Chapter 3 Transmission Lines and Waveguides

Transmission Lines (TLs)

→ Two or more conductors

→ Can support TEM (transverse electromagnetic) waves as well as TE (transverse electric) or TM (transverse magnetic)

→ Upper frequency limit for TLs usually set by lowest frequency where TE & TM waves are possible

→ work down to DC (No cut-off frequency)

Waveguides

→ Single conductor (e.g., pipe) possible

→ Only TE & TM waves are possible; must have a magnetic or electric field in direction of wave propagation

→ Impedance changes w/ frequency

→ Phase velocity changes w/ frequency

Dispersion

→ Have lower cut-off frequencies, i.e., they only work for $f > f_c$

3.1 General Solutions for TEM, TE, & TM Waves

→ We will assume that the TL/waveguide is along the z-direction, i.e., nothing changes as you go back and forth in the z-direction (axial direction)

→ Assume time-harmonic fields with $e^{j\omega t}$ dependence

→ Assume wave propagates in +z-direction, i.e., $e^{-j\beta z}$ term(s)

→ to get −z-direction waves, substitute $-\beta$ for β

→ To incorporate losses, substitute γ for $j\beta$

→ Assume we have no sources w/in TL/waveguide

$$\Downarrow$$

Faraday's Law $\quad \bar{\nabla} \times \bar{E} = -j\omega\mu\bar{H}$

Ampere's Law $\quad \bar{\nabla} \times \bar{H} = j\omega\epsilon\bar{E}$

→ Each of these eqns has <u>three</u> vector components (e.g., x, y, & z)

3.1 cont.

From Faraday's Law, we get

$$\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu H_x \tag{3.3a}$$ x-terms

$$-j\beta E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \tag{3.3b}$$ y-terms

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \tag{3.3c}$$ z-terms

From Ampere's Law, we get

$$\frac{\partial H_z}{\partial y} + j\beta H_y = j\omega\epsilon E_x \tag{3.4a}$$ x-terms

$$-j\beta H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y \tag{3.4b}$$ y-terms

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z \tag{3.4c}$$ z-terms

As it turns out, the tranverse, i.e, x & y components, of the electric and magnetic fields can be determined from E_z & H_z, the axial field components.

For example, solve (3.4b) for E_y:

$$E_y = \frac{1}{j\omega\epsilon}\left(-j\beta H_x - \frac{\partial H_z}{\partial x}\right)$$

3.1 cont.

Next, substitute this expression for E_y into (3.3a):

$$\frac{\partial E_z}{\partial y} + j\beta \frac{1}{j\omega\epsilon}\left(-j\beta H_x - \frac{\partial H_z}{\partial x}\right) = -j\omega\mu H_x$$

$$\frac{\partial E_z}{\partial y} - \frac{\beta}{\omega\epsilon}\frac{\partial H_z}{\partial x} = \left(\frac{-\beta^2}{j\omega\epsilon} - j\omega\mu\right)H_x$$

$\times\,\omega\epsilon$

$$\omega\epsilon\frac{\partial E_z}{\partial y} - \beta\frac{\partial H_z}{\partial x} = \left(+j\beta^2 - j\omega^2\mu\epsilon\right)H_x$$

$$H_x = \frac{1}{j\beta^2 - j\omega^2\mu\epsilon}\left[\omega\epsilon\frac{\partial E_z}{\partial y} - \beta\frac{\partial H_z}{\partial x}\right]$$

$$= \frac{j}{\omega^2\mu\epsilon - \beta^2}\left[\omega\epsilon\frac{\partial E_z}{\partial y} - \beta\frac{\partial H_z}{\partial x}\right]$$

(3.7) Define <u>wave number</u> $\equiv k = \omega\sqrt{\mu\epsilon} = \frac{2\pi}{\lambda}$

(3.6) Define <u>cutoff wave number</u> $\equiv k_c = \sqrt{k^2 - \beta^2}$

Note: To account for dielectric losses, ϵ can be complex $\epsilon_c = \epsilon_0\epsilon_r(1 - j\tan\delta)$
$= \epsilon' - j\epsilon''$

Using $k^2 = \omega^2\mu\epsilon$ and $k_c^2 = k^2 - \beta^2$,

we get:

3.1 cont.

(3.5a) $$H_x = \frac{j}{K_c^2}\left(\omega\epsilon \frac{\partial E_z}{\partial y} - \beta \frac{\partial H_z}{\partial x}\right)$$

Similarly

(3.5b) $$H_y = \frac{-j}{K_c^2}\left(\omega\epsilon \frac{\partial E_z}{\partial x} + \beta \frac{\partial H_z}{\partial y}\right)$$

(3.5c) $$E_x = \frac{-j}{K_c^2}\left(\beta \frac{\partial E_z}{\partial x} + \omega\mu \frac{\partial H_z}{\partial y}\right)$$

(3.5d) $$E_y = \frac{j}{K_c^2}\left(-\beta \frac{\partial E_z}{\partial y} + \omega\mu \frac{\partial H_z}{\partial x}\right)$$

TEM Waves

→ These are waves where $E_z = 0$ and $H_z = 0$, i.e., no field in direction of wave propagation.

⇓

From (3.5 a-d) above, we would get

$H_x = H_y = E_x = E_y$ unless $K_c^2 = K^2 - \beta^2 = 0$

$\Rightarrow K^2 = \beta^2$. In fact, if we go back to

(3.3a) $$\cancel{\frac{\partial E_z}{\partial y}}^{\,0} + j\beta E_y = -j\omega\mu H_x$$

and (3.4b) $$-j\beta H_x - \cancel{\frac{\partial H_z}{\partial x}}^{\,0} = j\omega\epsilon E_y$$

3.1 cont.

Solving (3.3a) for $H_x = \frac{j\beta}{-j\omega\mu} E_y$ and substitute into (3.4b)

$$-j\beta\left(\frac{j\beta}{-j\omega\mu} E_y\right) = j\omega\epsilon E_y$$

$$j\beta^2 E_y = j\omega^2\mu\epsilon E_y$$

$$\Rightarrow \beta^2 = \omega^2\mu\epsilon = k^2 \, !$$

The fact that $k_c^2 = 0$ implies that TEM waves have a cutoff frequency of zero!

$\Rightarrow$ We can get TEM waves when we have at least two separate (lossless) conductors, i.e., transmission lines

$\Rightarrow$ With E_z + H_z both zero, we can find the voltage between the two conductors as

$$V_{12} = \int_1^2 \bar{E} \cdot d\bar{\ell}$$

and the current on a conductor is related to the magnetic field by Ampere's Law $I = \oint_c \bar{H} \cdot d\bar{\ell}$

$\Downarrow$

TL Theory!!

3.1 cont.

$\Rightarrow$ Another possibility for TEM waves are EM plane waves propagating through a lossless medium (often assume two infinite conductive planes separated by infinity as our 'TL')

Define a <u>wave impedance</u> $\equiv Z_{TEM} = \frac{E_x}{H_y} = \frac{-E_y}{H_x}$

which yield (3.5) $\underline{Z_{TEM} = \sqrt{\mu/\epsilon} = \eta}$

Note: $Z_{TEM} \neq Z_0$ (characteristic impedance) which also depends on the TL geometry.

3.1 cont.

TE Waves (AKA: H-waves) Transverse Electric

→ These are waves where $E_z = 0$ but $H_z \neq 0$, i.e., the electric field only has tranverse components. Eqn's (3.5a-d) now become

(3.19a) $H_x = \dfrac{-j\beta}{k_c^2} \dfrac{\partial H_z}{\partial x}$

(3.19b) $H_y = \dfrac{-j\beta}{k_c^2} \dfrac{\partial H_z}{\partial y}$

(3.19c) $E_x = \dfrac{-j\omega\mu}{k_c^2} \dfrac{\partial H_z}{\partial y}$

(3.19d) $E_y = \dfrac{j\omega\mu}{k_c^2} \dfrac{\partial H_z}{\partial x}$

Just need H_z

→ Here $k_c \neq 0$ and $\beta = \sqrt{k^2 - k_c^2}$ is a function of both frequency and geometry of the TL or waveguide.

