfrac{|V_2^-|^2}{2Z_{0,2}}}{\dfrac{|V_1^+|^2}{2Z_{0,1}}} = \frac{\dfrac{|V_2^-|^2}{Z_{0,2}}}{\dfrac{|V_1^+|^2}{Z_{0,1}}}$$

is NOT the same.

$\Rightarrow$ We would like a new definition of S-parameters that will keep $|S_{ij}|^2$ (in this case $|S_{21}|^2$) as a way to get relative power flow between ports w/ matched loads

For the equation above, what if we say $|S_{21}| = \left.\dfrac{|V_2^-|/\sqrt{Z_{0,2}}}{|V_1^+|/\sqrt{Z_{0,1}}}\right|_{V_2^+=0}$?

4.3 cont.

This leads to defining normalized 'wave' amplitudes toward the ports as

$$a_n = \frac{V_n^+}{\sqrt{Z_{0,n}}},$$

and normalized 'wave' amplitudes away from the ports as

$$b_n = \frac{V_n^-}{\sqrt{Z_{0,n}}}.$$

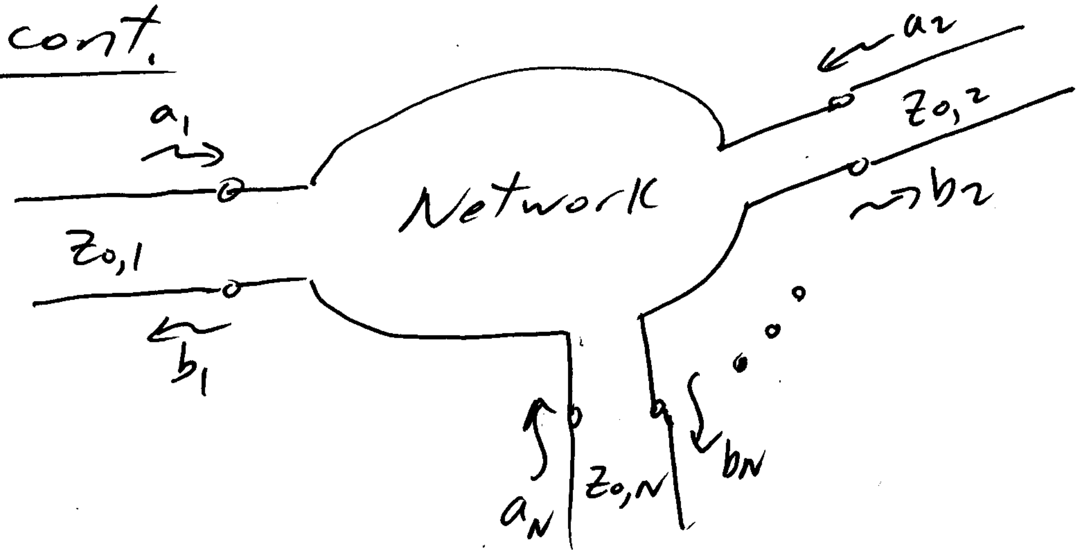
⇒ We can now define generalized S-parameters as

$$S_{ij} = \frac{b_i}{a_j}\bigg|_{a_k = 0 \text{ for all } k \neq j}$$

where we still match all other ports to prevent waves being incident (matches are NOT necessarily the same).

⇒ Further, we define the matrix equation $[b] = [S][a]$

4.3 cont.



Now, at some port n

$$V_n = V_n^+ + V_n^-$$

$$I_n = \frac{1}{Z_{0,n}}\left[V_n^+ - V_n^-\right]$$

Substituting in $a_n\sqrt{Z_{0,n}} = V_n^+$ and $b_n\sqrt{Z_{0,n}} = V_n^-$,

we get $V_n = a_n\sqrt{Z_{0,n}} + b_n\sqrt{Z_{0,n}}$

$$I_n = \frac{1}{Z_{0,n}}\left[a_n\sqrt{Z_{0,n}} - b_n\sqrt{Z_{0,n}}\right]$$

$\Downarrow$ solve for a_n and b_n

$$a_n = \frac{1}{2\sqrt{Z_{0,n}}}\left[V_n + Z_{0,n}I_n\right]$$

$$b_n = \frac{1}{2\sqrt{Z_{0,n}}}\left[V_n - Z_{0,n}I_n\right]$$

4.3 cont.

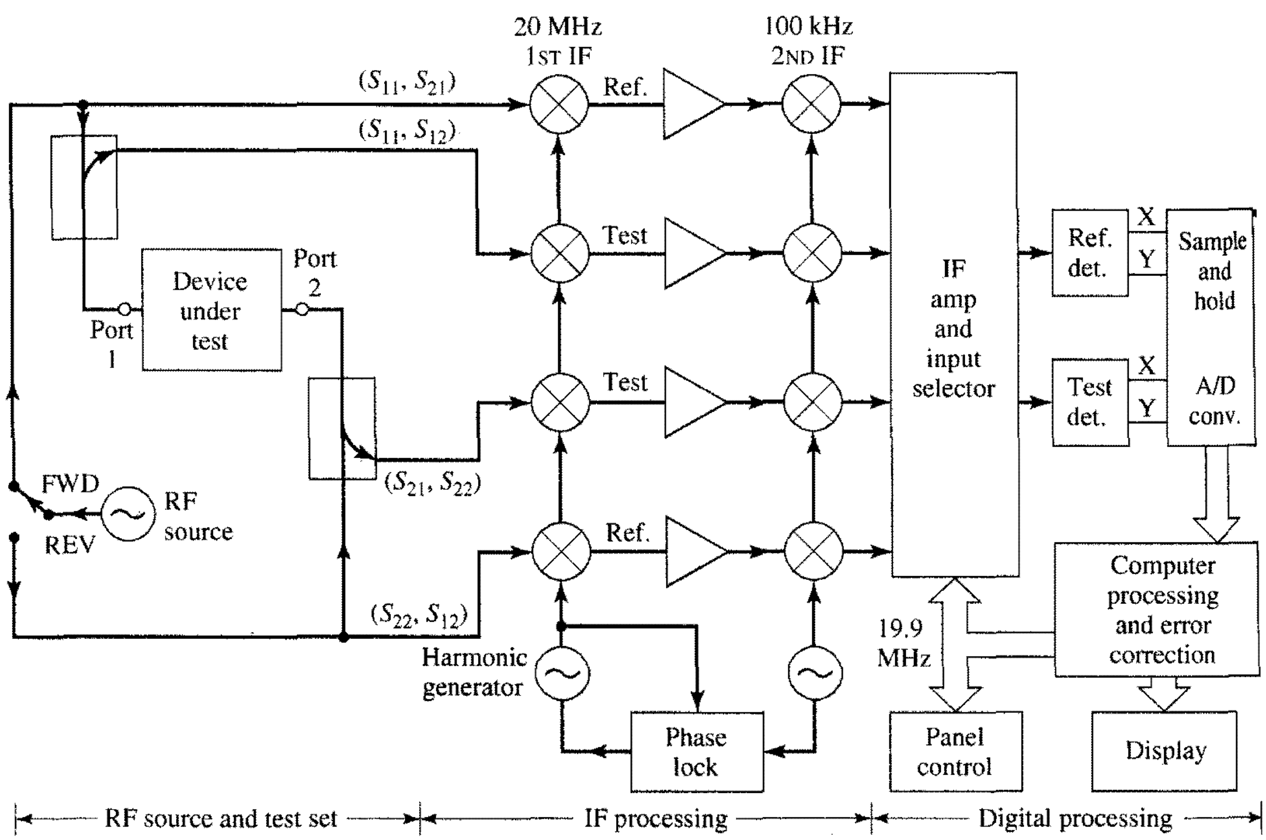
We will not use a_n, b_n, and/or generalized S-parameters much because our VNAs will have same characteristic impedance at both ports as will most of our devices. However, they needed to be mentioned as they turn up in many discussions/writings on microwave network theory.

4.3 cont.

How can we measure/test S-parameters?

1) Scalar Network Analyzer (SNA)
   * Measure only magnitude of S-parameters

2) Vector Network Analyzer (VNA)
   * Measures both magnitude + phase of S-parameters
   * Performance Network Analyzers (PNA) are also VNAs, marketing ploy



***Microwave Engineering* (Fourth Edition), Pozar, Wiley, 2012, ISBN 978-0-470-63155-3.**

4.3 cont.

Note that a VNA is a very complicated transmitter and receiver system w/ very sophisticated signal processing capabilities.

⇒ Measurements not done directly at RF/microwave frequencies, use mixers to move down to much lower frequencies

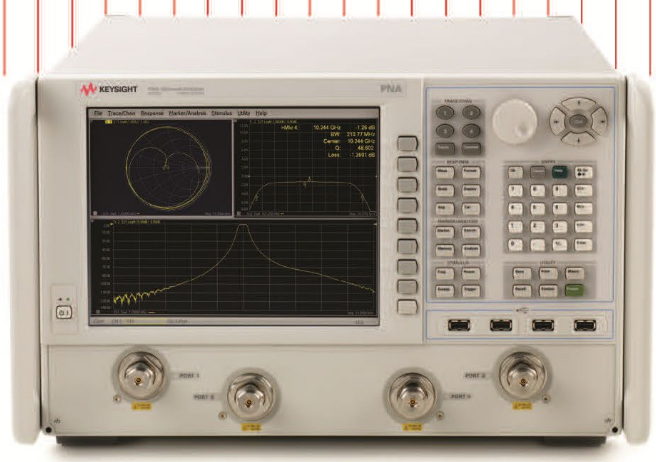
⇒ One big reason/need for signal processing capabilities is calibration, i.e., want to remove as many sources of error or unwanted signals

The following section features slides from a Keysight Technologies 'Network Analyzer Basics' training deliverable 5965-7917E.pdf

Note: Hewlett-Packard $\xrightarrow[1999]{\text{spun-off}}$ Agilent $\xrightarrow[\text{spun-off}]{2014}$ Keysight Technologies

Keysight Technologies
Network Analyzer Basics

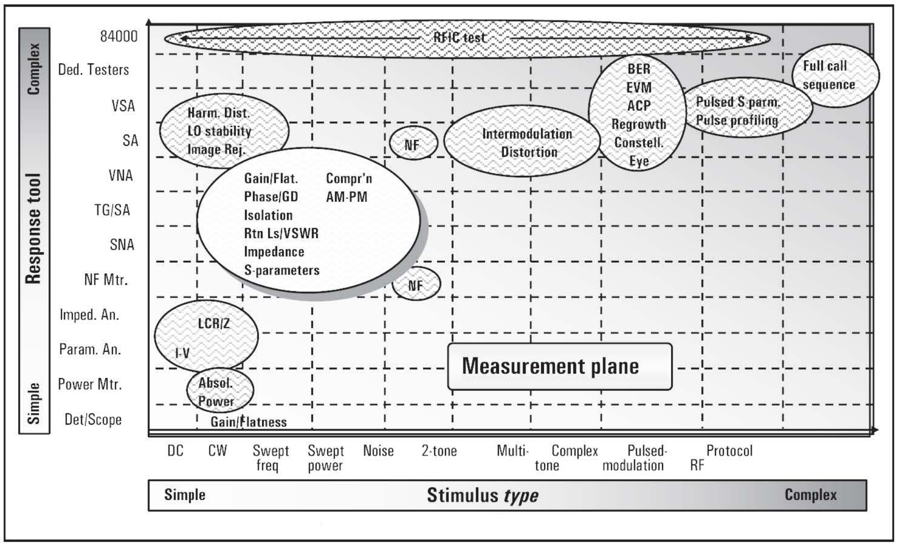
Training Deliverable



KEYSIGHT TECHNOLOGIES

Keysight Technologies Network Analyzer Basics, 5965-7917E.pdf, page 1.

7



Device Test Measurement Model

Here is a key to many of the abbreviations used at right:

Response

84000	8400 series high-volume RFIC tester
Ded. Testers	Dedicated (usually one-box) testers
VSA	Vector signal analyzer
SA	Spectrum analyzer
VNA	Vector signal analyzer
TG/SA	Tracking generator/spectrum analyzer
SNA	Scalar network analyzer
NF Mtr.	Noise-figure meter
Imped. An.	Impedance analyzer (LCR meter)
Power Mtr.	Power meter
Det./Scope	Diode detector/oscilloscope

Measurement

ACP	Adjacent channel power
AM-PM	AM to PM conversion
BER	Bit-error rate
Compr'n	Gain compression
Constell.	Constellation diagram
EVM	Error-vector magnitude
Eye	Eye diagram
GD	Group delay
Harm. Dist.	Harmonic distortion
NF	Noise figure
Regrowth	Spectral regrowth
Rtn Ls	Return loss
VSWR	Voltage standing wave ratio

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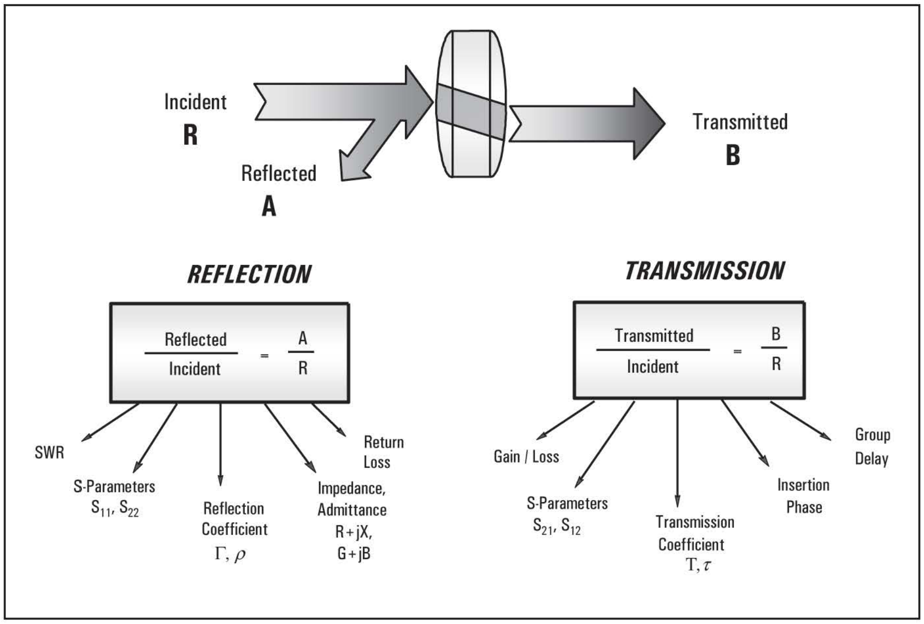

High-Frequency Device Characterization

Now that we fully understand the relationship of electromagnetic waves, we must also recognize the terms used to describe them. Common network analyzer terminology has the incident wave measured with the R (for reference) receiver. The reflected wave is measured with the A receiver and the transmitted wave is measured with the B receiver. With amplitude and phase information of these three waves, we can quantify the reflection and transmission characteristics of our device under test (DUT). Some of the common measured terms are scalar in nature (the phase part is ignored or not measured), while others are vector (both magnitude and phase are measured). For example, return loss is a scalar measurement of reflection, while impedance results from a vector reflection measurement. Some, like group delay, are purely phase-related measurements.

