Chapter 1 Electromagnetic Theory

1.1 Intro

We often hear of radio (RF) + microwave engineering. What frequencies +/or wavelengths are we talking about?

Text - RF 30 MHz to 3 GHz for frequency
10 m to 10 cm for wavelength

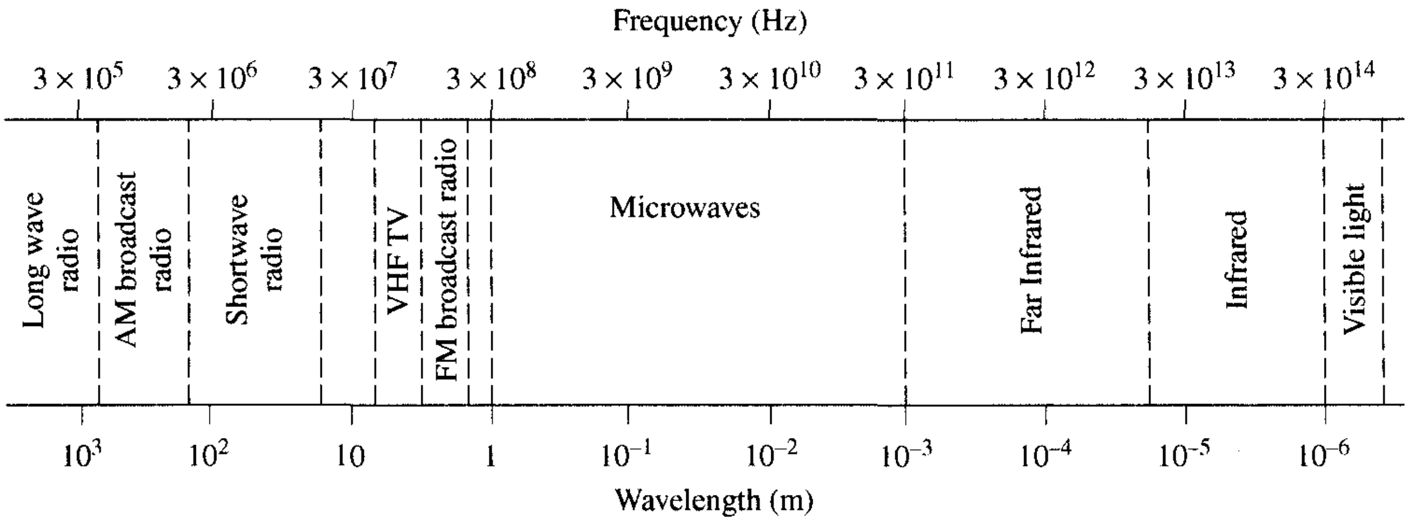
Microwave 3 GHz to 300 GHz for frequency
10 cm to 1 mm for wavelength

* However, in practice, it is common to refer to work above 1 GHz as microwave engineering.

⇒ show Figure 1.1 EM Spectrum ⇐

* Another 'realm' is the 'millimeter-wave' region which typically is thought to run from ~30 GHz to 300 GHz for f
10 mm to 1 mm for λ

* Above 300 GHz, we start entering the optical world of far infrared, infrared, ...



Typical Frequencies

AM broadcast band	535–1605 kHz
Short wave radio band	3–30 MHz
FM broadcast band	88–108 MHz
VHF TV (2–4)	54–72 MHz
VHF TV (5–6)	76–88 MHz
UHF TV (7–13)	174–216 MHz
UHF TV (14–83)	470–890 MHz
US cellular telephone	824–849 MHz
	869–894 MHz
European GSM cellular	880–915 MHz
	925–960 MHz
GPS	1575.42 MHz
	1227.60 MHz
Microwave ovens	2.45 GHz
US DBS	11.7–12.5 GHz
US ISM bands	902–928 MHz
	2.400–2.484 GHz
	5.725–5.850 GHz
US UWB radio	3.1–10.6 GHz