→ Further, the wave impedance for the TE waves is $Z_{TE} = \dfrac{E_x}{H_y} = \dfrac{-E_y}{H_x} = \dfrac{\omega\mu}{\beta} = \dfrac{k\eta}{\beta}$ (3.22)

(dependent on frequency)

→ Solutions for TE waves depend on finding H_z from Helmholtz wave equation.

→ TE waves are possible for enclosed single conductors (waveguides) as well as multiple conductors.

3.1 cont.

TM Waves (AKA: E-waves) Transverse Magnetic

→ These are waves where $H_z = 0$ but $E_z \neq 0$, i.e., the magnetic field only has transverse components. Equations (3.5a-d) now become

(3.23a) $$H_x = \frac{j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial y}$$

(3.23b) $$H_y = \frac{-j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial x}$$

(3.23c) $$E_x = \frac{-j\beta}{k_c^2}\frac{\partial E_z}{\partial x}$$

(3.23d) $$E_y = \frac{-j\beta}{k_c^2}\frac{\partial E_z}{\partial y}$$

Just need E_z

→ Again $k_c \neq 0$ and $\beta = \sqrt{k^2 - k_c^2}$ is a function of both frequency + geometry of the TL or waveguide.

→ The wave impedance for the TM waves is
$$Z_{TM} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\beta}{\omega\epsilon} = \frac{\beta\eta}{k} \quad (3.26)$$
(dependent on frequency)

→ Solutions for TM waves depend on finding E_z from Helmholtz wave equation.

→ TM waves are possible for enclosed single conductors (waveguides) as well as between multiple conductors.

3.1 cont.

Attenuation Due to Dielectric Loss

Total attenuation constant $\equiv \alpha = \alpha_d + \alpha_c$

α_d: dielectric losses; α_c: conductor losses

→ The conductor losses are TL or waveguide specific as they depend on how the fields are distributed, i.e., more losses where surface currents are high.

→ The dielectric losses can be found from the propagation constant when the waveguide or TL has a homogeneous dielectric 'filling'.

propagation constant $\equiv \gamma = \alpha_d + j\beta = \sqrt{k_c^2 - k^2}$

$$= \sqrt{k_c^2 - \omega^2 \mu \epsilon} \qquad \text{assume } \mu = \mu_0,\ \epsilon = \epsilon_c$$

$$= \sqrt{k_c^2 - \omega^2 \mu_0 \epsilon_0 \epsilon_r (1 - j\tan\delta)}$$

Define a real wave number $k = \omega\sqrt{\mu_0 \epsilon_0 \epsilon_r}$

$$k^2 = \omega^2 \mu_0 \epsilon_0 \epsilon_r$$

$$\gamma = \sqrt{k_c^2 - k^2 + jk^2 \tan\delta}$$

Assuming $\tan\delta \ll 1$ (true for practical dielectrics), we can use approximation $\sqrt{a^2 + x^2} \simeq a + \frac{1}{2}\left(\frac{x^2}{a}\right)$ $x \ll 1$ with $a^2 = k_c^2 - k^2$ to get

3.1 cont.

$$\gamma \simeq \sqrt{k_c^2 - k^2} + j\frac{k^2 \tan\delta}{2\sqrt{k_c^2 - k^2}}$$

Now, use (3.6) $k_c^2 = k^2 - \beta^2 \Rightarrow -\beta^2 = k_c^2 - k^2$

$\hookrightarrow j\beta = \sqrt{k_c^2 - k^2}$

to arrive at

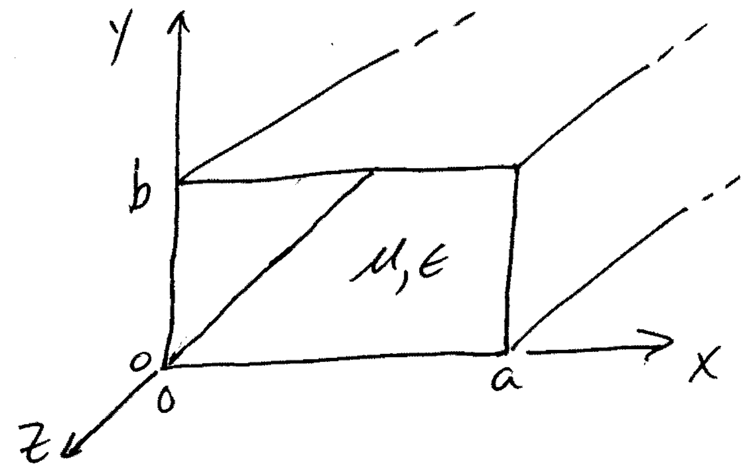
$$\gamma \simeq \underbrace{\frac{k^2 \tan\delta}{2\beta}}_{\alpha_d} + j\beta \quad (3.28)$$

$\alpha_d = \dfrac{k^2 \tan\delta}{2\beta}$ (Np/m) (3.29) for TE/TM waves

For TEM waves, where $k_c = 0 \Rightarrow k = \beta$,

we get $\alpha_d = \dfrac{k \tan\delta}{2}$ (Np/m) (3.30)

3.3 Rectangular Waveguides



$a > b$

TE Modes $\rightarrow E_z = 0$, need H_z

Start w/ Helmholtz wave equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\right) H_z = 0$$

For z-direction wave propagation, assume

$$H_z = h_z e^{-j\beta z} \quad \text{where } h_z(x,y)$$

which, when put into the wave eq'n yields

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \beta^2 + k^2\right) h_z = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) h_z = 0$$

Using the method of separation of variables, let $h_z(x,y) = X(x)\,Y(y)$

3.3 cont.

This leads to

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + k_c^2 = 0$$

which holds true if each of the first two terms is equal to a constant, i.e.,

$-k_x^2$ + $-k_y^2$, $-k_x^2 - k_y^2 + k_c^2 = 0$

$$\text{or} \quad k_c^2 = k_x^2 + k_y^2 \qquad (3.77)$$

and

$$(3.76a) \quad \frac{d^2X}{dx^2} + k_x^2 X = 0$$

$$(3.76b) \quad \frac{d^2Y}{dy^2} + k_y^2 Y = 0$$

Eq'ns (3.76a) + (3.76b) are recognizable as one-dimensional wave eq'ns w/ solutions

$$X(x) = A e^{-jk_x x} + B e^{+jk_x x} = A\cos(k_x x) + B\sin(k_x x)$$

$$Y(y) = C e^{-jk_y y} + D e^{jk_y y} = C\cos(k_y y) + D\sin(k_y y)$$

Therefore, the general solution for $H_z(x,y,z)$ is

$$H_z(x,y,z) = h_z(x,y)e^{-j\beta z} = X(x)Y(y)e^{-j\beta z}$$

$$H_z(x,y,z) = \left[A\cos(k_x x) + B\sin(k_x x)\right]\left[C\cos(k_y y) + D\sin(k_y y)\right]e^{-j\beta z}$$

3.3 cont.