Ratioed reflection is often shown as A/R and ratioed transmission is often shown as B/R, relating to the measurement receivers used in the network analyzer

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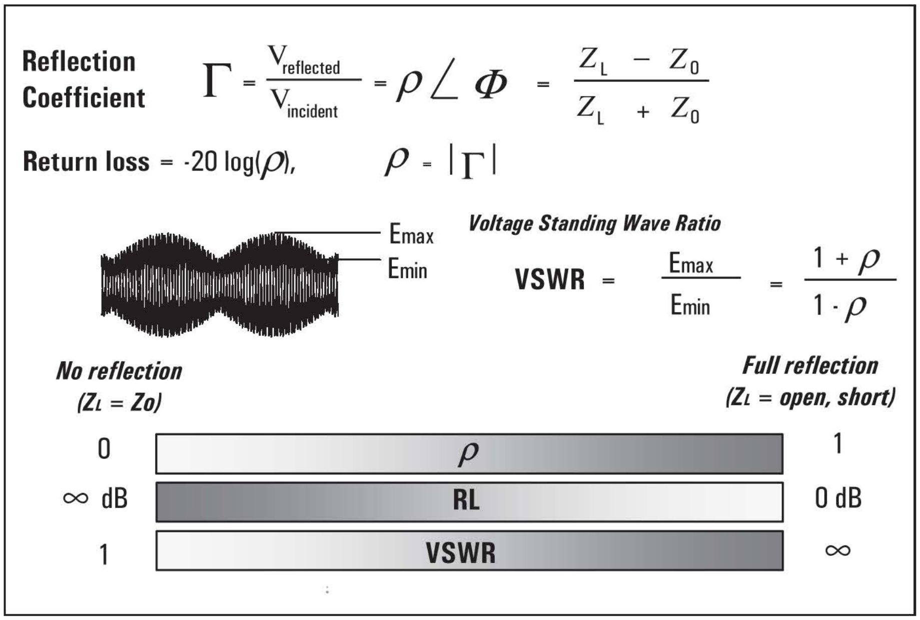

Reflection Parameters

Let's now examine reflection measurements. The first term for reflected waves is reflection coefficient gamma (Γ). Reflection coefficient is the ratio of the reflected signal voltage to the incident signal voltage. It can be calculated as shown above by knowing the impedances of the transmission line and the load. The magnitude portion of gamma is called rho (ρ). A transmission line terminated in Z_0 will have all energy transferred to the load; hence $V_{refl} = 0$ and $\rho = 0$. When Z_L is not equal to Z_0, some energy is reflected and ρ is greater than zero. When Z_L is a short or open circuit, all energy is reflected and $\rho = 1$. The range of possible values for ρ is therefore zero to one.

Since it is often very convenient to show reflection on a logarithmic display, the second way to convey reflection is return loss. Return loss is expressed in terms of dB, and is a scalar quantity. The definition for return loss includes a negative sign so that the return loss value is always a positive number (when measuring reflection on a network analyzer with a log magnitude format, ignoring the minus sign gives the results in terms of return loss). Return loss can be thought of as the number of dB that the reflected signal is below the incident signal. Return loss varies between infinity for a Z_0 impedance and 0 dB for an open or short circuit.

As we have already seen, two waves traveling in opposite directions on the same transmission line cause a "standing wave". This condition can be measured in terms of the voltage-standing-wave ratio (VSWR or SWR for short). VSWR is defined as the maximum value of the RF envelope over the minimum value of the envelope. This value can be computed as $(1+\rho)/(1-\rho)$. VSWR can take ??????

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Transmission Coefficient = $T = \dfrac{V_{Transmitted}}{V_{Incident}} = \tau\angle\phi$

Insertion Loss (dB) = $-\ 20\ \mathrm{Log}\left|\dfrac{V_{Trans}}{V_{Inc}}\right| = -\ 20\log\tau$

Gain (dB) = $20\ \mathrm{Log}\left|\dfrac{V_{Trans}}{V_{Inc}}\right| = 20\log\tau$

Transmission Parameters

Transmission coefficient _ is defined as the transmitted voltage divided by the incident voltage. If $|V_{trans}| > |V_{inc}|$, the DUT has gain, and if $|V_{trans}| < |V_{inc}|$, the DUT exhibits attenuation or insertion loss. When insertion loss is expressed in dB, a negative sign is added in the definition so that the loss value is expressed as a positive number. The phase portion of the transmission coefficient is called insertion phase. There is more to transmission than simple gain or loss. In communications systems, signals are time varying –they occupy a given bandwidth and are made up of multiple frequency components. It is important then to know to what extent the DUT alters the makeup of the signal, thereby causing signal distortion. While we often think of distortion as only the result of nonlinear networks, we will see shortly that linear networks can also cause signal distortion.

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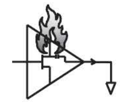
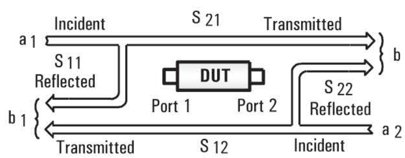


- relatively easy to **obtain** at high frequencies
  - -- measure voltage traveling waves with a vector network analyzer
  - -- don't need shorts/opens which can cause active devices to oscillate or self-destruct
- relate to **familiar** measurements (gain, loss, reflection coefficient ...)
- can **cascade** S-parameters of multiple devices to predict system performance
- can **compute** H, Y, or Z parameters from S-parameters if desired
- can easily import and use S-parameter files in our **electronic-simulation** tools



$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

Why Use S-Parameters?

At high frequencies, it is very hard to measure total voltage and current at the device ports. One cannot simply connect a voltmeter or current probe and get accurate measurements due to the impedance of the probes themselves and the difficulty of placing the probes at the desired positions. In addition, active devices may oscillate or self-destruct with the connection of shorts and opens.

Clearly, some other way of characterizing high-frequency networks is needed that doesn't have these drawbacks. That is why scattering or S-parameters were developed. S-parameters have many advantages over the previously mentioned H, Y or Z-parameters. They relate to familiar measurements such as gain, loss, and reflection coefficient. They are defined in terms of voltage traveling waves, which are relatively easy to measure. S-parameters don't require connection of undesirable loads to the device under test. The measured S-parameters of multiple devices can be cascaded to predict overall system performance. If desired, H, Y, or Z-parameters can be derived from S-parameters. And very important for RF design, S-parameters are easily imported and used for circuit simulations in electronic-design automation (EDA) tools like the Keysight Technologies, Inc. Advanced Design System (ADS). S-parameters are the shared language between simulation and measurement.

An N-port device has N^2 S-parameters. So, a two-port device has four S-parameters. The numbering convention for S-parameters is that the first number following the "S" is the port where the signal emerges, and the second number is the port where the signal is applied. So, S_{21} is a measure of the signal coming out port 2 relative to the RF stimulus entering port 1. When the numbers are the same (e.g., S_{11}), it indicates a reflection measurement, as the input and output ports are the same. The incident terms (a_1, a_2) and output terms (b_1, b_2) represent voltage traveling waves.

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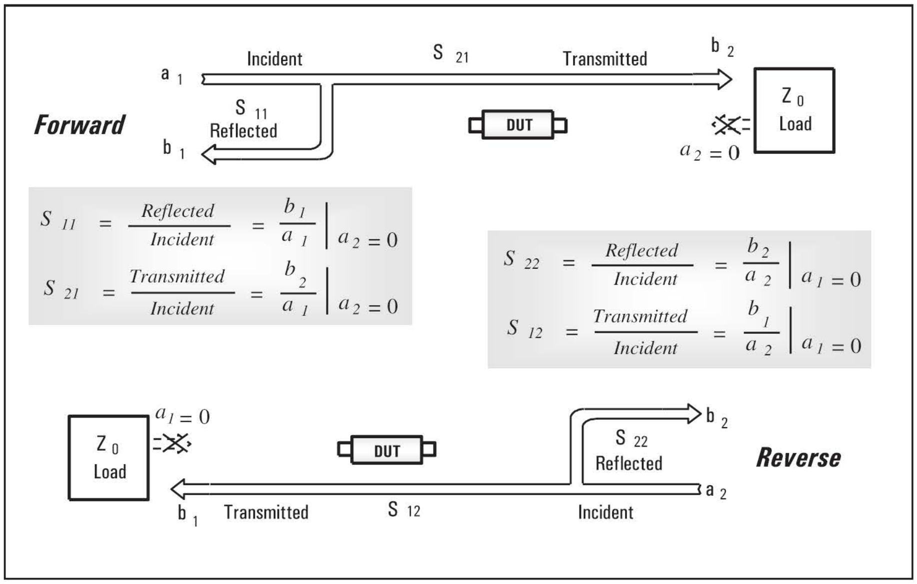

Measuring S-Parameters

S_{11} and S_{21} are determined by measuring the magnitude and phase of the incident, reflected and transmitted voltage signals when the output is terminated in a perfect Z_0 (a load that equals the characteristic impedance of the test system). This condition guarantees that a_2 is zero, since there is no reflection from an ideal load. S_{11} is equivalent to the input complex reflection coefficient or impedance of the DUT, and S_{21} is the forward complex transmission coefficient. Likewise, by placing the source at port 2 and terminating port 1 in a perfect load (making a_1 zero), S_{22} and S_{12} measurements can be made. S_{22} is equivalent to the output complex reflection coefficient or output impedance of the DUT, and S_{12} is the reverse complex transmission coefficient.

The accuracy of S-parameter measurements depends greatly on how good a termination we apply to the load port (the port not being stimulated). Anything other than a perfect load will result in a_1 or a_2 not being zero (which violates the definition for S-parameters). When the DUT is connected to the test ports of a network analyzer and we don't account for imperfect test-port match, we have not done a very good job satisfying the condition of a perfect termination. For this reason, two-port error correction, which corrects for source and load match, is very important for accurate S-parameter measurements (two-port correction is covered in the calibration section).

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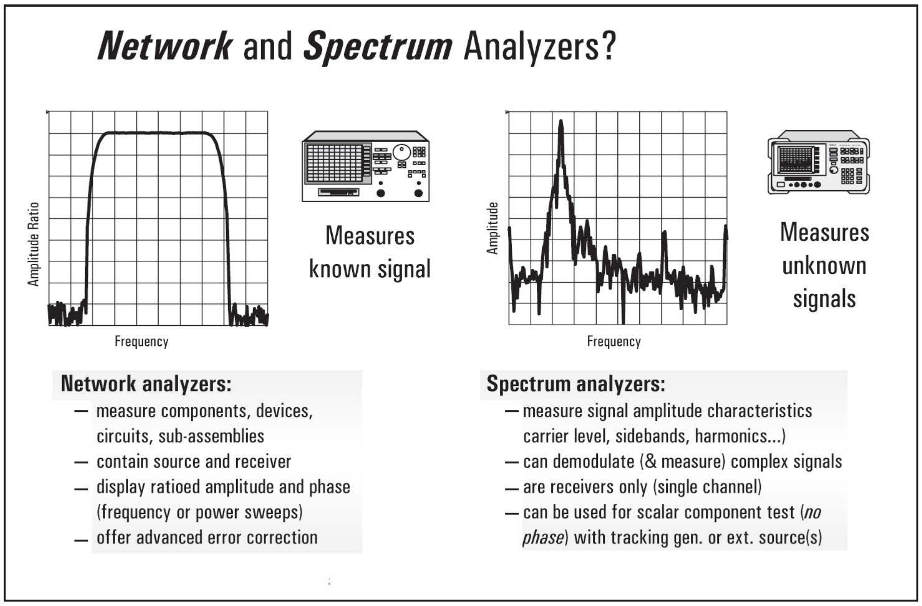
S11 = forward reflection coefficient ***(input match)***
S22 = reverse reflection coefficient ***(output match)***
S21 = forward transmission coefficient ***(gain or loss)***
S12 = reverse transmission coefficient ***(isolation)***

Remember, S-parameters are inherently complex, linear quantities -- however, we often express them in a log-magnitude format

Equating S-Parameters with Common Measurement Terms

S-parameters are essentially the same parameters as some of the terms we have mentioned before, such as input match and insertion loss. It is important to separate the fundamental definition of S-parameters and the format in which they are often displayed. S-parameters are inherently complex, linear quantities. They are expressed as real-and-imaginary or magnitude-and-phase pairs. However, it isn't always very useful to view them as linear pairs. Often we want to look only at the magnitude of the S-parameter (for example, when looking at insertion loss or input match), and often, a logarithmic display is most useful. A log-magnitude format lets us see far more dynamic range than a linear format.

Keysight Technologies Network Analyzer Basics, 5965-7917E.pdf, top of page 24.



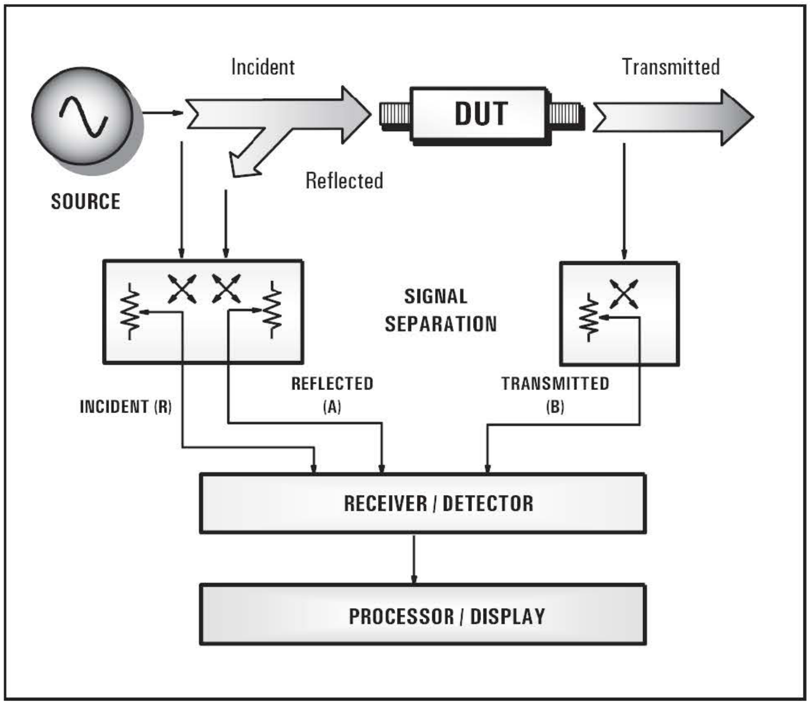
What is the Differnce Between Network and Spectrum Analyzers?

Now that we have seen some of the measurements that are commonly done with network and spectrum analyzers, it might be helpful to review the main differences between these instruments. Although they often both contain tuned receivers operating over similar frequency ranges, they are optimized for very different measurement applications.