Approximate Band Designations

Medium frequency	300 kHz–3 MHz
High frequency (HF)	3 MHz–30 MHz
Very high frequency (VHF)	30 MHz–300 MHz
Ultra high frequency (UHF)	300 MHz–3 GHz
L band	1–2 GHz
S band	2–4 GHz
C band	4–8 GHz
X band	8–12 GHz
Ku band	12–18 GHz
K band	18–26 GHz
Ka band	26–40 GHz
U band	40–60 GHz
V band	50–75 GHz
E band	60–90 GHz
W band	75–110 GHz
F band	90–140 GHz

FIGURE 1.1 The electromagnetic spectrum.

Microwave Engineering (Fourth Edition), Pozar, Wiley, 2012, ISBN 978-0-470-63155-3.

1.1 cont.

Challenges of microwave engineering -

1) Lumped elements must physically be very small wrt applicable wavelengths, else they behave as distributed elements. I.e., low frequency elements do NOT behave as expected.

2) Voltage & current can be difficult to measure. In fact, voltage is NOT well defined over distances that are electrically large (i.e., appreciable fraction of a wavelength).

3) Instead of wires, we use transmission lines and waveguides to channel/direct/guide signals

4) Design & construction of circuits is more challenging as quantities change with position as well as time when circuit dimensions an appreciable fraction of a wavelength

5) Material properties can/will change with frequency in the microwave realm (may/may not be a challenge), e.g., losses climb.

1.1 cont.

Advantages of microwave engineering –

1) Circuits/devices shrink, allowing for much smaller items. E.g., look @ early portable/cell phones versus modern ones.

2) Most antenna sizes are directly proportional to wavelength for a given directivity or gain. E.g., a 15dBi horn antenna @ 1 GHz (L-band) will be 10x larger than a 15dBi horn @ 10 GHz (X-Band)

3) More absolute bandwidth is available since 1% of 20 GHz is far more than 1% of 100MHz!
⇒ Bigger/larger/faster data transfer

4) Microwave EM signals typically travel in straight paths (line of sight) and do NOT bounce off ionosphere (as much). Think about AM & short wave radio @ night.

5) Much better resolution (detail) for sensing/RADAR @ microwave frequencies. A 'blob' @ low frequencies becomes individual planes, helicopters, ... Radar Cross Sections (RCS) are also proportional to physical size in wavelengths

1.1 cont.

Advantages cont.

6) Less background EM/electrical noise @ higher frequencies since there are fewer sources + propagation attenuation is higher.

7) EM material properties are changing due to atomic, molecular, + nuclear resonances which can be useful for remote sensing + medical imaging / treatment (MRI, microwave ablation, ...)

Applications –

1) RADAR
2) Cell phones
3) Microwave ovens
4) Communication links (i.e., DirectTV)
5) GPS
6) medical
7) wi-fi

⋮

1.1 cont.

History

→ James Clerk Maxwell's Equations ~1873 laid foundation for concept of EM waves

→ 1880's + 1890's further theoretical developments (Oliver Heaviside) and experimental proof (Heinrich Hertz)

→ ~1901 commercialization begins w/ Marconi's transatlantic wireless transmission

→ 1900-1940 most work focused on RF world due to limitations in equipment (vacuum tubes)

→ WWII years saw huge burst of microwave engineering work. E.g., MIT Radiation Laboratory, aperture antennas, high power + frequency sources such as magnetron + Klystron, waveguides, ...

→ 1950's + 1960's saw advent of transistors, diodes, ...

→ 1970's - on computers + numerical modeling start to have big impact as well as continued advances in materials - GaAs, fiber optics, ...

1.2 Maxwell's Equations

While we won't be using these extensively in this course, they are the underlying principles on which we are relying.