From (3.19c), $E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y}$

$$E_x = \frac{-j\omega\mu}{k_c^2}\left[A\cos(k_x x) + B\sin(k_x x)\right]\left[-k_y C\sin(k_y y) + k_y D\cos(k_y y)\right]e^{-j\beta z}$$

and, from (3.19d), $E_y = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x}$

$$E_y = \frac{j\omega\mu}{k_c^2}\left[-k_x A\sin(k_x x) + k_x B\cos(k_x x)\right]\left[C\cos(k_y y) + D\sin(k_y y)\right]e^{-j\beta z}$$

Now, we can use the boundary condition that $E_{tan} = 0$ at the walls of the rectangular waveguide (conductors)

First, $E_x = 0$ @ $y = 0$:

$$0 = \frac{-j\omega\mu}{k_c^2}\left[A\cos(k_x x) + B\sin(k_x x)\right]\left[-k_y C\overset{0}{\sin(0)} + k_y D\overset{1}{\cos(0)}\right]e^{-j\beta z}$$

can only hold true if $\underline{D = 0}$. This reduces E_x to $E_x = \frac{+j\omega\mu k_y}{k_c^2}\left[A\cos(k_x x) + B\sin(k_x x)\right]C\sin(k_y y)e^{-j\beta z}$

Next, $E_x = 0$ @ $y = b$:

$$0 = \frac{j\omega\mu k_y}{k_c^2}\left[A\cos(k_x x) + B\sin(k_x x)\right]C\sin(k_y b)e^{-j\beta z}$$

can only hold true if $\sin(k_y b) = 0$. This in turn implies $k_y b = n\pi \quad n = 0, 1, 2, \ldots$

or $\underline{k_y = \frac{n\pi}{b} \quad n = 0, 1, 2, \ldots}$

3.3 cont

Moving on to E_y, $E_y = 0$ @ $x = 0$ means

$$E_y = 0 = \frac{j\omega\mu}{k_c^2}\left[-k_x A \sin(0)^{0} + k_x B \cos(0)^{1}\right] C \cos\left(\frac{n\pi}{b} y\right) e^{-j\beta z}$$

can only hold true if $\underline{B = 0}$. This reduces E_y to

$$E_y = \frac{j\omega\mu}{k_c^2}(-k_x) A \sin(k_x x) C \cos\left(\frac{n\pi}{b} y\right) e^{-j\beta z}$$

Applying the condition that $E_y = 0$ @ $x = a$

$$E_y = 0 = \frac{-j\omega\mu k_x}{k_c^2} A \sin(k_x a) C \cos\left(\frac{n\pi}{b} y\right) e^{-j\beta z}$$

Can only hold true if $\sin(k_x a) = 0$. This implies that $k_x a = m\pi \quad m = 0, 1, 2, \ldots$ or

$$\underline{k_x = \frac{m\pi}{a} \quad m = 0, 1, 2, \ldots}$$

Putting these into our TE field component equations yields:

$$H_z = \underbrace{AC}_{A_{mn}} \cos\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-j\beta z}$$

(3.81) $$H_z = A_{mn} \cos\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-j\beta z}$$

(3.82a) $$E_x = \frac{+j\omega\mu n\pi}{k_c^2 b} A_{mn} \cos\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-j\beta z}$$

(3.82b) $$E_y = \frac{-j\omega\mu m\pi}{k_c^2 a} A_{mn} \sin\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-j\beta z}$$

3.3 cont.

Using (3.19a) + (3.19b) w/ H_z in (3.81) gives

(3.82c) $$H_x = \frac{j\beta m\pi}{k_c^2 a} A_{mn} \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta z}$$

(3.82d) $$H_y = \frac{j\beta n\pi}{k_c^2 b} A_{mn} \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta z}$$

where

$$k_c^2 = k_{c,mn}^2 = k_x^2 + k_y^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$m = 0, 1, 2, \ldots$
$n = 0, 1, 2, \ldots$
$\underline{m = n \neq 0}$

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

For a propagating mode, β is a <u>real</u> #.

This implies $k > k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$.

By definition, $k = \omega\sqrt{\mu\epsilon} = 2\pi f\sqrt{\mu\epsilon}$. We will say the frequency where $k = k_c$ is the <u>cutoff frequency</u> $f_{c_{mn}}$ for the particular TE_{mn} mode as any frequency below $f_{c_{mn}}$ will cause β to be imaginary $(-j\alpha)$ and the $e^{-j\beta z}$ term to be attenuative $(e^{-\alpha z})$!

(3.84) $$f_{c_{mn}} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$m = 0, 1, 2, \ldots;\ n = 0, 1, 2, \ldots;\ m = n \neq 0$

3.3 cont.

Along with this we get

$$\beta = \beta_{mn} = \sqrt{k^2 - k_{c,mn}^2}$$ $m = 0, 1, 2, \ldots$, $n = 0, 1, 2, \ldots$, $m = n \neq 0$

Why can't $m = n = 0$?

$\Rightarrow$ That would make $k_{c,00}^2 = 0$ and implies that $E_x = E_y = H_x = H_y \to \infty$!

(unless $A_{00} = 0$ where all fields are zero).

What is the lowest cutoff frequency and lowest mode?

Since $a > b$, the lowest cutoff frequency & mode is TE_{10} (i.e., $m=1, n=0$) where

$$f_{c,10} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{1\pi}{a}\right)^2 + 0} = \frac{1}{2a\sqrt{\mu\epsilon}} \quad (3.85)$$

w/ (3.91) $\beta_{10} = \sqrt{\omega^2\mu\epsilon - \left(\frac{\pi}{a}\right)^2}$ & $k_{c,10} = \pi/a$ (3.90)

guided wavelength: $\lambda_{g,10} = \dfrac{2\pi}{\beta_{10}} > \lambda = \dfrac{2\pi}{k}$ (free wavelength)

& phase velocity: $v_{p,10} = \dfrac{\omega}{\beta_{10}} > v_p = \dfrac{1}{\sqrt{\mu\epsilon}}$ (free phase velocity)

3.3 cont.

Given that the TE_{10} mode has the lowest cutoff frequency, most rectangular waveguides are operated/used only over the range of frequencies where it alone exists. Thus, the particular fields are of interest

(3.89a) $H_z = A_{10} \cos\left(\frac{\pi x}{a}\right) e^{-j\beta z}$

(3.89b) $E_y = \frac{-j\omega\mu a}{\pi} A_{10} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}$

(3.89c) $H_x = \frac{j\beta a}{\pi} A_{10} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}$

(3.89d) $E_x = E_z = H_y = 0$

Power flow for the TE_{10} mode is

$$P_{10} = \frac{1}{2} \mathrm{Re} \int_{x=0}^{a} \int_{y=0}^{b} (\bar{E} \times \bar{H}^*) \cdot \hat{z} \, dy \, dx$$

$$= \frac{\omega\mu a^3 |A_{10}|^2 b}{4\pi^2} \mathrm{Re}\{\beta_{10}\} \qquad (3.92)$$

Hence, if β_{10} is NOT real $\Rightarrow$ No power flow!

3.3 cont.

TM Modes $\rightarrow H_z = 0$, need E_z

Start w/ Helmholtz equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\right)E_z = 0$$

where $E_z = e_z e^{-j\beta z}$. Following a similar solution process leads to

$$E_z = \left[A\cos(k_x x) + B\sin(k_x x)\right]\left[C\cos(k_y y) + D\sin(k_y y)\right]e^{-j\beta z}$$

Apply the $E_{tan} = 0$ boundary conditions that $E_z = 0$ @ $x = 0$ & $x = a$
& $y = 0$ & $y = b$

leads to $A = C = 0$ and $k_x = \frac{m\pi}{a}$ $m = 1, 2, \ldots$
$k_y = \frac{n\pi}{b}$ $n = 1, 2, \ldots$

and

$$E_z = B_{mn}\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta z} \quad (3.100)$$

Note the neither m <u>or</u> n can be zero, else $E_z = 0$ and we have all the TM fields equal to zero (Null solution)

3.3 cont.