Network analyzers are used to measure components, devices, circuits, and sub-assemblies. They contain both a source and multiple receivers, and generally display *ratioed* amplitude and phase information (frequency or power sweeps). A network analyzer is always looking at a *known* signal (in terms of frequency), since it is a stimulus/response system. With network analyzers, it is harder to get an (accurate) trace on the display, but very easy to interpret the results. With vector-error correction, network analyzers provide much higher measurement accuracy than spectrum analyzers.

Spectrum analyzers are most often used to measure signal characteristics such as carrier level, sidebands, harmonics, phase noise, etc., on unknown signals. They are most commonly configured as a single-channel receiver, without a source. Because of the flexibility needed to analyze signals, spectrum analyzers generally have a much wider range of IF bandwidths available than most network analyzers. Spectrum analyzers are often used with external sources for nonlinear stimulus/response testing. When combined with a tracking generator, spectrum analyzers can be used for scalar component testing (magnitude versus frequency, but no phase measurements). With spectrum analyzers, it is easy to get a trace on the display, but interpreting the results can be much more difficult than with a network analyzer.

Keysight Technologies Network Analyzer Basics, 5965-7917E.pdf, page 26.



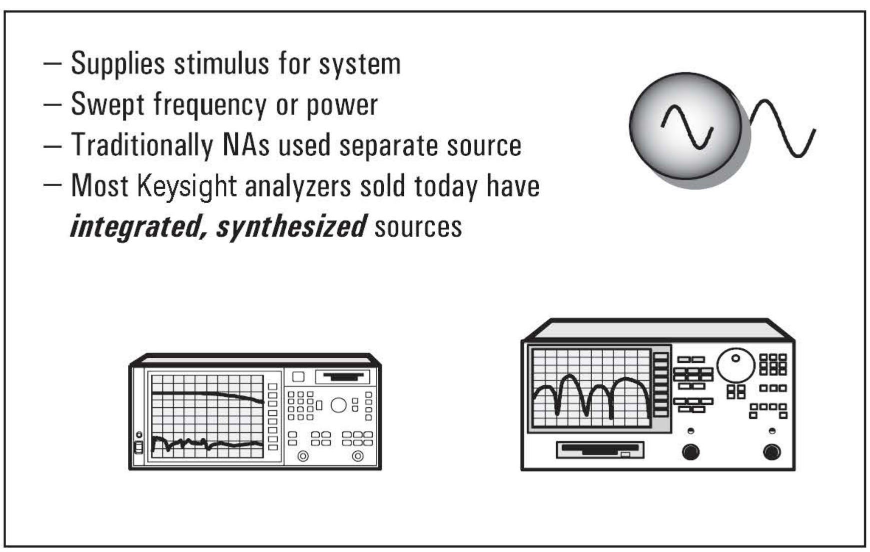
Generalized Network Analyzer Block Diagram

Here is a generalized block diagram of a network analyzer, showing the major signal-processing sections. In order to measure the incident, reflected and transmitted signal, four sections are required:

- Source for stimulus
- Signal-separation devices
- Receivers that downconvert and detect the signals
- Processor/display for calculating and reviewing the results

We will briefly examine each of these sections. More detailed information about the signal separation devices and receiver section are in the appendix.

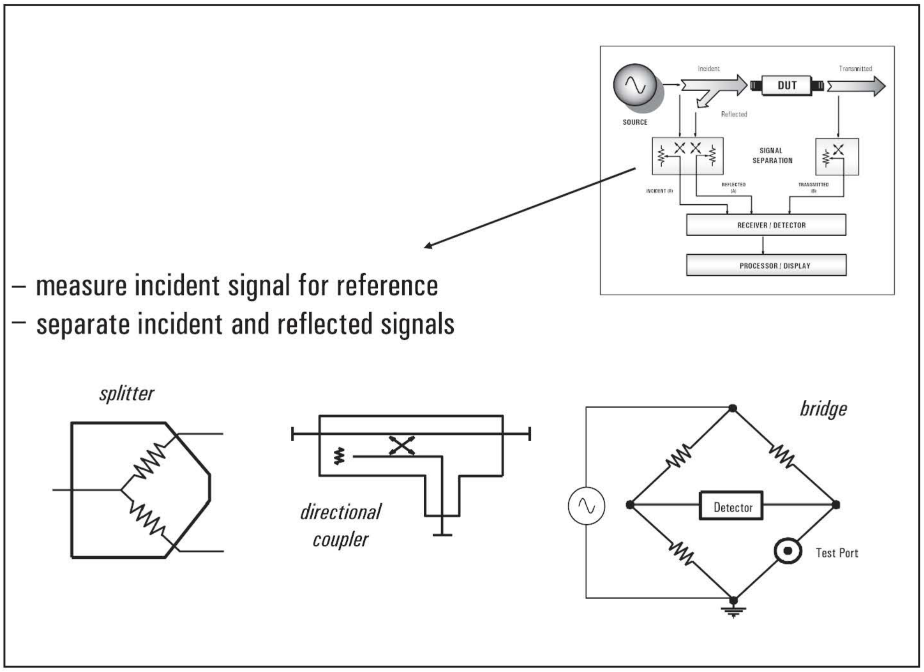
Keysight Technologies Network Analyzer Basics, 5965-7917E.pdf, bottom page 27.



Source

The signal source supplies the stimulus for our stimulus-response test system. We can either sweep the frequency of the source or sweep its power level. Traditionally, network analyzers used a separate source. These sources were either based on open-loop voltage-controlled oscillators (VCOs) which were cheaper, or more expensive synthesized sweepers which provided higher performance, especially for measuring narrowband devices. Excessive phase noise on open-loop VCOs degrades measurement accuracy considerably when measuring narrowband components over small frequency spans. Most network analyzers that Keysight sells today have integrated, synthesized sources, providing excellent frequency resolution and stability.

Keysight Technologies Network Analyzer Basics, 5965-7917E.pdf, page 28.

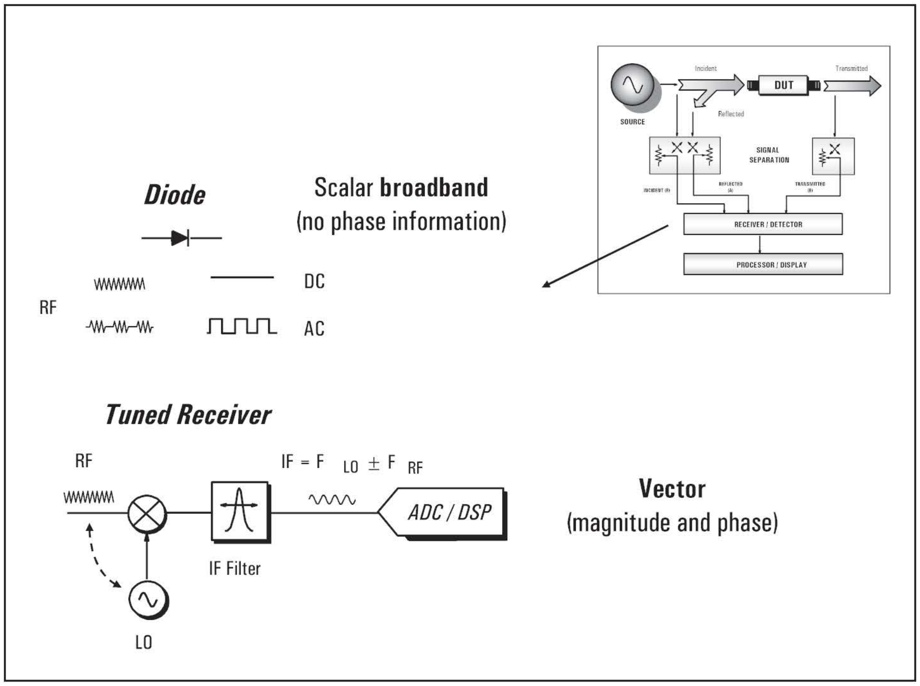


Signal Separation

The next major area we will cover is the signal separation block. The hardware used for this function is generally called the "test set". The test set can be a separate box or integrated within the network analyzer. There are two functions that our signal-separation hardware must provide. The first is to measure a portion of the incident signal to provide a reference for ratioing. This can be done with splitters or directional couplers. Splitters are usually resistive. They are non-directional devices (more on directionality later) and can be very broadband. The trade-off is that they usually have 6 dB or more of loss in each arm. Directional couplers have very low insertion loss (through the main arm) and good isolation and directivity. They are generally used in microwave network analyzers, but their inherent high-pass response makes them unusable below 40 MHz or so.

The second function of the signal-splitting hardware is to separate the incident (forward) and reflected (reverse) traveling waves at the input of our DUT. Again, couplers are ideal in that they are directional, have low loss, and high reverse isolation. However, due to the difficulty of making truly broadband couplers, bridges are often used instead. Bridges work down to DC, but have more loss, resulting in less signal power delivered to the DUT. See the appendix for a more complete description of how a directional bridge works.

Keysight Technologies Network Analyzer Basics, 5965-7917E.pdf, page 29.

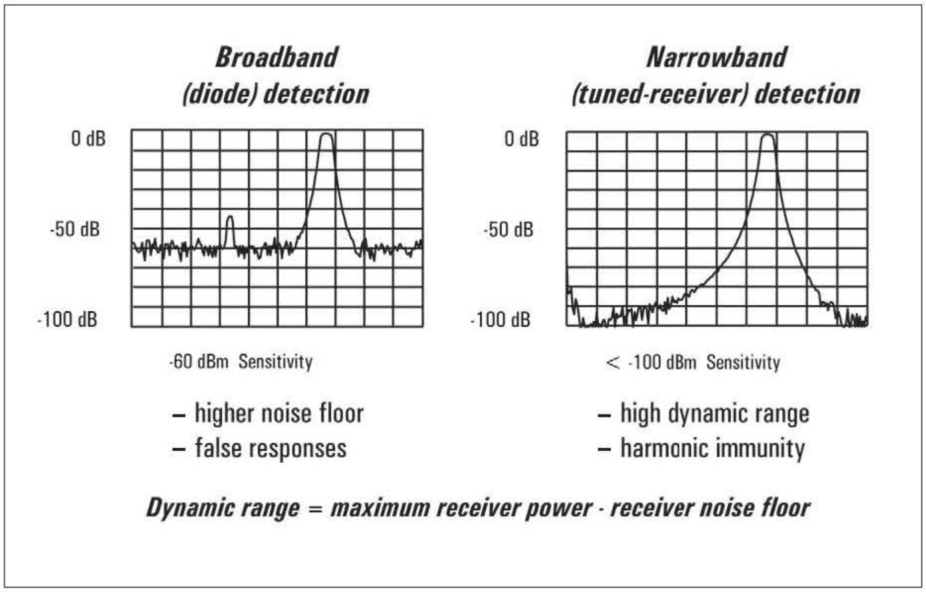


Detector Types

The next portion of the network analyzer we'll look at is the signal-detection block. There are two basic ways of providing signal detection in network analyzers. Diode detectors convert the RF signal level to a proportional DC level. If the stimulus signal is amplitude modulated, the diode strips the RF carrier from the modulation (this is called AC detection). Diode detection is inherently scalar, as phase information of the RF carrier is lost.

The tuned receiver uses a local oscillator (LO) to mix the RF down to a lower "intermediate" frequency (IF). The LO is either locked to the RF or the IF signal so that the receivers in the network analyzer are always tuned to the RF signal present at the input. The IF signal is bandpass filtered, which narrows the receiver bandwidth and greatly improves sensitivity and dynamic range. Modern analyzers use an analog-to-digital converter (ADC) and digital-signal processing (DSP) to extract magnitude and phase information from the IF signal. The tuned-receiver approach is used in vector network analyzers and spectrum analyzers.

Keysight Technologies Network Analyzer Basics, 5965-7917E.pdf, page 31.

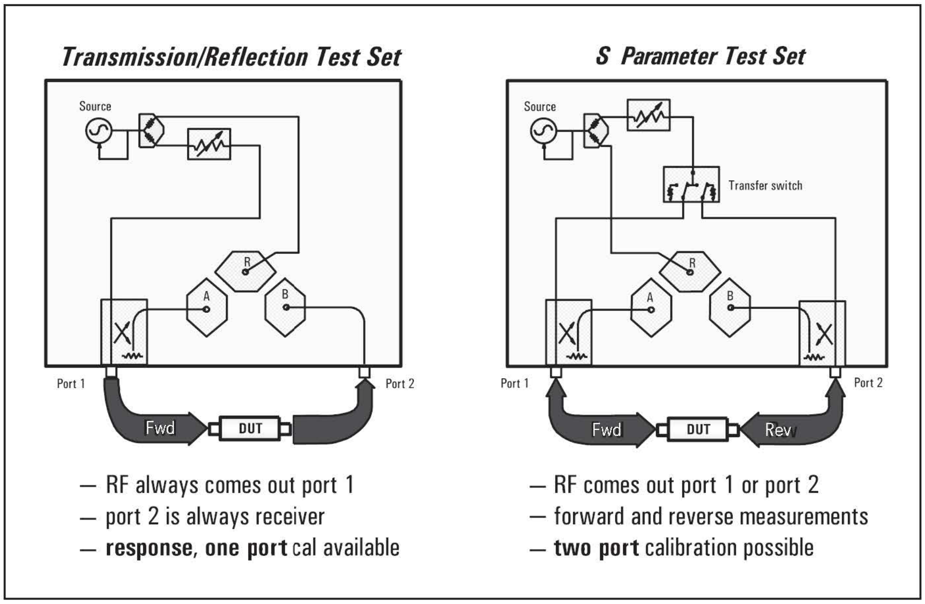


Comparision of Receiver Techniques

Dynamic range is generally defined as the maximum power the receiver can accurately measure minus the receiver noise floor. There are many applications requiring large dynamic range. One of the most common is measuring filter stopband performance. As you can see here, at least 80 dB dynamic range is needed to properly characterize the rejection characteristics of this filter. The plots show a typical narrowband filter measured on an 8757 scalar network analyzer and on an 8510 vector network analyzer. Notice that the filter exhibits 90 dB of rejection but the scalar analyzer is unable to measure it because of its higher noise floor.

In the case where the scalar network analyzer was used with broadband diode detection, a harmonic from the source created a "false" response. For example, at some point on a broadband sweep, the second harmonic of the source might fall within the passband of the filter. If this occurs, the detector will register a response, even though the stopband of the filter is severely attenuating the frequency of the fundamental. This response from the second harmonic would show on the display at the frequency of the fundamental. On the tuned receiver, a false signal such as this would be filtered away and would not appear on the display. Note that source subharmonics and spurious outputs can also cause false display responses.

Keysight Technologies Network Analyzer Basics, 5965-7917E.pdf, page 34.