Using (3.23a-d), we get the remaining (transverse) TM field components to be

(3.101a) $$E_x = \frac{-j\beta m\pi}{a k_c^2} B_{mn} \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta z}$$

(3.101b) $$E_y = \frac{-j\beta n\pi}{b k_c^2} B_{mn} \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta z}$$

(3.101c) $$H_x = \frac{j\omega\epsilon n\pi}{b k_c^2} B_{mn} \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta z}$$

(3.101d) $$H_z = \frac{-j\omega\epsilon m\pi}{a k_c^2} B_{mn} \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta z}$$

w/ $$\beta_{mn} = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$$k_{c,mn}^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$f_{c,mn} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

where $m = 1, 2, 3, \ldots$ and $n = 1, 2, 3, \ldots$

The lowest TM mode is TM_{11} where

$$f_{c,11} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{(\pi/a)^2 + (\pi/b)^2}$$

TABLE 3.2 Summary of Results for Rectangular Waveguide

Quantity	TE_{mn} Mode	TM_{mn} Mode
k	$\omega\sqrt{\mu\epsilon}$	$\omega\sqrt{\mu\epsilon}$
k_c	$\sqrt{(m\pi/a)^2+(n\pi/b)^2}$	$\sqrt{(m\pi/a)^2+(n\pi/b)^2}$
β	$\sqrt{k^2-k_c^2}$	$\sqrt{k^2-k_c^2}$
λ_c	$\frac{2\pi}{k_c}$	$\frac{2\pi}{k_c}$
λ_g	$\frac{2\pi}{\beta}$	$\frac{2\pi}{\beta}$
v_p	$\frac{\omega}{\beta}$	$\frac{\omega}{\beta}$
α_d	$\frac{k^2\tan\delta}{2\beta}$	$\frac{k^2\tan\delta}{2\beta}$
E_z	0	$B\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}$
H_z	$A\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$	0
E_x	$\frac{j\omega\mu n\pi}{k_c^2 b}A\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}$	$\frac{-j\beta m\pi}{k_c^2 a}B\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}$
E_y	$\frac{-j\omega\mu m\pi}{k_c^2 a}A\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$	$\frac{-j\beta n\pi}{k_c^2 b}B\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$
H_x	$\frac{j\beta m\pi}{k_c^2 a}A\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$	$\frac{j\omega\epsilon n\pi}{k_c^2 b}B\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$
H_y	$\frac{j\beta n\pi}{k_c^2 b}A\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}$	$\frac{-j\omega\epsilon m\pi}{k_c^2 a}B\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}$
Z	$Z_{TE}=\frac{k\eta}{\beta}$	$Z_{TM}=\frac{\beta\eta}{k}$

Microwave Engineering (Fourth Edition), Pozar, Wiley, 2012, ISBN 978-0-470-63155-3.

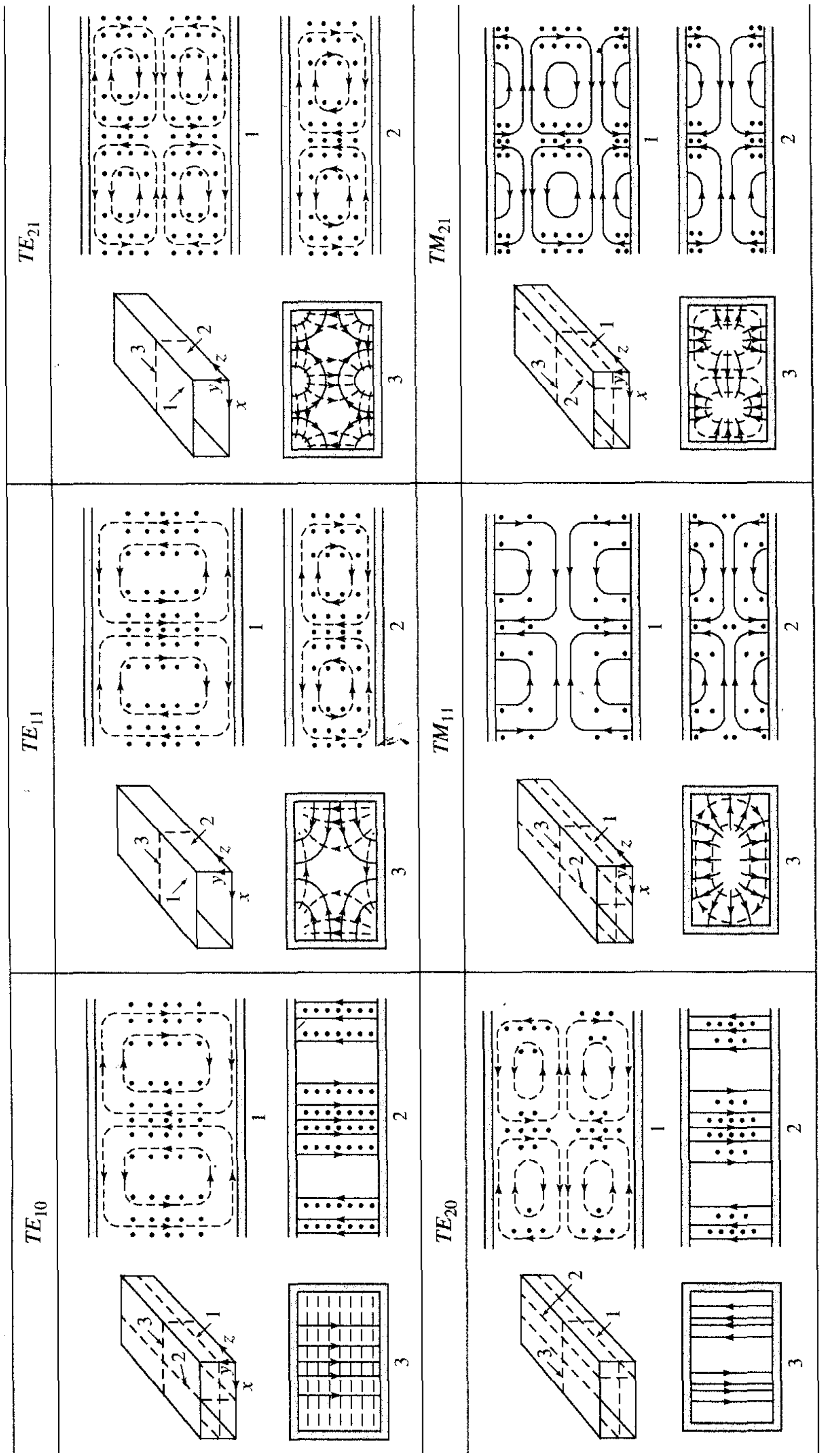


FIGURE 3.9 Field lines for some of the lower order modes of a rectangular waveguide.

Reprinted with permission from S. Ramo, J. R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics*. Copyright © 1965 by John Wiley & Sons, Inc. Table 8.02.

3.3 cont.

ex. Let's consider the X-band rectangular waveguide WR90 where $a = 0.9'' = 2.286$ cm and $b = 0.4'' = 1.016$ cm and having an air dielectric (ϵ_0, μ_0)

TE $$f_{c,mn} = \frac{1}{2\pi\sqrt{\mu_0\epsilon_0}}\sqrt{\left(\frac{m\pi}{0.02286}\right)^2 + \left(\frac{n\pi}{0.01016}\right)^2}$$

$m = n \neq 0$ $\quad$ $m = 0, 1, \ldots$ $\quad$ $n = 0, 1, \ldots$

TM $$f_{c,mn} = \frac{1}{2\pi\sqrt{\mu_0\epsilon_0}}\sqrt{\left(\frac{m\pi}{0.02286}\right)^2 + \left(\frac{n\pi}{0.01016}\right)^2}$$

$m = 1, 2, \ldots$

$n = 1, 2, \ldots$

TE Modes

m	n	$f_{c,mn}$ (GHz)
1	0	6.557
2	0	13.114
0	1	14.754
1	1	16.145

TM Mode

m	n	$f_{c,mn}$ (GHz)
1	1	16.145
2	1	19.740
1	2	30.227
2	2	32.290

From 6.557 to 13.114 GHz, <u>only</u> the TE_{10} mode will propagate in WR90. X-band is normally defined as 8 to 12 GHz.

3.3 cont.

ex. cont. Let's further consider a few quantities @ $f = 9$ & 10 GHz.

$f = 9$ GHz

$K = 2\pi f \sqrt{\mu_0 \epsilon_0} = 188.626 \ \text{rad/m}$

$\beta_{10} = \sqrt{K^2 - K_c^2} = \sqrt{188.626^2 - \left(\frac{\pi}{0.02286}\right)^2} = \underline{129.203 \ \frac{\text{rad}}{\text{m}}}$

$\lambda_{g,10} = \frac{2\pi}{\beta_{10}} = \frac{2\pi}{129.203} = \underline{4.863 \text{ cm}}$

$v_{p,10} = \frac{\omega}{\beta_{10}} = \frac{2\pi(9\times10^9)}{129.203} = \underline{4.3767 \times 10^8 \ \text{m/s}}$

$Z_{TE,10} = \frac{K\eta}{\beta} = \frac{188.626\sqrt{\mu_0/\epsilon_0}}{129.203} = \frac{188.626(376.7303)}{129.203}$

$= \underline{550 \Omega}$

$f = 10$ GHz

$K = 2\pi f \sqrt{\mu_0 \epsilon_0} = \underline{209.5845 \ \frac{\text{rad}}{\text{m}}}$

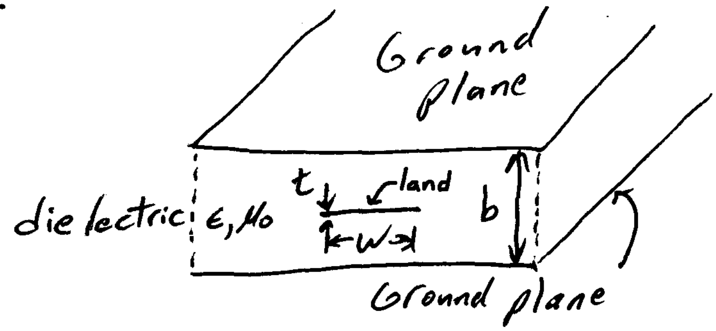
$\beta_{10} = \sqrt{K^2 - K_c^2} = \underline{158.238 \ \frac{\text{rad}}{\text{m}}}$

$\lambda_{g,10} = \frac{2\pi}{158.238} = \underline{3.971 \text{ cm}}$

$v_{p,10} = \frac{2\pi(10\times10^9)}{158.238} = \underline{3.9707 \times 10^8 \ \text{m/s}}$ (Dispersion!)

$Z_{TE,10} = \frac{209.6(376.7)}{158.24} = \underline{498.97 \Omega}$

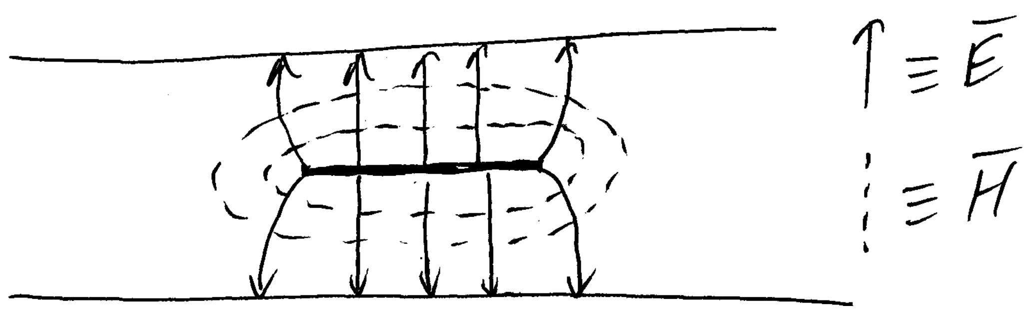
3.7 Stripline



* As shown, we have a thin (thickness t) conducting strip (land) centered between two ground planes (theoretically infinite) separated by a dielectric substrate (ϵ, μ_0) of thickness b.
* Stripline can support TEM waves as well as TE + TM waves (avoid).
* To ensure only TEM waves, the rule is to keep $b \lesssim \lambda/4$ where λ is the 'free' wavelength in the dielectric,

$$\lambda = \lambda_0/\sqrt{\epsilon_r} = \frac{v_p}{f} = \frac{1}{f\sqrt{\epsilon_r \epsilon_0 \mu_0}} = \frac{c}{f\sqrt{\epsilon_r}}.$$

Typical TEM field lines



3.7 cont.

In the TEM mode, per section 3.1,

(3.176) $$v_p = \frac{1}{\sqrt{\mu_0 \epsilon_r \epsilon_0}} = \frac{c}{\sqrt{\epsilon_r}} \quad (m/s)$$

(3.177) $$\beta = \frac{\omega}{v_p} = \omega\sqrt{\mu_0 \epsilon_r \epsilon_0} = \omega\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon_r} \quad \left(\frac{rad}{m}\right)$$
$$= k_0 \sqrt{\epsilon_r}$$

From TL theory, $v_p = \frac{1}{\sqrt{LC}}$ for lossless TLs.

Thus,

(3.178) $$Z_0 = \sqrt{L/C} = \frac{\sqrt{LC}}{C} = \frac{1/v_p}{C} = \frac{1}{v_p C}$$

Since we can get v_p from (3.176) if we know ϵ_r, the challenge is to find the per-unit-length capacitance C.

⇒ Not easy to get an analytical solution, can come close w/ complex conformal mapping.

⇒ Can get very accurate numerical solutions from a variety of methods (e.g., FE, MoM, FDTD)

3.7 cont.

For quick practical designs of stripline, curve fitting has been used to get formulas that work w/ ~1% accuracy.

$$(3.179a) \quad Z_0 = \frac{30\pi}{\sqrt{\epsilon_r}} \frac{b}{W_e + 0.441b} = \frac{30\pi}{\sqrt{\epsilon_r}} \frac{1}{\frac{W_e}{b} + 0.441}$$

where the effective strip width W_e is found using

$$(3.179b) \quad \frac{W_e}{b} = \frac{W}{b} - \begin{cases} 0 & W/b > 0.35 \\ (0.35 - W/b)^2 & W/b < 0.35 \end{cases}$$

The equations (3.179a+b) neglect the thickness t of the strip, i.e., assume $t \simeq 0$.

From (3.179a+b), it is apparent that as $\frac{W_e}{b}$ and W/b get larger, the characteristic impedance Z_0 decreases. This makes sense as a wider strip will have more capacitance per unit length.

3.7 cont.

ex. Find Z_o and W_e for a 2mm wide strip in a 6mm thick Teflon ($\epsilon_r = 2.1$) dielectric between two ground planes.

$W = 2mm$ and $b = 6mm$

$W/b = \frac{2}{6} = 0.\overline{3} < 0.35$

Per (3.179b), $\frac{W_e}{b} = 0.\overline{3} - (0.35 - 0.\overline{3})^2$
$= 0.33305\overline{5}$

$W_e = 0.33305\overline{5}(6mm) = \underline{\underline{1.9983\overline{3}\ mm}}$

Per (3.179a), $Z_o = \frac{30\pi}{\sqrt{\epsilon_r}} \frac{b}{W_e + 0.441b}$

$= \frac{30\pi}{\sqrt{2.1}} \frac{6}{1.9983\overline{3} + 0.441(6)}$

$$\underline{\underline{Z_o = 84.02\ \Omega}}$$

Bonus: $v_p = c/\sqrt{\epsilon_r} = \underline{2.069 \times 10^8\ m/s}$

we want $b \leq \lambda/4 = \frac{v_p}{4f}$

$f_{\substack{TEM\\ONLY}} < f_{limit} = \frac{v_p}{4b} = \frac{2.069 \times 10^8}{4(6 \times 10^{-3})} = \underline{\underline{8.62\ GHz}}$

3.7 cont.

What if we need to design a stripline for a particular Z_0 given a substrate (i.e., ϵ_r & b)? Inverting (3.179) leads to

(3.180a) $$\frac{W}{b} = \begin{cases} x & \sqrt{\epsilon_r}\,Z_0 < 120\Omega \\ 0.85 - \sqrt{0.6 - x} & \sqrt{\epsilon_r}\,Z_0 > 120\Omega \end{cases}$$

where (3.180b) $$x = \frac{30\pi}{\sqrt{\epsilon_r}\,Z_0} - 0.441$$

What about attenuation for real world conductors & dielectrics?