T/R Versus S-Parameter Test Sets

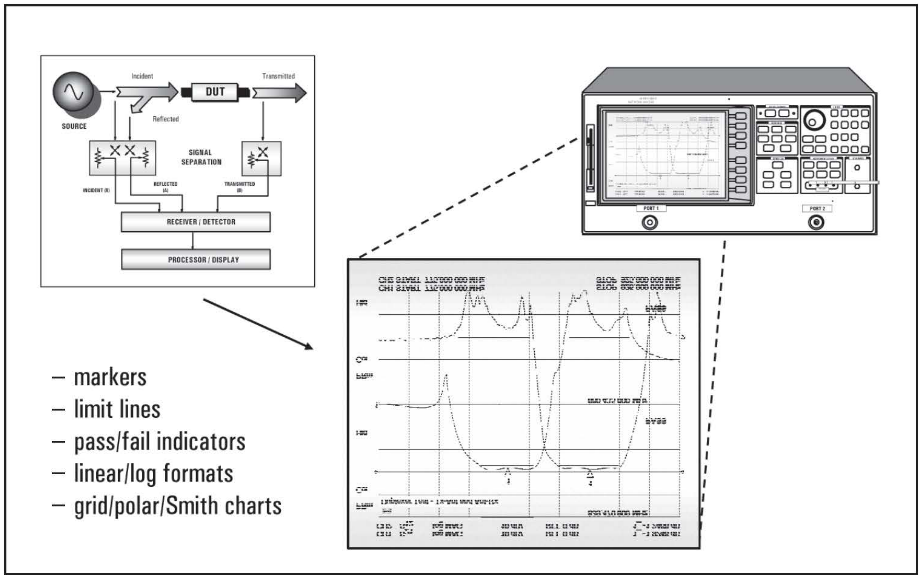
There are two basic types of test sets that are used with network analyzers. For transmission/reflection (T/R) test sets, the RF power always comes out of test port one and test port two is always connected to a receiver in the analyzer. To measure reverse transmission or output reflection of the DUT, we must disconnect it, turn it around, and re-connect it to the analyzer. T/R-based network analyzers offer only response and one-port calibrations, so measurement accuracy is not as good as that which can be achieved with S-parameter test sets. However, T/R-based analyzers are more economical. For the 8712, 8753 and 8720 families, Keysight uses the ET suffix to denote a T/R analyzer, and the ES suffix to denote an S-parameter analyzer.

S-parameter test sets allow both forward and reverse measurements on the DUT, which are needed to characterize all four S-parameters. RF power can come out of either test port one or two, and either test port can be connected to a receiver. S-parameter test sets also allow full two-port (12-term) error correction, which is the most accurate form available. S-parameter network analyzers provide more performance than T/R-based analyzers, but cost more due to extra RF components in the test set.

There are two different types of transfer switches that can be used in an S-parameter test set: solid-state and mechanical. Solid-state switches have the advantage of infinite lifetimes (assuming they are not damaged by too much power from the DUT). However, they are more lossy so they reduce the maximum output power of the network analyzer. Mechanical switches have very low loss and therefore allow higher output powers. Their main disadvantage is that eventually they wear out (after 5 million cycles or so). When using a network analyzer with mechanical switches, measurements are generally done in single-sweep mode, so the transfer switch is not continuously switching.

S-parameter test sets can have either a 3-receiver (shown on slide) or 4-receiver architecture. The 8753 series and standard 8720 series analyzers have a 3-receiver architecture. Option 400 adds a fourth receiver to 8720 series analyzers, to allow true TRL calibration. The 8510C family and the PNA Series uses a 4-receiver architecture. More detailed information of the two architecture is available in the appendix.

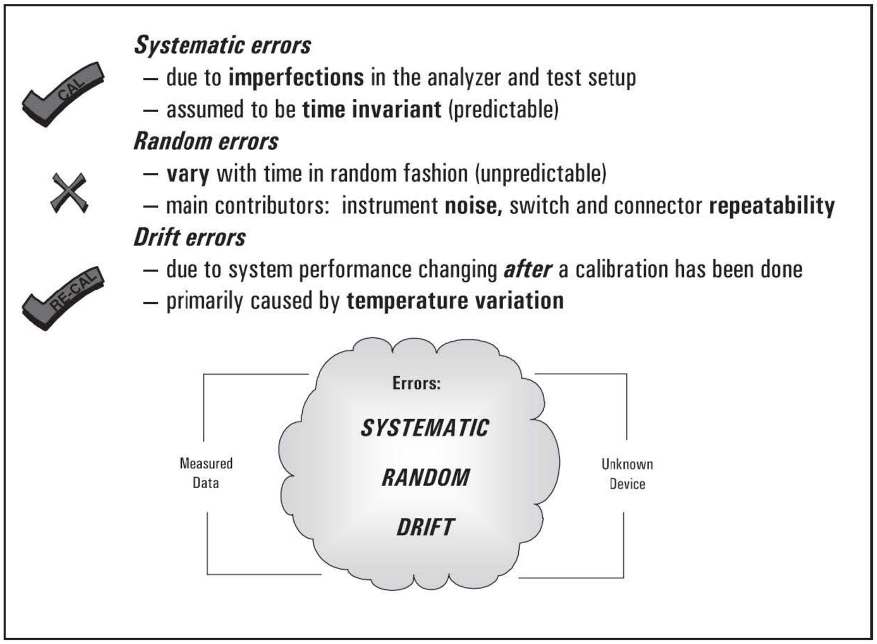
Keysight Technologies Network Analyzer Basics, 5965-7917E.pdf, page 36.



Processor/Display

The last major block of hardware in the network analyzer is the display/processor section. This is where the reflection and transmission data is formatted in ways that make it easy to interpret the measurement results. Most network analyzers have similar features such as linear and logarithmic sweeps, linear and log formats, polar plots, Smith charts, etc. Other common features are trace markers, limit lines, and pass/fail testing. Many of Keysight's network analyzers have specialized measurement features tailored to a particular market or application. One example is the E5100A/B, which has features specific to crystal-resonator manufacturers.

Keysight Technologies Network Analyzer Basics, 5965-7917E.pdf, top page 37.



Measurement Error Modeling

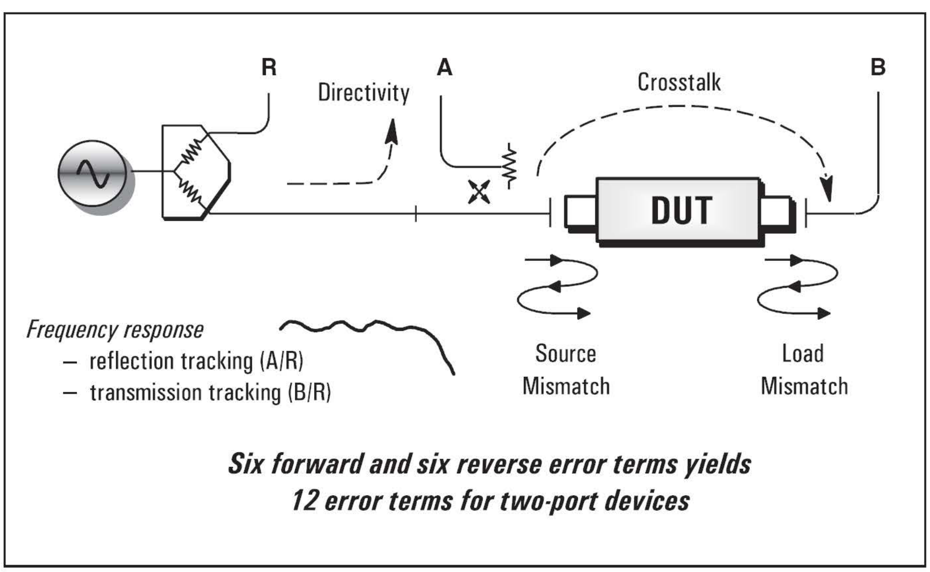
Let's look at the three basic sources of measurement error: systematic, random and drift.

Systematic errors are due to imperfections in the analyzer and test setup. They are repeatable (and therefore predictable), and are assumed to be time invariant. Systematic errors are characterized during the calibration process and mathematically removed during measurements.

Random errors are unpredictable since they vary with time in a random fashion. Therefore, they cannot be removed by calibration. The main contributors to random error are instrument noise (source phase noise, sampler noise, IF noise).

Drift errors are due to the instrument or test-system performance changing *after* a calibration has been done. Drift is primarily caused by temperature variation and it can be removed by further calibration(s). The timeframe over which a calibration remains accurate is dependent on the rate of drift that the test system undergoes in the user's test environment. Providing a stable ambient temperature usually goes a long way towards minimizing drift.

Keysight Technologies Network Analyzer Basics, 5965-7917E.pdf, bottom page 40.



Systematic Measurement Errors

Shown here are the major systematic errors associated with network measurements. The errors relating to signal leakage are directivity and crosstalk. Errors related to signal reflections are source and load match. The final class of errors are related to frequency response of the receivers, and are called reflection and transmission tracking. The full two-port error model includes all six of these terms for the forward direction and the same six (with different data) in the reverse direction, for a total of twelve error terms. This is why we often refer to two-port calibration as twelve-term error correction

Keysight Technologies Network Analyzer Basics, 5965-7917E.pdf, page 41.

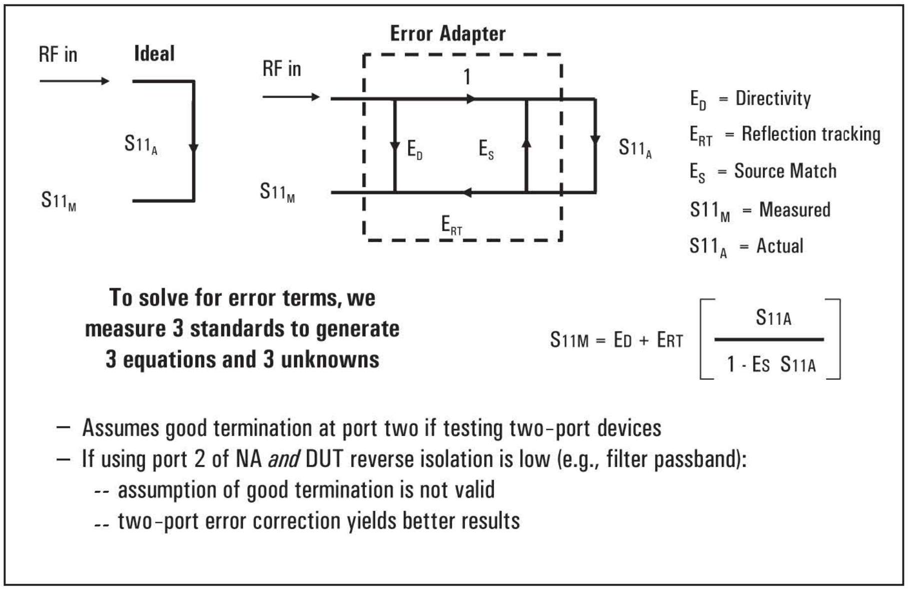
– Process of characterizing systematic error terms
  -- measure **known standards**
  -- remove effects from subsequent measurements
– **1-port calibration** *(reflection measurements)*
  -- only 3 systematic error terms measured
  -- directivity, source match, and reflection tracking
– **Full 2-port calibration** *(reflection and transmission measurements)*
  -- 12 systematic error terms measured
  -- usually requires 12 measurements on four known standards (SOLT)
– Standards defined in **cal kit definition** file
  -- network analyzer contains standard cal kit definitions
  -- **CAL KIT DEFINITION MUST MATCH ACTUAL CAL KIT USED!**
  -- User-built standards must be characterized and entered into user cal-kit

What is Vector-Error Correction?

Vector-error correction is the process of characterizing systematic error terms by measuring known calibration standards, and then removing the effects of these errors from subsequent measurements.

One-port calibration is used for reflection measurements and can measure and remove three systematic error terms (directivity, source match, and reflection tracking). Full two-port calibration can be used for both reflection and transmission measurements, and all twelve systematic error terms are measured and removed. Two-port calibration usually requires twelve measurements on four known standards (short-open-load-through or SOLT). Some standards are measured multiple times (e.g., the through standard is usually measured four times). The standards themselves are defined in a cal-kit definition file, which is stored in the network analyzer. Keysight network analyzers contain all of the cal-kit definitions for our standard calibration kits. In order to make accurate measurements, the cal-kit definition MUST MATCH THE ACTUAL CALIBRATION KIT USED! If userbuilt calibration standards are used (during fixtured measurements for example), then the user must characterize the calibration standards and enter the information into a user cal-kit file. Sources of more information about this topic can be found in the appendix.

Keysight Technologies Network Analyzer Basics, 5965-7917E.pdf, page 43.

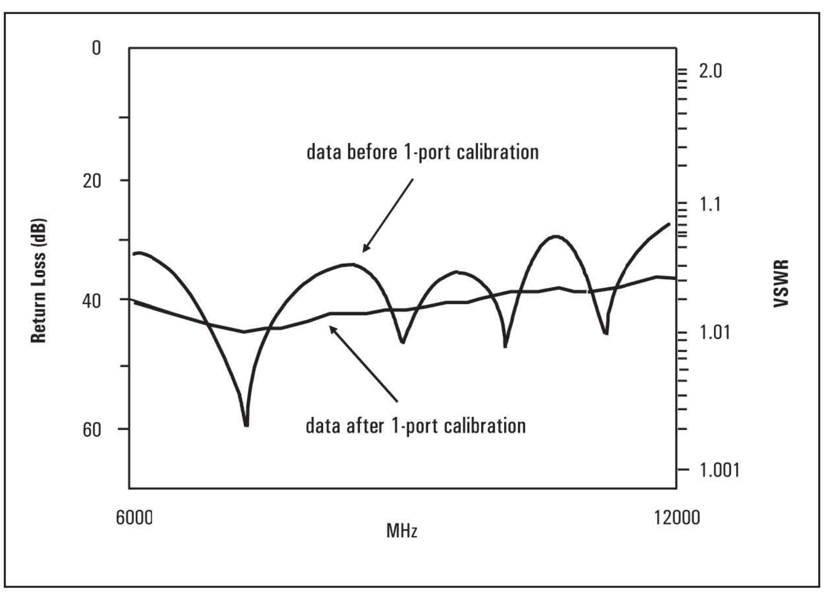


Reflection: One-Port Model

Taking the simplest case of a one-port reflection measurement, we have three systematic errors and one equation to solve in order to calculate the actual reflection coefficient from the measured value. In order to do this, we must first calculate the individual error terms contained in this equation. We do this by creating three more equations with three unknowns each, and solving them simultaneously. The three equations come from measuring three known calibration standards, for example, a short, an open, and a Z_0 load. Solving the equations will yield the systematic error terms and allow us to derive the actual reflection S-parameter of the device from our measurements.

When measuring reflection two-port devices, a one-port calibration assumes a good termination at port two of the device. If this condition is met (by connecting a load calibration standard for example), the one-port calibration is quite accurate. If port two of the device is connected to the network analyzer and the reverse isolation of the DUT is low (for example, filter passbands or cables), the assumption of a good load termination is not valid. In these cases, two-port error correction provides more accurate measurements. An example of a two-port device where load match is not important is an amplifier. The reverse isolation of the amplifier allows one-port calibration to be used effectively. An example of the measurement error that can occur when measuring a two-port filter using a one-port calibration will be shown shortly.