Dielectric attenuation constant $\alpha_d = \dfrac{k \tan\delta}{2}$ (3.30)
(TEM mode)

where $k = \omega\sqrt{\mu_0 \epsilon_r \epsilon_0}$

Conductor attenuation constant α_c is trickier.

(3.181) $$\alpha_c = \begin{cases} \dfrac{2.7\times10^{-3} R_s \epsilon_r Z_0}{30\pi(b-t)} A & \sqrt{\epsilon_r}\,Z_0 < 120\Omega \\ \dfrac{0.16 R_s}{Z_0 b} B & \sqrt{\epsilon_r}\,Z_0 > 120\Omega \end{cases}$$

where $$A = 1 + \frac{2W}{b-t} + \frac{1}{\pi}\frac{b+t}{b-t}\ln\left(\frac{2b-t}{t}\right)$$

$$B = 1 + \frac{b}{(0.5W + 0.7t)}\left[0.5 + \frac{0.414t}{W} + \frac{1}{2\pi}\ln\left(\frac{4\pi W}{t}\right)\right]$$

(1.125) $$R_s = \frac{1}{\sigma\delta_s} = \sqrt{\frac{\omega\mu}{2\sigma}}$$

EE 481/581 Stripline Example

We wish to build a 77 Ω stripline using a 0.5 oz. copper foil land ($5.8*10^7$ S/m, 17 μm thickness) sandwiched between two FR-4 (ε_r = 4.06, tan δ = 0.021 at 2.5 GHz, 1.6 mm thick/each) PCBs with copper ground plane backs for use at 2.5 GHz.

$f := 2.5\cdot 10^{9}$ Hz $\omega := 2\cdot\pi\cdot f$ $c := 2.99792458\cdot 10^{8}$ m/s

$\varepsilon 0 := 8.8541878\cdot 10^{-12}$ F/m $\mu 0 := 4\cdot\pi\cdot 10^{-7}$ H/m

$\varepsilon r := 4.06$ $loss\_tan := 0.021$ $\sigma\_cu := 5.8\cdot 10^{7}$ S/m

$t := 17\cdot 10^{-6}$ m $b := 2\cdot 1.6\cdot 10^{-3}$ m $Z0 := 77$ Ω

Calculate/define stripline parameters

(3.176) $vp := \dfrac{c}{\sqrt{\varepsilon r}}$ $\boxed{vp = 1.48784\times 10^{8}}$ m/s

(3.177) $\beta := \dfrac{\omega}{vp}$ $\boxed{\beta = 105.57527}$ rad/m, phase constant

$k := \omega\cdot\sqrt{\mu 0\cdot\varepsilon r\cdot\varepsilon 0}$ $k = 105.57527$ rad/m, same as β

$\lambda g := \dfrac{vp}{f}$ $\boxed{\lambda g = 0.059514}$ m, guided wavelength

Find $\sqrt{\varepsilon r}\cdot Z0 = 155.1507$ Note that this is greater than 120 Ω

(3.180b) $x := \dfrac{30\cdot\pi}{\sqrt{\varepsilon r}\cdot Z0} - 0.441$ $x = 0.16646$

(3.180a) $W := \left(0.85 - \sqrt{0.6 - x}\right)\cdot b$ $\boxed{W = 6.13\times 10^{-4}}$ m

(3.30) $\alpha d := \dfrac{k\cdot loss\_tan}{2}$ $\boxed{\alpha d = 1.10854}$ Np/m, not good

(1.125) $Rs\_cu := \sqrt{\dfrac{\omega\cdot\mu 0}{2\cdot\sigma\_cu}}$ $\boxed{Rs\_cu = 0.01304}$ Ω, surface resistance

$$B := 1 + \frac{b}{(0.5\cdot W + 0.7\cdot t)}\cdot\left(0.5 + \frac{0.414\cdot t}{W} + \frac{1}{2\cdot\pi}\cdot\ln\left(\frac{4\cdot\pi\cdot W}{t}\right)\right) \qquad B = 15.92365$$

(3.181) $\alpha c := 0.16\cdot\dfrac{Rs\_cu}{Z0\cdot b}\cdot B$ $\boxed{\alpha c = 0.13488}$ Np/m

$\alpha := \alpha d + \alpha c$ $\boxed{\alpha = 1.24342}$ Np/m

$\alpha dB := \alpha\cdot(20\cdot\log(e))$ $\boxed{\alpha dB = 10.8}$ dB/m, ouch

$\gamma := \alpha + j\cdot\beta$ $\boxed{\gamma = 1.24342 + 105.57527i}$ 1/m, prop. constant

3.8 Microstrip Line

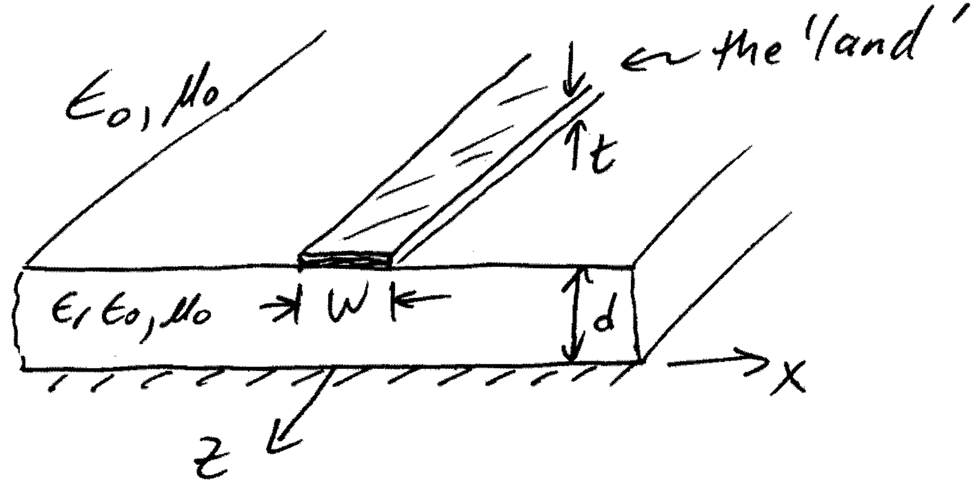
→ Very popular TL widely used in planar microwave circuits for interconnections

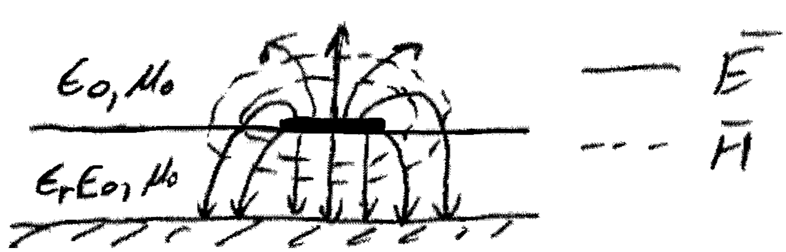
→ Many advantages

* Robust
* Easy to fabricate (milling, printing, ...)
* Easy to integrate w/ surface mount devices
* Inexpensive

→ Limitations

* quasi-TEM or hybrid TE-TM waves
* lossier than other TLs
* low power
* Some dispersion
* some EMI/EMC concerns





3.8 cont.

→ As is obvious, the EM waves associated with microstrip are in two differing dielectric materials, i.e., the substrate ($\epsilon_r \epsilon_0$) and air (ϵ_0).

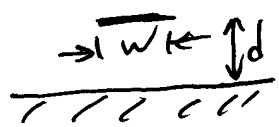
→ This obviously affects the electric fields as, for $d < \lambda$, the majority will be in the dielectric, but a non-neglible fraction will be in air ($d < y$)

→ For example, in the air, the phase velocity of an EM wave is $\sim c$. However, in the dielectric, an EM wave would have $v_p = c/\sqrt{\epsilon_r}$. What happens at interface $y = d$?