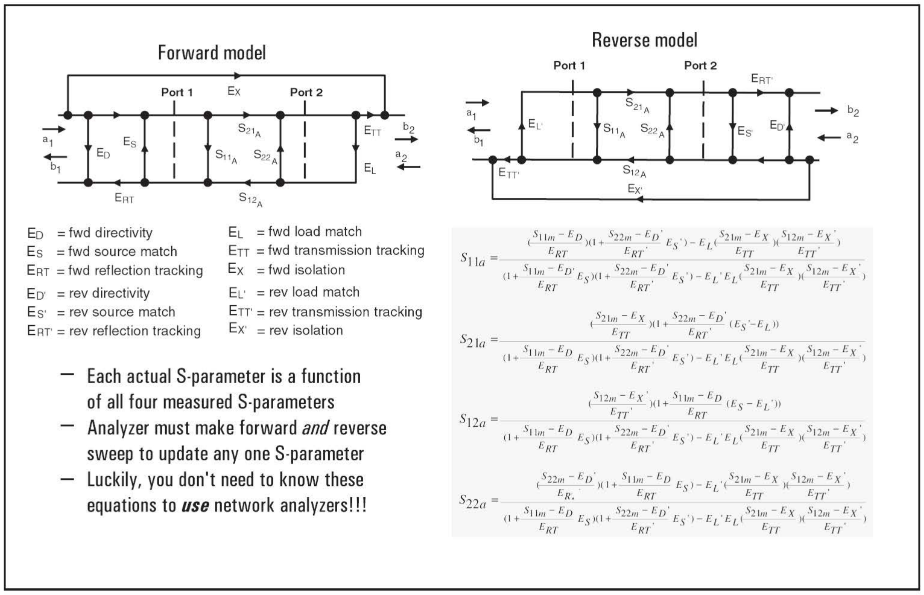
Keysight Technologies Network Analyzer Basics, 5965-7917E.pdf, page 44.



Before and After One-Port Calibration

Shown here is a plot of reflection with and without one-port calibration. Without error correction, we see the classic ripple pattern caused by the systematic errors interfering with the measured signal. The error-corrected trace is much smoother and better represents the device's actual reflection performance.

Keysight Technologies Network Analyzer Basics, 5965-7917E.pdf, page 45.



Two-Port Error Correction

Two-port error correction is the most accurate form of error correction since it accounts for all of the major sources of systematic error. The error model for a two-port device is shown above. Shown below are the equations to derive the actual device S-parameters from the measured S-parameters, once the systematic error terms have been characterized. Notice that each actual S-parameter is a function of all four measured S-parameters. The network analyzer must make a forward and reverse sweep to update any one S-parameter. Luckily, you don't need to know these equations to use network analyzers!!!

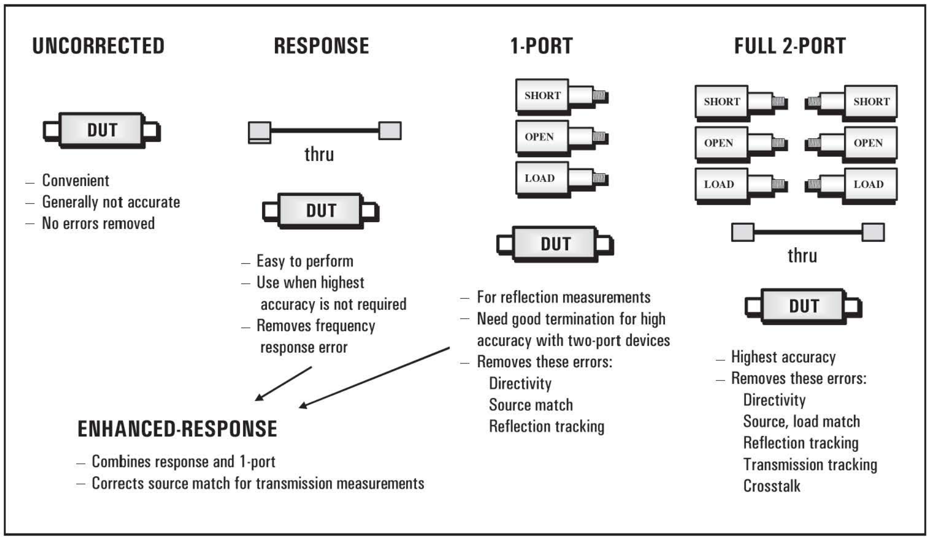
$$S_{11a} = \frac{\left(\frac{S_{11m}-E_D}{E_{RT}}\right)\left(1+\frac{S_{22m}-E_D'}{E_{RT}'}E_S'\right)-E_L\left(\frac{S_{21m}-E_X}{E_{TT}}\right)\left(\frac{S_{12m}-E_X'}{E_{TT}'}\right)}{\left(1+\frac{S_{11m}-E_D}{E_{RT}}E_S\right)\left(1+\frac{S_{22m}-E_D'}{E_{RT}'}E_S'\right)-E_L'E_L\left(\frac{S_{21m}-E_X}{E_{TT}}\right)\left(\frac{S_{12m}-E_X'}{E_{TT}'}\right)}$$

$$S_{21a} = \frac{\left(\frac{S_{21m}-E_X}{E_{TT}}\right)\left(1+\frac{S_{22m}-E_D'}{E_{RT}'}(E_S'-E_L)\right)}{\left(1+\frac{S_{11m}-E_D}{E_{RT}}E_S\right)\left(1+\frac{S_{22m}-E_D'}{E_{RT}'}E_S'\right)-E_L'E_L\left(\frac{S_{21m}-E_X}{E_{TT}}\right)\left(\frac{S_{12m}-E_X'}{E_{TT}'}\right)}$$

$$S_{12a} = \frac{\left(\frac{S_{12m}-E_X'}{E_{TT}'}\right)\left(1+\frac{S_{11m}-E_D}{E_{RT}}(E_S-E_L')\right)}{\left(1+\frac{S_{11m}-E_D}{E_{RT}}E_S\right)\left(1+\frac{S_{22m}-E_D'}{E_{RT}'}E_S'\right)-E_L'E_L\left(\frac{S_{21m}-E_X}{E_{TT}}\right)\left(\frac{S_{12m}-E_X'}{E_{TT}'}\right)}$$

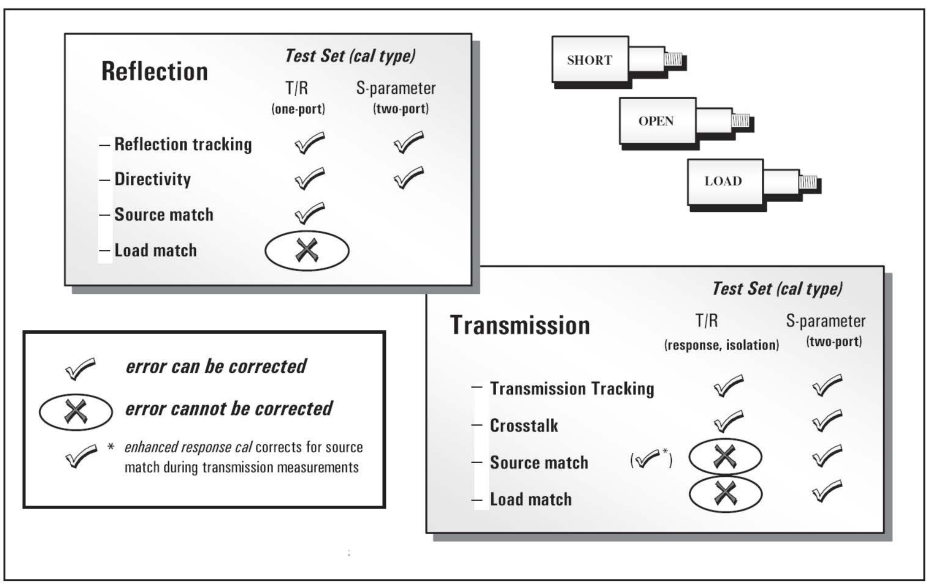
$$S_{22a} = \frac{\left(\frac{S_{22m}-E_D'}{E_{RT}'}\right)\left(1+\frac{S_{11m}-E_D}{E_{RT}}E_S\right)-E_L'\left(\frac{S_{21m}-E_X}{E_{TT}}\right)\left(\frac{S_{12m}-E_X'}{E_{TT}'}\right)}{\left(1+\frac{S_{11m}-E_D}{E_{RT}}E_S\right)\left(1+\frac{S_{22m}-E_D'}{E_{RT}'}E_S'\right)-E_L'E_L\left(\frac{S_{21m}-E_X}{E_{TT}}\right)\left(\frac{S_{12m}-E_X'}{E_{TT}'}\right)}$$

Keysight Technologies Network Analyzer Basics, 5965-7917E.pdf, page 46.



Errors and Calibration Standards

A network analyzer can be used for uncorrected measurements, or with any one of a number of calibration choices, including response calibrations and one- or two-port vector calibrations. A summary of these calibrations is shown above. We will explore the measurement uncertainties associated with the various calibration types in this section.



Calibration Summary

This summary shows which error terms are accounted for when using analyzers with T/R test sets (models ending with ET) and S-parameter test sets (models ending with ES). Notice that load match is the key error term than cannot be removed with a T/R-based network analyzer.

The following examples show how measurement uncertainty can be estimated when measuring two-port devices with a T/R-based network analyzer. We will also show how 2-port error correction provides the least measurement uncertainty.

Keysight Technologies Network Analyzer Basics, 5965-7917E.pdf, page 48.

Microwave Laboratory Equipment and Practices

You will discover that making microwave/RF circuit measurements is quite different than measurements at lower frequencies. For example:

- We will use a vector network analyzer (VNA) to make both magnitude and phase measurements of *S* parameters rather than other equipment used with DC and lower frequency circuits. By its nature, a VNA makes frequency domain measurements.
- We will take precautions against electrostatic electricity.
- We will use precision connectors and special tightening tools (torque wrenches) in order to get accurate and repeatable measurements.
- Calibrations will be done before taking measurements to remove systematic errors from data.

For this class, we will use a Keysight E5063A ENA Series Network Analyzer (see below). This VNA has a possible measurement bandwidth from 100 kHz to 8.5 GHz.

Note that the E5063A is a two port VNA with type N connectors. However, we will typically use a Type N to SMA adaptor to match our cables. For one port calibrations, we will use an Agilent 85033E 3.5 mm Calibration Kit. (below).

Types of Coaxial Connectors

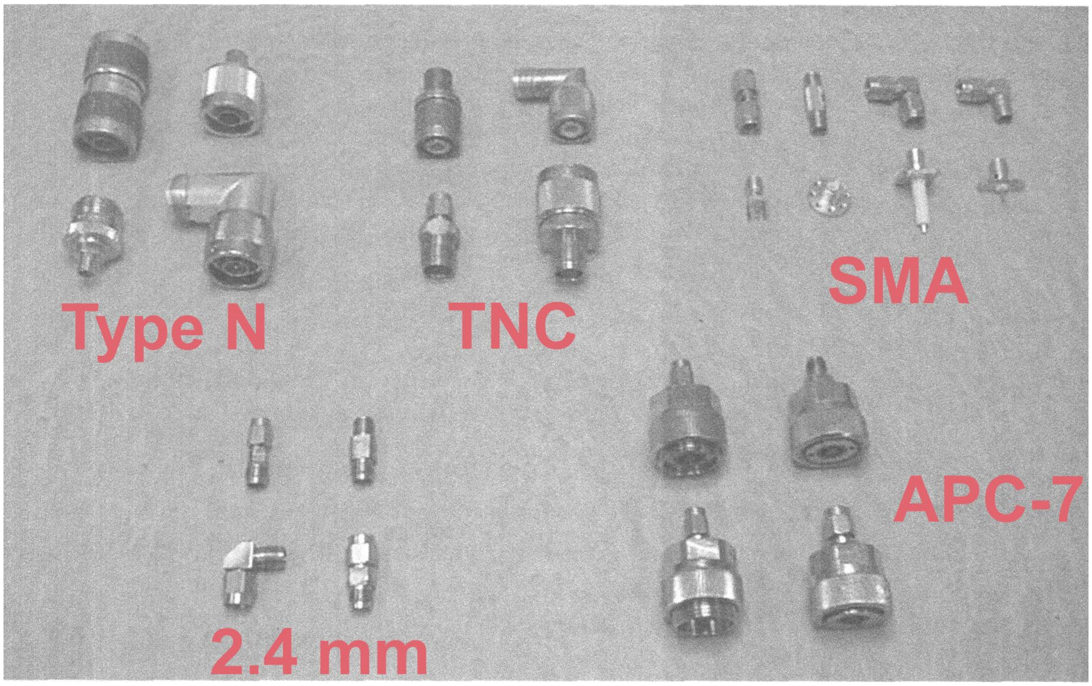
There are many types of coaxial connectors used to serve a variety of uses, i.e., frequency range, voltage, power handling capacity, PCB mounting, etcetera. Some **commercially** used connectors include-

- **BNC (Bayonet Neill–Concelman, Bayonet Navy Connector, or British Naval Connector)-** 1950s vintage. These are usually made to match cables with characteristic impedances of either 50 Ω or 75 Ω. They are usually used for frequencies lower than 4 GHz (<2 GHz in my experience) and voltages below 500 V.
- **TNC (Threaded Neill–Concelman or Threaded Navy Connector)-** 1950s vintage. These are usually made to match cables with characteristic impedances of 50 Ω or 75 Ω. They are usually used for frequencies lower than 11 GHz (<8 GHz in my experience) and voltages below 500 V.
- **Type F (sometimes called RCA)**. 1950s vintage. Typically used with 75 Ω RG-6 or RG-59 coaxial cables for television applications. They are usually used for frequencies lower than 2 GHz and voltages below 500 V.
- **Type N**. 1940s vintage. Named after Paul Neill of Bell Labs. Typically used with 50 Ω or 75 Ω coaxial cables. Originally, they were usually used for frequencies lower than 11 GHz, but now can go to 18 GHz. They can handle voltages below 1000 V_{rms} and power levels up to 500 W at 2 GHz.
- **SMA (SubMiniature version A)-** 1960s vintage. Typically used with 50 Ω coaxial cables. They were usually used for frequencies up to 18 GHz, but some can go to 26.5 GHz. They can handle voltages below 375 to 500 V_{rms} (depends on cable). While SMA connectors use a Teflon or PTFE insulation, they are compatible with 3.5 mm connectors. Probably, the most common/popular microwave connector.

Some **metrology-grade** (i.e., high precision, lab settings) air-dielectric connectors include-

- **APC-7 (Amphenol Precision Connector-7 mm or 7 mm)-** Typically used with 50 Ω coaxial cables for frequencies up to 18 GHz. Genderless.
- **APC-3.5 (Amphenol Precision Connector-3.5 mm or 3.5 mm, sometimes called K type)-** Typically used with 50 Ω coaxial cables for frequencies up to 26.5 GHz. Compatible with SMA connectors.
- **2.92 mm (sometimes called K type)-** Typically used with 50 Ω coaxial cables for frequencies up to 40 GHz. Compatible with SMA connectors.
- **2.4 mm (sometimes called V type)-** Typically used with 50 Ω coaxial cables for frequencies up to 50 GHz. Not compatible with SMA connectors.
- **1.85 mm (sometimes called V type)-** Typically used with 50 Ω coaxial cables for frequencies up to 67 GHz. Not compatible with SMA connectors.

The picture below shows a few of these connectors.



Microwave Engineering (Fourth Edition), Pozar, Wiley, 2012, p. 134, ISBN 978-0-470-63155-3.

When using threaded RF/microwave connectors, users will typically finger tighten the connector before using a torque wrench to finish tightening. For example, the torque wrench (gold handle) from the Agilent 85033E 3.5 mm Calibration Kit is for a torque of 0.9 N·m or 8 in-lbs. Torque wrenches are used to get repeatable connections. In many cases, open-ended box wrenches are used to provide mechanical support to the connector and resistance to the torque wrenches. It is possible to over tighten a connector which can cause damage.

Some considerations when making RF/microwave coaxial connections-

- Be electrostatically grounded (i.e., wrist strap). In addition, I like to tap the shield/exterior of the connectors together before making the connection.
- Inspect connectors to ensure there is no debris inside. If there is debris, remove using canned air (at an angle) or lint-free cleaning ‘Q tips’ with rubbing alcohol.

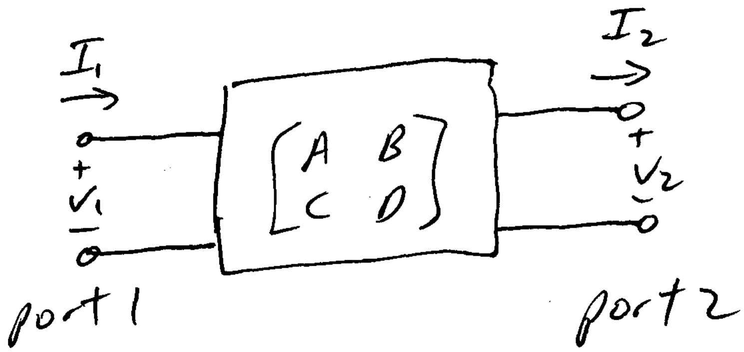
- Carefully line up the connectors (and cables) before beginning to finger tighten. If you feel any resistance, back off, and try again. Do NOT force the connection as you may cross thread the connector which destroys/ruins it!
- Finger tighten the connectors. Then, use the appropriate torque wrench as well as open-ended wrench (if needed) to finish tightening the connection. I.e., hold the torque wrench beyond the ring/groove and tighten until you feel it <u>begin</u> to ‘break’ at the knee or hear it click. The open-ended wrench is used to hold the non-moving part of the connection.
- Use the appropriate torque wrench as well as open-ended wrench (if needed) when disconnecting a connection to avoid inadvertently damaging the connector(s).

4.4 The Transmission (ABCD) Matrix

So far, we have seen [Z], [Y], + [S] matrices that are good for n-port networks.

* [Z] parameters are good for networks connected in series.
* [Y] parameters are good for networks connected in parallel.
* [S] parameters are well-suited for microwave measurements and describing current + voltage waves

A common scenario in microwave work is the head-to-tail or cascade connection of two-port networks. For this, the 2x2 transmission or [ABCD] matrix was developed.



4.4 cont.

Note: Unlike [Z] & [Y] parameters, I_2 comes out of port 2.

We define the relationship of the total currents + voltages at the two ports as

$$V_1 = A V_2 + B I_2$$

$$I_1 = C V_2 + D I_2$$

or

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad (4.69)$$

where each parameter is found as

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0 \text{ (open ckt)}} \quad (V/V)$$

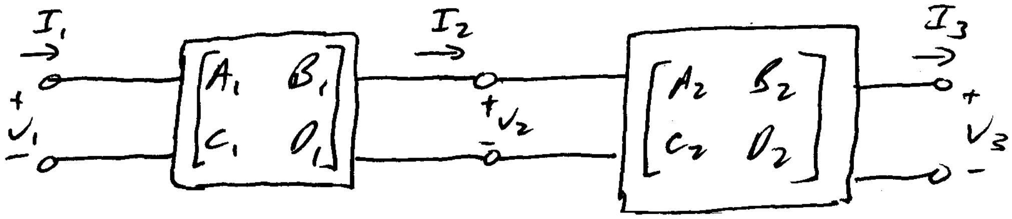
$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0 \text{ (short ckt)}} \quad \left(\frac{V}{A} = \Omega\right)$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0 \text{ (open ckt)}} \quad (A/V = S \text{ or } \mho)$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0 \text{ (short ckt)}} \quad (A/A)$$

4.4 cont.

Let's consider the cascade connection



By definition

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad (4.70a)$$

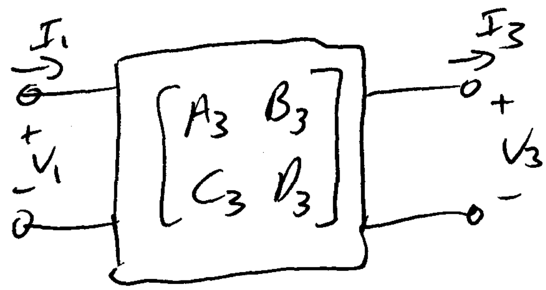
and

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} \qquad (4.70b)$$

If we substitute the RHS of (4.70b) into (4.70a), we get

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}}_{\begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix}} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} \qquad (4.71)$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$



$\Rightarrow$ We can replace two separate two-ports with a single equivalent two-port!

4.4 cont.

left ⟶ right

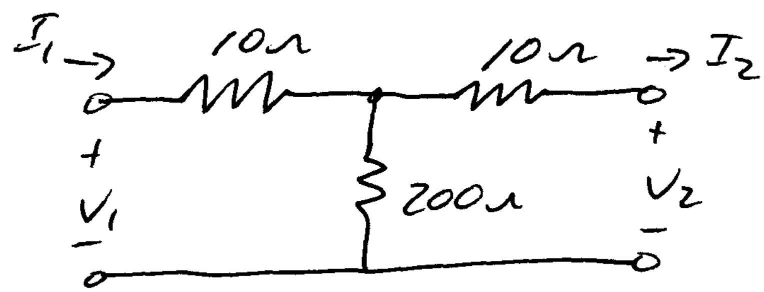
In general, $\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdots \begin{bmatrix} A_N & B_N \\ C_N & D_N \end{bmatrix}$

* Order of matrix multiplication <u>must</u> be the same as the order that the applicable two-port networks are cascaded as matrix multiplication is NOT commutative in general, i.e.,

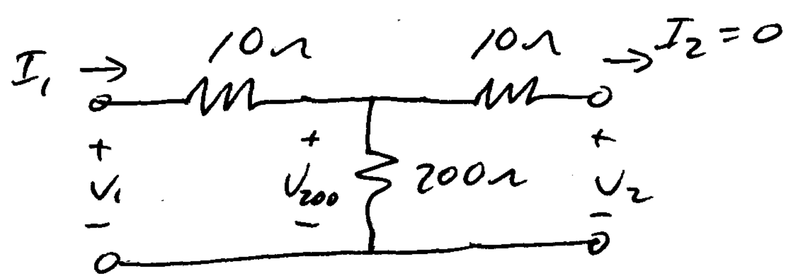
$$[A][B] \neq [B][A]$$

4.4 cont.

example - Let's find the [ABCD] parameters for our resistive T-network.



$\boxed{A \,\&\, C}$ Set $I_2 = 0$ (open)



By voltage division, $V_{200} = V_1 \frac{200}{200+10} = V_1 \frac{20}{21}$

By KVL, $-V_{200} + 0(10) + V_2 = 0 \Rightarrow V_2 = V_{200} = V_1 \frac{20}{21}$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{21}{20} = \underline{1.05\left(V/V\right)}$$

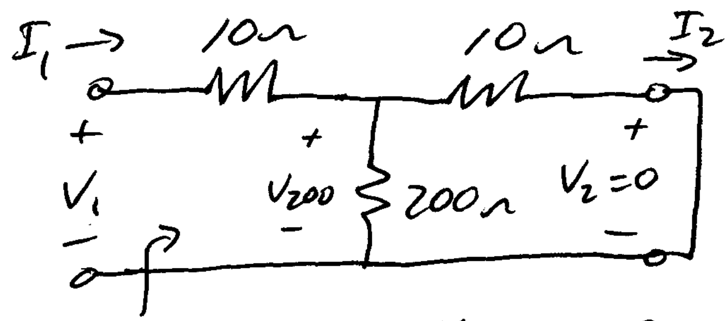
By Ohm's Law, $I_1 = \frac{V_1}{10+200}$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{\frac{V_1}{210}}{V_1\,{}^{20}\!/_{21}} = \underline{0.005\,(S)}$$

4.4 cont.

example cont.

$\boxed{B + D}$ Set $V_2 = 0$ (short)



$$R_{eq} = 10 + 10 // 200 = 19.52381\,\Omega$$

$$V_{200} = V_1 \frac{200 // 10}{R_{eq}} = V_1 \frac{9.52381}{19.52381}$$

$$I_2 = \frac{V_{200}}{10} = V_1 \frac{9.52381}{195.2381}$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{V_1}{V_1 \frac{9.52381}{195.2381}} = \underline{20.5\ (\Omega)}$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{V_1/R_{eq}}{V_1 \frac{9.52381}{195.2381}} = \frac{195.2381}{19.52381(9.52381)}$$

$$\underline{D = 1.05\ (A/A)}$$

$$\boxed{[ABCD] = \begin{bmatrix} 1.05 & 20.5\,\Omega \\ 0.005\,S & 1.05 \end{bmatrix}}$$

4.4 cont.

As one might expect from the equations

$$(4.69) \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad \text{and}$$

$$(4.25) \quad \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix},$$

there is a relation between [ABCD] and [Z] parameters. Note: $I_{2,Z} = -I_{2,ABCD}$.

Using (4.25) with this current substitution yields

$$V_1 = I_1 Z_{11} - I_2 Z_{12} \qquad (A)$$

$$V_2 = I_1 Z_{21} - I_2 Z_{22} \qquad (B)$$

Then, we can find [ABCD] using (A) + (B).

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{I_1 Z_{11}}{I_1 Z_{21}} \Rightarrow \underline{A = \frac{Z_{11}}{Z_{21}}}$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{I_1 Z_{11} - I_2 Z_{12}}{I_2} = Z_{11}\frac{I_1}{I_2} - Z_{12}$$

$$= Z_{11}\frac{I_1}{I_1 \frac{Z_{21}}{Z_{22}}} - Z_{12} = \frac{Z_{11} Z_{22}}{Z_{21}} - Z_{12}$$

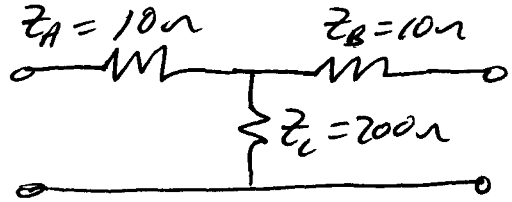
$$\underline{B = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}}$$

4.4 cont.

$$C = \frac{I_1}{V_2}\bigg|_{I_2=0} = \frac{I_1}{I_1 Z_{21}} \Rightarrow \underline{C = \frac{1}{Z_{21}}}$$

$$D = \frac{I_1}{I_2}\bigg|_{V_2=0} = \frac{I_2 \frac{Z_{22}}{Z_{21}}}{I_2} \Rightarrow \underline{D = \frac{Z_{22}}{Z_{21}}}$$

example - As a check, let's find the [Z]-parameters for our resistive T-network and then convert to [ABCD] parameters.



From earlier example

$$Z_{11} = Z_A + Z_C = 10 + 200 \Rightarrow Z_{11} = 210\,\Omega$$

$$Z_{12} = Z_C \Rightarrow Z_{12} = 200\,\Omega$$

$$Z_{21} = Z_C \Rightarrow Z_{21} = 200\,\Omega$$

$$Z_{22} = Z_B + Z_C = 10 + 200 \Rightarrow Z_{22} = 210\,\Omega$$

$$A = \frac{Z_{11}}{Z_{21}} = \frac{210}{200} = \underline{1.05} \quad \text{same!}$$

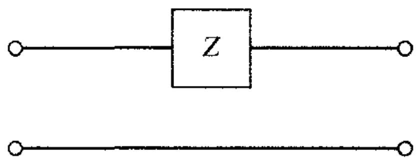
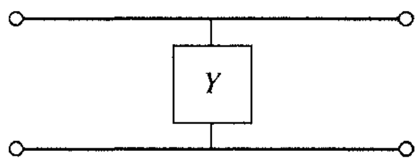
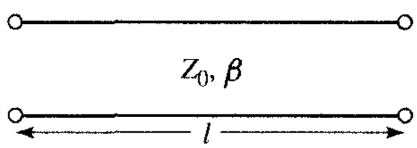
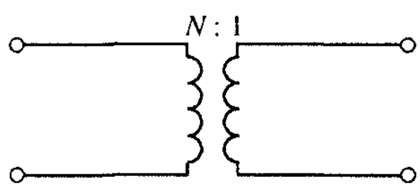
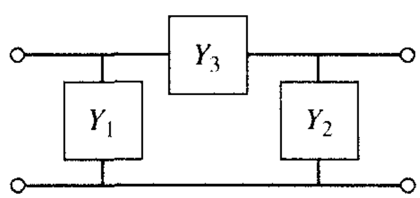
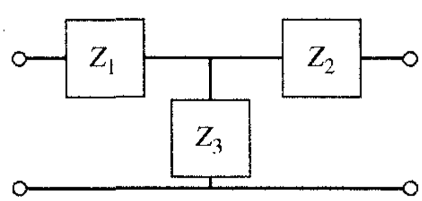
$$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} = \frac{210^2 - 200^2}{200} = \underline{20.5\,\Omega} \quad \text{same!}$$

$$C = \frac{1}{Z_{21}} = \frac{1}{200} = \underline{0.005\ S} \quad \text{same!}$$

$$D = \frac{Z_{22}}{Z_{21}} = \frac{210}{200} = \underline{1.05} \quad \text{same!}$$

- Table 4.1 (below) gives the [*ABCD*] parameters for some common two-port networks.