↳ leads to a quasi-TEM wave rather than a true TEM wave

Fortunately, at 'low' frequencies where $d \ll \lambda$, the quasi-TEM mode is <u>very</u> close to TEM in its behavior. At these frequencies, we will define an <u>effective</u> permittivity $\epsilon_e = \epsilon_{r,e} \cdot \epsilon_0$ where

$\underset{\text{air}}{\epsilon_0} < \epsilon_e < \underset{\substack{\text{substrate}\\\text{dielectric}}}{\epsilon}$ ⇒ $\epsilon_e = \epsilon_{r,e}\epsilon_0$



3.8 cont.

Based on extensive numerical simulations and experimentation, we have some curve-fitted equations that allow for quick analysis and design of microstrip.

(3.193) $$v_p = \frac{c}{\sqrt{\epsilon_{r,e}}}$$

(3.194) $$\beta = \omega\sqrt{\mu_0 \epsilon_{r,e} \epsilon_0} = \omega\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon_{r,e}} = k_0\sqrt{\epsilon_{r,e}} = \frac{\omega}{v_p}$$

(3.195) $$\epsilon_{r,e} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\frac{1}{\sqrt{1 + 12\,d/W}}$$

(3.196) $$Z_0 \cong \begin{cases} \dfrac{60}{\sqrt{\epsilon_{r,e}}}\ln\left(\dfrac{8d}{W} + \dfrac{W}{4d}\right) & W/d \le 1 \\[2ex] \dfrac{120\pi}{\sqrt{\epsilon_{r,e}}\left[\dfrac{W}{d} + 1.393 + 0.667\ln\left(\dfrac{W}{d} + 1.444\right)\right]} & \dfrac{W}{d} \ge 1 \end{cases}$$

(Ω)

For design of a microstrip w/ a given Z_0 on a substrate w/ ϵ_r (usually will have value(s) for d as well)

(3.197) $$\frac{W}{d} = \begin{cases} \dfrac{8e^A}{e^{2A} - 2} & \dfrac{W}{d} < 2 \\[2ex] \dfrac{2}{\pi}\left[B - 1 - \ln(2B - 1) + \dfrac{\epsilon_r - 1}{2\epsilon_r}\left\{\ln(B - 1) + 0.39 - \dfrac{0.61}{\epsilon_r}\right\}\right] & \dfrac{W}{d} > 2 \end{cases}$$

where the constants A & B are given by

3.8 cont.

$$A = \frac{Z_0}{60}\sqrt{\frac{\epsilon_r+1}{2}} + \frac{\epsilon_r-1}{\epsilon_r+1}\left(0.23 + \frac{0.11}{\epsilon_r}\right)$$

$$B = \frac{377\pi}{2Z_0\sqrt{\epsilon_r}}$$

What about dielectric and conductor losses?

→ Dielectric losses are complicated by the $\bar{E}$ fields being partly in air (~lossless) and in dielectric (lossier). A filling factor $\frac{\epsilon_r(\epsilon_{r,e}-1)}{\epsilon_{r,e}(\epsilon_r-1)}$ is used to modify eq'n (3.30) for TEM waves to get

$$(3.198)\quad \alpha_d = \frac{k_0\epsilon_r(\epsilon_{r,e}-1)\tan\delta}{2\sqrt{\epsilon_{r,e}}(\epsilon_r-1)} \quad (Np/m)$$

where $k_0 = \omega\sqrt{\mu_0\epsilon_0} = \omega/c$

→ Conduction losses are characterized by

$$(3.199)\quad \alpha_c = \frac{R_s}{Z_0 W} \quad (Np/m)$$

where $R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}}$ ← conductor!

Overall, $\alpha = \alpha_c + \alpha_d$

3.8 cont.

Where might we need to be concerned about undesired modes occuring for microstrip?

Possibilities: * Surface waves (see section 3.6) in dielectric sheet above ground plane

* Parallel plate waveguide mode(s) between land & ground plane

(3.201) for TM_1 surface wave is

$$f_{T1} \simeq \frac{c}{2\pi d}\sqrt{\frac{2}{\epsilon_r - 1}}\ \tan^{-1}(\epsilon_r)$$

(3.202) for TE_1 surface wave

$$f_{T2} \simeq \frac{c}{4d\sqrt{\epsilon_r - 1}}$$

(3.203) for transverse resonance of land

$$f_{T3} \simeq \frac{c}{\sqrt{\epsilon_r}\,(2W + d)}$$

(3.204) for parallel plate waveguide mode

$$f_{T4} \simeq \frac{c}{2d\sqrt{\epsilon_r}}$$

Usually, choose the lowest of these threshold frequencies to be safe. They are roughly in order of smallest to largest.

3.8 cont.

Ex. Analyze a microstrip TL built from 1/4" plexiglass ($\epsilon_r = 2.60$, $\tan\delta = 0.006$) with a land 1/4" wide made of 2oz copper tape ($\sigma_{cu} = 5.8 \times 10^7$ S/m, $t = 70\mu m$) operated at 3.3 GHz. (Note: 1/4" = 6.35 mm)

(3.195) $$\epsilon_{r,e} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\frac{1}{\sqrt{1+12\, d/W}}$$

$$= \frac{2.6+1}{2} + \frac{2.6-1}{2}\frac{1}{\sqrt{1+12\left(\frac{0.25}{0.25}\right)}} \Rightarrow \underline{\epsilon_{r,e} = 2.02188}$$

(3.193) $$v_p = \frac{c}{\sqrt{\epsilon_{r,e}}} = \frac{2.997925 \times 10^8}{\sqrt{2.02188}} \Rightarrow \underline{v_p = 2.10835 \times 10^8 \text{ m/s}}$$

$$\lambda_g = v_p/f = \frac{2.10835 \times 10^8}{3.3 \times 10^9} \Rightarrow \underline{\lambda_g = 6.389 \text{ cm}}$$

(3.194) $$\beta = \omega\sqrt{\mu_0 \epsilon_{r,e} \epsilon_0} = \frac{\omega}{c}\sqrt{\epsilon_{r,e}} = \frac{2\pi(3.3\times10^9)}{2.9979\times10^8}\sqrt{2.02188}$$

$$= \frac{\omega}{v_p} \Rightarrow \qquad \underline{\beta = 98.345 \text{ rad/m}}$$

Since $W = d = 0.25"$, we can use either eq'n in (3.196)

$$Z_0 = \frac{60}{\sqrt{\epsilon_{r,e}}}\ln\left(8\,d/W + \frac{W}{4d}\right) = \frac{60}{\sqrt{2.02188}}\ln\left(8 + \frac{1}{4}\right) = 89.043\,\Omega$$

or

$$Z_0 = \frac{120\pi}{\sqrt{\epsilon_{r,e}}\left[W/d + 1.393 + 0.667\ln(W/d + 1.444)\right]}$$

$$= \frac{120\pi}{\sqrt{2.02188}\left[1 + 1.393 + 0.667\ln(1 + 1.444)\right]} = 88.699\,\Omega$$

3.8 cont.

Ex. cont. Since both answers are close, use simple average

$$Z_0 = \tfrac{1}{2}\left[89.04 + 88.7\right] \Rightarrow \underline{Z_0 = 88.9\,\Omega}$$

$$(3.198)\quad \alpha_d = \frac{k_0 \epsilon_r(\epsilon_{r,e}-1)\tan\delta}{2\sqrt{\epsilon_{r,e}}\,(\epsilon_r - 1)} = \frac{\frac{2\pi(3.3\times10^9)}{2.9979\times10^8}\,2.6(2.02188-1)0.006}{2\sqrt{2.02188}\,(2.6-1)}$$

$$\underline{\alpha_d = 0.2423\ Np/m}$$

$$(3.199)\quad \alpha_c = \frac{R_s}{Z_0 W} = \frac{\sqrt{\frac{2\pi(3.3\times10^9)4\pi\times10^{-7}}{2(5.8\times10^7)}}}{88.9(6.35\times10^{-3})} = \frac{0.01499}{88.9(6.35\times10^{-3})}$$

$$\underline{\alpha_c = 0.02656\ Np/m}$$

$$\alpha = \alpha_c + \alpha_d = 0.2423 + 0.0266 = \underline{0.2689\ \tfrac{Np}{m} = 2.335\ \tfrac{dB}{m}}$$

Threshold frequencies

$$(3.201)\quad f_{T1} = \frac{c}{2\pi d}\sqrt{\frac{2}{\epsilon_r - 1}} = \frac{2.9979\times10^8}{2\pi\, 6.35\times10^{-3}}\sqrt{\frac{2}{2.6-1}} \Rightarrow \underline{f_{T1} = 8.4\ GHz}$$

$$(3.202)\quad f_{T2} = \frac{c}{4d\sqrt{\epsilon_r - 1}} = \frac{2.9979\times10^8}{4(0.635\times10^{-2})\sqrt{2.6-1}} \Rightarrow \underline{f_{T2} = 9.3\ GHz}$$

$$(3.203)\quad f_{T3} = \frac{c}{\sqrt{\epsilon_r}\,(2W+d)} = \frac{2.9979\times10^8}{\sqrt{2.6}\;3(6.35\times10^{-3})} \Rightarrow \underline{f_{T3} = 9.8\ GHz}$$

$$(3.204)\quad f_{T4} = \frac{c}{2d\sqrt{\epsilon_r}} = \frac{2.9979\times10^8}{2(6.35\times10^{-3})\sqrt{2.6}} \Rightarrow \underline{f_{T4} = 14.6\ GHz}$$

$\Rightarrow$ 3.3 GHz should be safe!

3.10 Wave Velocities and Dispersion

velocity of light in a medium (infinite in extent) $\equiv \dfrac{1}{\sqrt{\mu\epsilon}}$

phase velocity $\equiv v_p = \dfrac{\omega}{\beta}$

group velocity (speed of narrowband signal) $\equiv v_g = \left(\dfrac{d\beta}{d\omega}\right)^{-1}\Bigg|_{\omega=\omega_0}$ (3.222)

$\Rightarrow$ for a TEM wave, all three may be the same. Not so for other waves.

ex. Rectangular air-filled waveguide

Per (3.83), $\beta = \sqrt{k_0^2 - k_c^2} = \sqrt{(\omega/c)^2 - k_c^2} < k_0 = \omega/c$

$$\frac{d\beta}{d\omega} = \frac{1}{2}\left[(\omega/c)^2 - k_c^2\right]^{-1/2}(2)(\omega/c^2) = \frac{\omega/c}{c\sqrt{(\omega/c)^2 - k_c^2}}$$

$$= \frac{k_0}{c\beta}$$

velocity of light in air $= c = \dfrac{1}{\sqrt{\mu_0\epsilon_0}}$

phase velocity $= \omega/\beta = \dfrac{k_0 c}{\beta} = v_p$

group velocity $= \left(\dfrac{d\beta}{d\omega}\right)^{-1} = \dfrac{c\beta}{k_0} = v_g$

we have $v_g < c < v_p$ for rectangular waveguide!

3.11 Summary of TLs and Waveguides

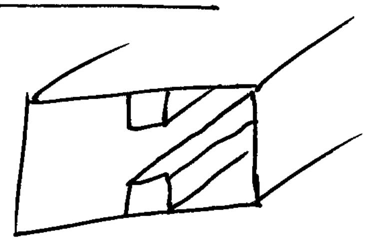
TABLE 3.6 Comparison of Common Transmission Lines and Waveguides

Characteristic	Coax	Waveguide	Stripline	Microstrip
Modes: Preferred	TEM	TE_{10}	TEM	Quasi-TEM
Other	TM,TE	TM,TE	TM,TE	Hybrid TM,TE
Dispersion	None	Medium	None	Low
Bandwidth	High	Low	High	High
Loss	Medium	Low	High	High
Power capacity	Medium	High	Low	Low
Physical size	Large	Large	Medium	Small
Ease of fabrication	Medium	Medium	Easy	Easy
Integration with	Hard	Hard	Fair	Easy

Microwave Engineering **(Fourth Edition), Pozar, Wiley, 2012, ISBN 978-0-470-63155-3.**

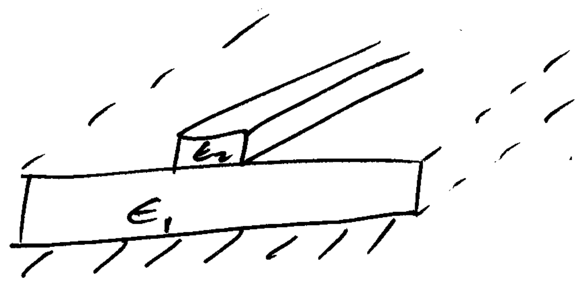
Other TLs and waveguides

→ Ridge waveguide



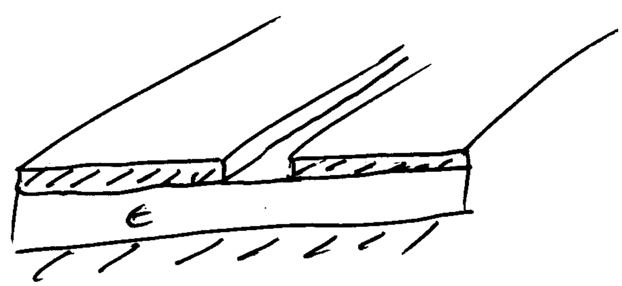
* More BW
* Better (flatter) impedance
* Lower power

→ Dielectric waveguide



* $\epsilon_2 > \epsilon_1$
* good @ mm-wave & optical frequencies

→ Slotline

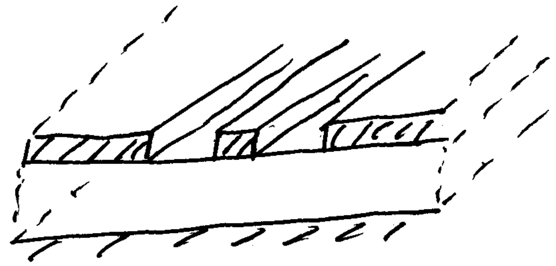


* quasi-TEM

3.11 cont.

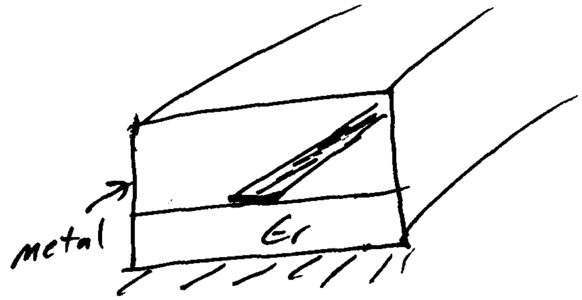
→ Co-planar waveguide

* even-mode & odd-mode quasi-TEM waves

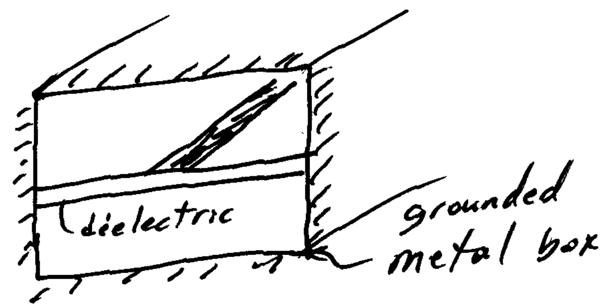


→ covered microstrip

* often put microstrip into metal enclosures, this has effects



→ Suspended stripline



Notes: 0.5 oz copper ~17 μm thick

1 oz copper ~35 μm thick

2 oz copper ~70 μm thick

based on copper weight to cover 1 ft^2

Dimensions sometimes are given in "mils"

1 mil = 0.001" = 25.4 μm