TABLE 4.1 ***ABCD*** **Parameters of Some Useful Two-Port Circuits**

Circuit	*ABCD* Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos\beta\ell$ $C = jY_0 \sin\beta\ell$	$B = jZ_0 \sin\beta\ell$ $D = \cos\beta\ell$
	$A = N$ $C = 0$	$B = 0$ $D = \frac{1}{N}$
	$A = 1 + \frac{Y_2}{Y_3}$ $C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3}$	$B = \frac{1}{Y_3}$ $D = 1 + \frac{Y_1}{Y_3}$
	$A = 1 + \frac{Z_1}{Z_3}$ $C = \frac{1}{Z_3}$	$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$ $D = 1 + \frac{Z_2}{Z_3}$

- Table 4.2 (following page) gives conversions between [*S*], [*Z*], [*Y*], and [*ABCD*] parameters for two-port networks.

TABLE 4.2 Conversions Between Two-Port Network Parameters

	S	Z	Y	$ABCD$
S_{11}	S_{11}	$\frac{(Z_{11}-Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0-Y_{11})(Y_0+Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\frac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D}$
S_{12}	S_{12}	$\frac{2Z_{12}Z_0}{\Delta Z}$	$\frac{-2Y_{12}Y_0}{\Delta Y}$	$\frac{2(AD-BC)}{A+B/Z_0+CZ_0+D}$
S_{21}	S_{21}	$\frac{2Z_{21}Z_0}{\Delta Z}$	$\frac{-2Y_{21}Y_0}{\Delta Y}$	$\frac{2}{A+B/Z_0+CZ_0+D}$
S_{22}	S_{22}	$\frac{(Z_{11}+Z_0)(Z_{22}-Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0+Y_{11})(Y_0-Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\frac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D}$
Z_{11}	$Z_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{11}	$\frac{Y_{22}}{\|Y\|}$	$\frac{A}{C}$
Z_{12}	$Z_0\frac{2S_{12}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{12}	$\frac{-Y_{12}}{\|Y\|}$	$\frac{AD-BC}{C}$
Z_{21}	$Z_0\frac{2S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{21}	$\frac{-Y_{21}}{\|Y\|}$	$\frac{1}{C}$
Z_{22}	$Z_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{22}	$\frac{Y_{11}}{\|Y\|}$	$\frac{D}{C}$
Y_{11}	$Y_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{22}}{\|Z\|}$	Y_{11}	$\frac{D}{B}$
Y_{12}	$Y_0\frac{-2S_{12}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{12}}{\|Z\|}$	Y_{12}	$\frac{BC-AD}{B}$
Y_{21}	$Y_0\frac{-2S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{21}}{\|Z\|}$	Y_{21}	$\frac{-1}{B}$
Y_{22}	$Y_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{11}}{\|Z\|}$	Y_{22}	$\frac{A}{B}$
A	$\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{11}}{Z_{21}}$	$\frac{-Y_{22}}{Y_{21}}$	A
B	$Z_0\frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{\|Z\|}{Z_{21}}$	$\frac{-1}{Y_{21}}$	B
C	$\frac{1}{Z_0}\frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{1}{Z_{21}}$	$\frac{-\|Y\|}{Y_{21}}$	C
D	$\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{22}}{Z_{21}}$	$\frac{-Y_{11}}{Y_{21}}$	D

$|Z| = Z_{11}Z_{22} - Z_{12}Z_{21}$; $|Y| = Y_{11}Y_{22} - Y_{12}Y_{21}$; $\Delta Y = (Y_{11}+Y_0)(Y_{22}+Y_0) - Y_{12}Y_{21}$; $\Delta Z = (Z_{11}+Z_0)(Z_{22}+Z_0) - Z_{12}Z_{21}$; $Y_0 = 1/Z_0$.

4.4 cont.

Another interesting property of [ABCD] parameters occurs for <u>reciprocal networks</u>. Using [Z] parameters, $Z_{12} = Z_{21}$ for a reciprocal network. This leads to

$$\underline{\mathrm{Det}\begin{bmatrix} A & B \\ C & D \end{bmatrix} = AD - BC = 1}$$

This can provide a useful check on results.

Second, again using [Z] parameters, for a <u>lossless network</u>, we know from (4.38) & (4.39) that all Z_{mn} are purely imaginary. Using the conversions, this implies

$$A = \frac{Z_{11}}{Z_{21}} \Rightarrow \text{real \#}$$

$$B = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}} = \frac{\text{real \#}}{\text{imag \#}} \Rightarrow \text{imaginary \#}$$

$$C = \frac{1}{Z_{21}} \Rightarrow \text{imaginary \#}$$

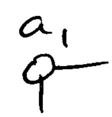
$$D = \frac{Z_{22}}{Z_{21}} \Rightarrow \text{real \#}$$

$\Rightarrow$ [ABCD] diagonal are real, off diagonal are imaginary.

4.5 Signal Flow Graphs

* Signal flow graphs are a graphical technique that is useful for analyzing signals flowing through networks that are linear.
* Signal flow graphs are used in several fields - microwave engineering w/ [S]-parameters, control systems, power systems, ...

For microwave engineering, the key parts of a signal flow graph are -

1) Nodes to represent system variables. Each port i of a microwave network will have two nodes, a_i and b_i. Node a_i is for the wave entering the port while node b_i is for the wave leaving the port. E.g. 

2) Branches are directional paths between nodes to represent signal flows.

E.g., $a_1 \xrightarrow{\text{gain}} a_2$

Note: Signals flow only in direction shown.

4.5 cont.

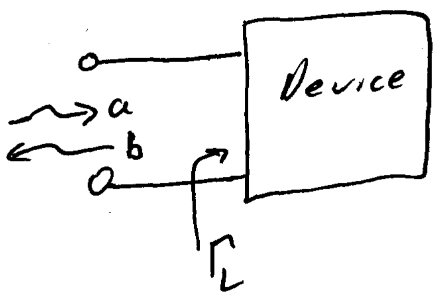
3) Each branch has a <u>gain</u> by which this signal/wave is multiplied as it passes through the branch. For microwave networks this will be the associated [S]-parameter or reflection coefficient. E.g., $b_1 \xrightarrow{S_{01}} b_0$ $\quad b_0 = S_{01} b_1$

4) When multiple branches enter a node, that node is equal to their sum

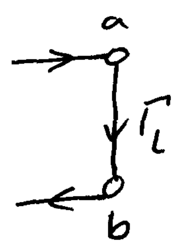
e.g.,

$$b_2 = S_{21} a_1 + S_{23} a_3$$

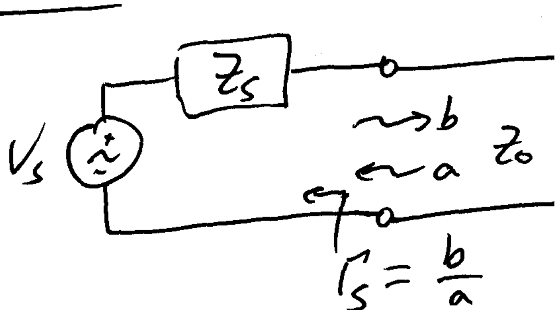
One-port network



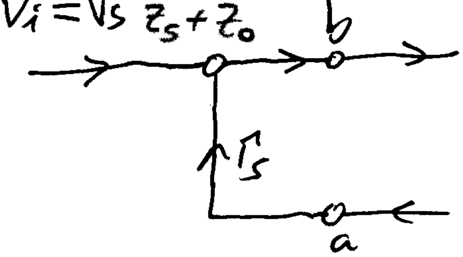
SFG



Source (looking into terminals)

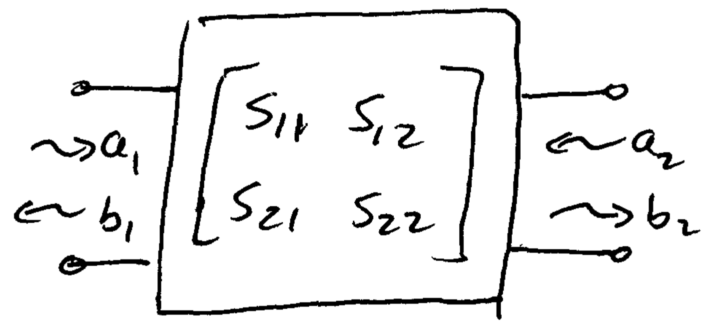


$$V_i = V_s \frac{Z_0}{Z_s + Z_0}$$



4.5 cont.

Two-port network

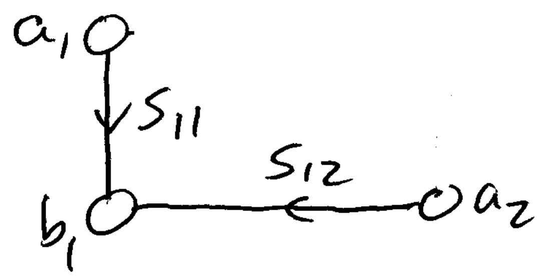


$$[b] = [S][a]$$

$$b_1 = S_{11} a_1 + S_{12} a_2$$

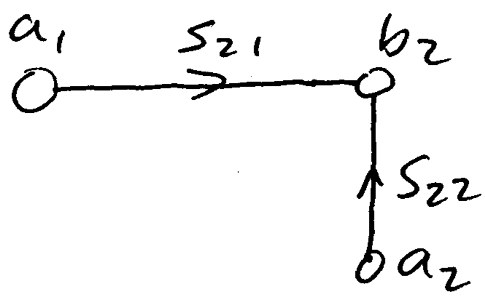
$$b_2 = S_{21} a_1 + S_{22} a_2$$

↓



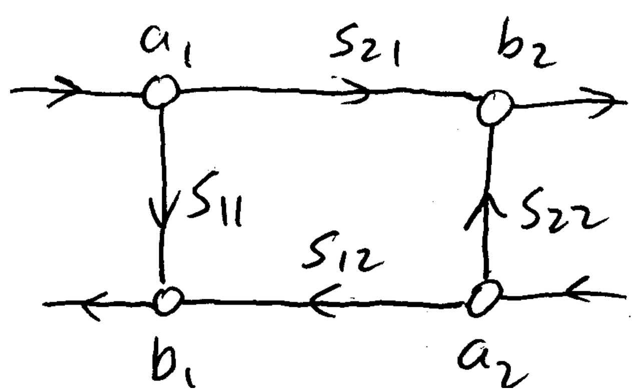
from b_1 eq'n

+



from b_2 eq'n

=

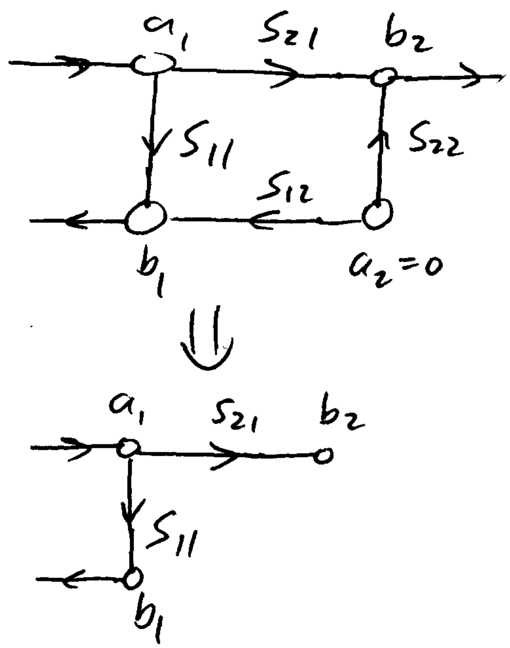


4.5 cont.

What happens if a port is matched?

→ Incident signal is then zero.

For example, if port 2 is matched for a two-port network. Then, $a_2 = 0$.



Signal Flow Graph (SFG) Algebra Rules

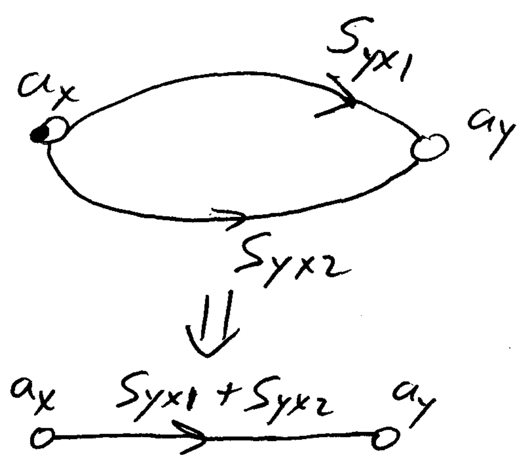
1) Series Rule Two branches who share a common intermediate node w/ one incoming and one outgoing branch (i.e., in series) can be combined into a single branch whose gain is the product of that of the two branches.

E.g. $$a_3 \xrightarrow{S_{43}} a_4 \xrightarrow{S_{54}} a_5 \quad \Rightarrow \quad a_3 \xrightarrow{S_{43}S_{54}} a_5$$

4.5 cont.

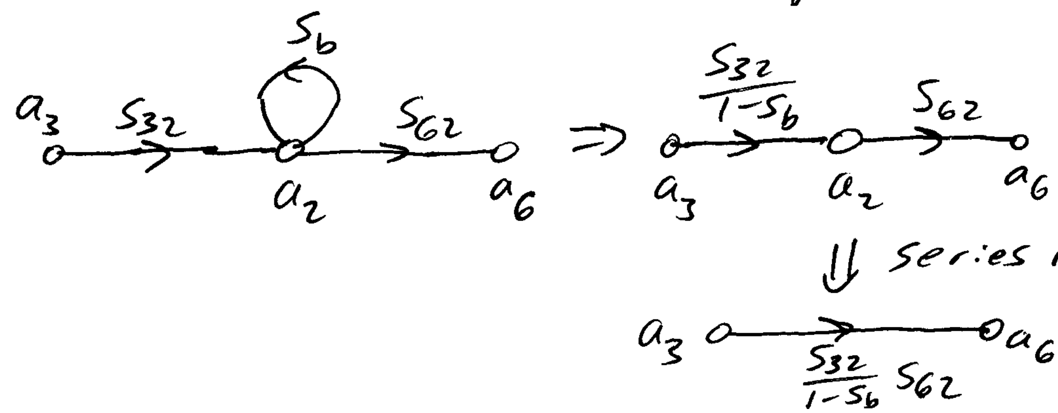
2) Parallel Rule Two branches from one node to another node (i.e., in parallel) can be combined into a single branch whose gain is the **sum** of that of the two branches.

E.g.



3) Self-Loop Rule When a node has a self-loop (i.e., loop that starts & ends at the same node), it may be eliminated by multiplying the gains of the branches entering the node by $\frac{1}{1-S}$ where S is the gain of the self-loop.

E.g.



4.5 cont.

Proof $a_2 = S_{32} a_3 + S_b a_2$ (A)

$a_6 = S_{62} a_2$ (B)

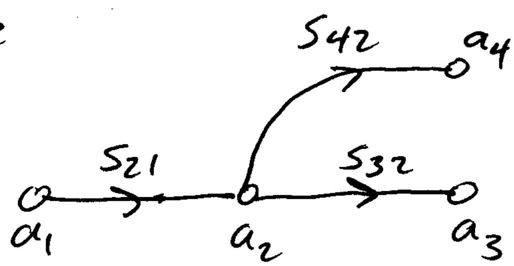
Solving (A) for a_2 yields

$$a_2 (1 - S_b) = S_{32} a_3$$

$$a_2 = \frac{S_{32}}{1 - S_b} a_3$$

Further, $a_6 = S_{62} \left(\frac{S_{32}}{1 - S_b} \right) a_3$

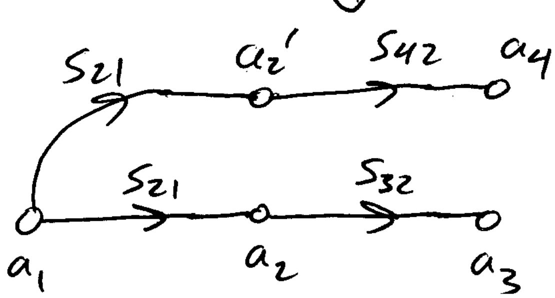
4) Splitting Rule A node may be split into two separate nodes so long as the SFG contains each combination of separate input & output branches that connect to the



$a_3 = S_{32} a_2 = S_{32} S_{21} a_1$

$a_4 = S_{42} a_2 = S_{42} S_{21} a_1$

$\Downarrow$ $\updownarrow$ same

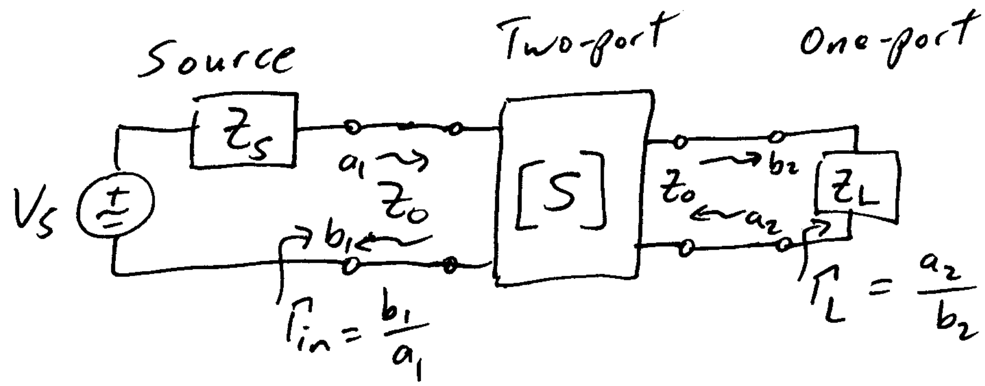


$a_4 = S_{21} S_{42} a_1$

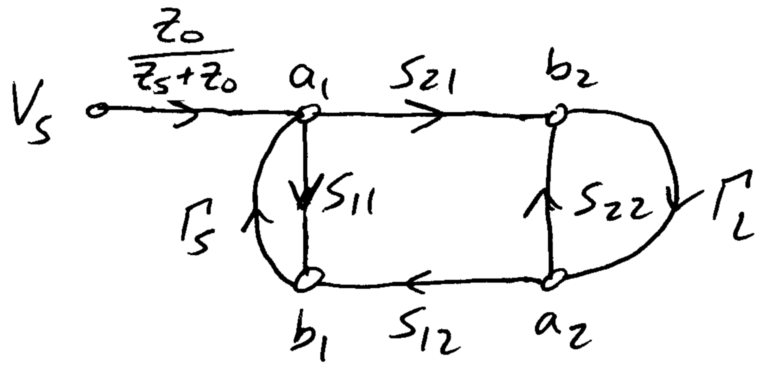
$a_3 = S_{21} S_{32} a_1$

4.5 cont.

example – For the microwave circuit shown, find Γ_{in} and V_L using SFG algebra.



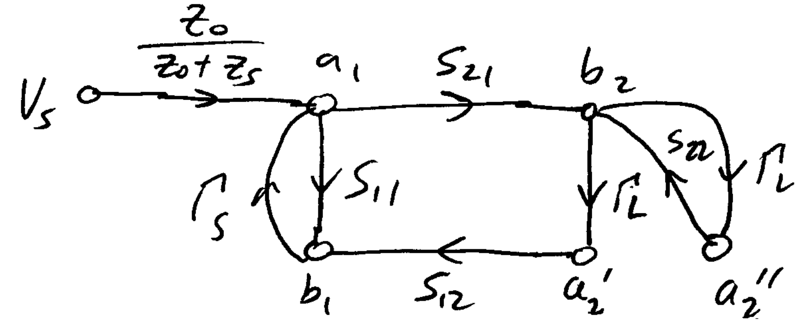
$\Downarrow$ SFG



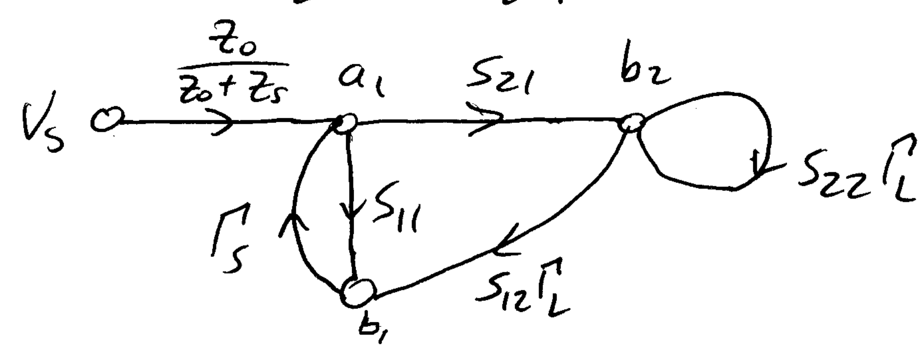
By definition, $\Gamma_{in} \equiv \frac{b_1}{a_1}$. So, we need to reduce the SFG to a point where there is a single branch to b_1 from a_1.

4.5 cont.

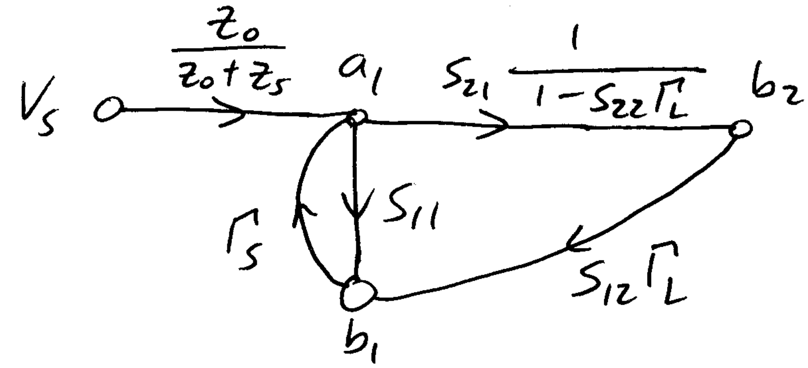
Step 1 Split node a_2 using splitting rule.



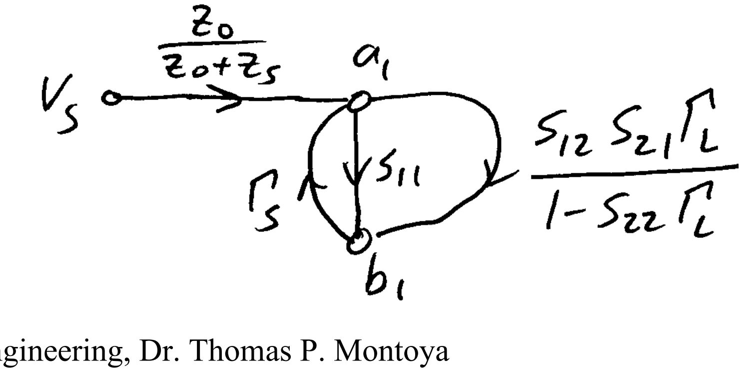
Step 2 Now, we can use the series rule 2x to eliminate a_2' and a_2''.



Step 3 Use self-loop rule.

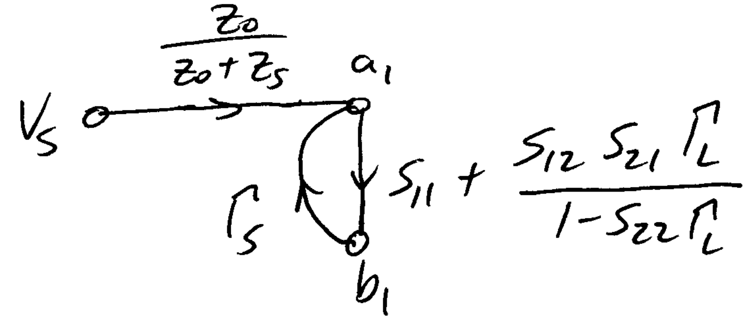


Step 4 Use series rule to eliminate b_2



4.5 cont.

Step 5 Use parallel rule



We can now write $b_1 = \left[S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}\right]a_1$.

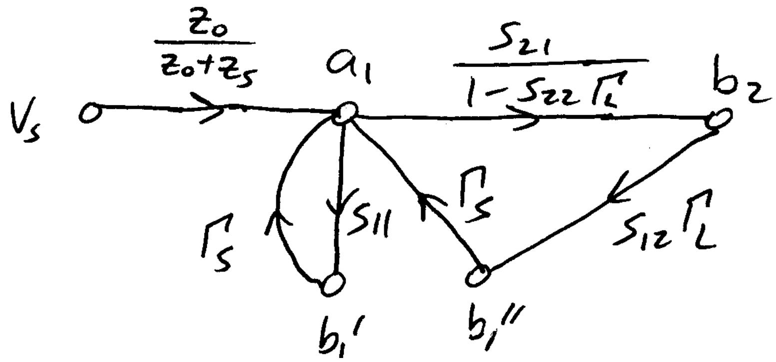
$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$$

Next, we want to find V_L. From TL theory,

$$V_L = a_2 + b_2 = b_2 + b_2\Gamma_L = b_2(1+\Gamma_L).$$

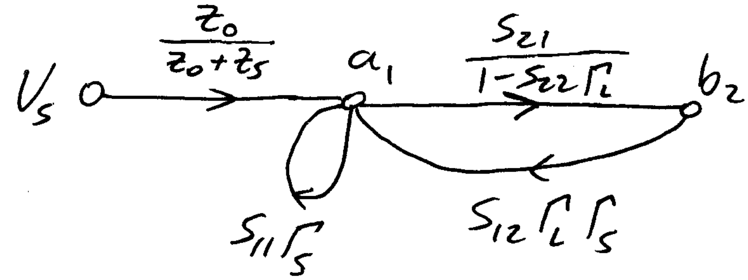
Therefore, we need to use the SFG to find b_2 in terms of V_S. We will start at the point we were at in step 3.

Step 6 Use splitting rule on node b_1.

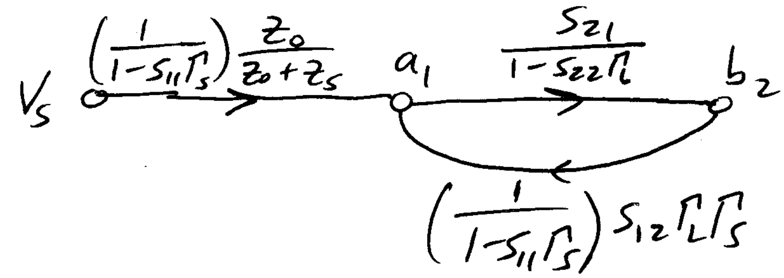


4.5 cont.

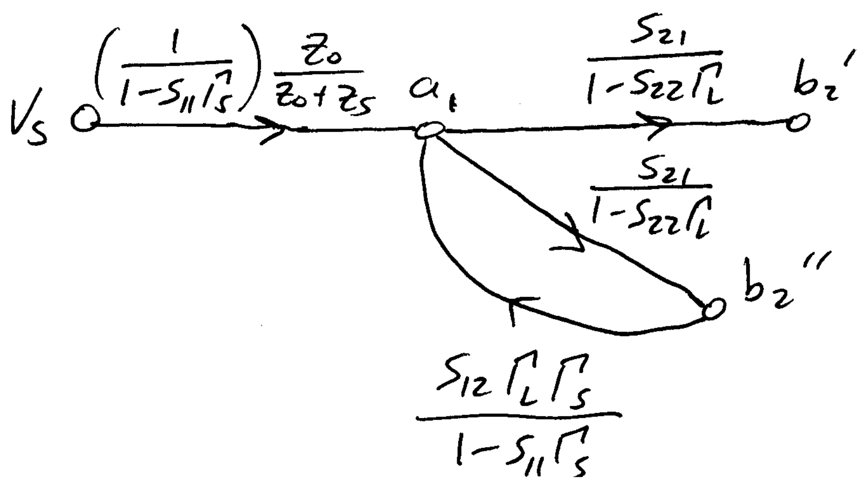
Step 7 Use series rule to eliminate b_1' & b_1''.



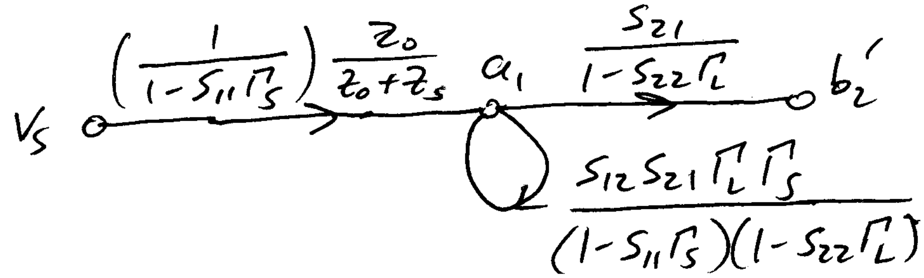
Step 8 Use self-loop rule on node a_1.
Note, there are two incoming branches.



Step 9 Split node b_2



Step 10 Apply series rule to eliminate b_2''



4.5 cont.

Step 11 Apply self-loop rule. Let $b_2' = b_2$

$$V_s \xrightarrow{\left(\dfrac{1}{1-\dfrac{S_{12}S_{21}\Gamma_L\Gamma_S}{(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)}}\right)\left(\dfrac{1}{1-S_{11}\Gamma_S}\right)\dfrac{Z_0}{Z_0+Z_S}} a_1 \xrightarrow{\dfrac{S_{21}}{1-S_{22}\Gamma_L}} b_2$$

Step 12 Apply series rule to get b_2

$$b_2 = \frac{Z_0 S_{21} V_S}{\left[(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L) - S_{12}S_{21}\Gamma_L\Gamma_S\right](Z_0+Z_S)}$$

Now, we can get $V_L = b_2(1+\Gamma_L)$

$$V_L = \frac{Z_0 S_{21}(1+\Gamma_L) V_S}{\left[(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L) - S_{12}S_{21}\Gamma_L\Gamma_S\right](Z_0+Z_S)}$$