⇒ Show Maxwell.pdf ⇐

Definitions –

$\bar{\mathcal{E}}$ or $\bar{E}$ → Electric field (V/m)

$\bar{\mathcal{H}}$ or $\bar{H}$ → Magnetic field (A/m)

$\bar{\mathcal{D}}$ or $\bar{D}$ → Electric flux density (C/m)

$\bar{\mathcal{B}}$ or $\bar{B}$ → Magnetic flux density (Wb/m or T)

$\bar{\mathcal{J}}$ or $\bar{J}$ → current density (A/m^2)

ρ_v → volume charge density (C/m^3)

ϵ → electrical permittivity (F/m)

μ → magnetic permeability (H/m)

σ → conductivity (S/m)

$\bar{\mathcal{M}}$ or $\bar{M}$ → fictitious magnetic current density (V/m^2) usually omitted

Phasors – we'll use $e^{+j\omega t}$ convention

$$\underset{\text{Time domain}}{\bar{\mathcal{E}}} = \mathrm{Re}\{\underset{\text{phasor}}{\bar{E}}\, e^{j\omega t}\}$$

Maxwell's Equations

Static fields:

	Integral Form	Differential Form
Faraday's Law	$\oint_c \bar{E}\cdot d\bar{l} = -\int_s \bar{M}\cdot d\bar{s}$	$\bar{\nabla}\times\bar{E} = -\bar{M}$
Ampere's Law	$\oint_c \bar{H}\cdot d\bar{l} = \int_s \bar{J}\cdot d\bar{s}$	$\bar{\nabla}\times\bar{H} = \bar{J}$
Gauss' Law	$\oint_s \bar{D}\cdot d\bar{s} = \int_V \rho_v \, dV$	$\bar{\nabla}\cdot\bar{D} = \rho_v$
	$\oint_s \bar{B}\cdot d\bar{s} = 0$	$\bar{\nabla}\cdot\bar{B} = 0$

Time-varying fields:

	Integral Form	Differential Form
Faraday's Law	$\oint_c \bar{\mathcal{E}}\cdot d\bar{l} = -\frac{d}{dt}\int_s \bar{\mathcal{B}}\cdot d\bar{s} - \int_s \bar{\mathcal{M}}\cdot d\bar{s}$	$\bar{\nabla}\times\bar{\mathcal{E}} = -\frac{\partial\bar{\mathcal{B}}}{\partial t} - \bar{\mathcal{M}}$
Ampere's Law	$\oint_c \bar{\mathcal{H}}\cdot d\bar{l} = \int_s \bar{\mathcal{J}}\cdot d\bar{s} + \int_s \frac{\partial\bar{\mathcal{D}}}{\partial t}\cdot d\bar{s}$	$\bar{\nabla}\times\bar{\mathcal{H}} = \bar{\mathcal{J}} + \frac{\partial\bar{\mathcal{D}}}{\partial t}$
Gauss' Law	$\oint_s \bar{\mathcal{D}}\cdot d\bar{s} = \int_V \rho_v \, dV$	$\bar{\nabla}\cdot\bar{\mathcal{D}} = \rho_v$
	$\oint_s \bar{\mathcal{B}}\cdot d\bar{s} = 0$	$\bar{\nabla}\cdot\bar{\mathcal{B}} = 0$

Time-varying fields, simple media, & stationary circuits:

	Integral Form	Differential Form
Faraday's Law	$\oint_c \bar{\mathcal{E}}\cdot d\bar{l} = -\mu\frac{d}{dt}\int_s \bar{\mathcal{H}}\cdot d\bar{s} - \int_s \bar{\mathcal{M}}\cdot d\bar{s}$	$\bar{\nabla}\times\bar{\mathcal{E}} = -\mu\frac{\partial\bar{\mathcal{H}}}{\partial t} - \bar{\mathcal{M}}$
Ampere's Law	$\oint_c \bar{\mathcal{H}}\cdot d\bar{l} = \sigma\int_s \bar{\mathcal{E}}\cdot d\bar{s} + \varepsilon\frac{d}{dt}\int_s \bar{\mathcal{E}}\cdot d\bar{s} + \int_s \bar{\mathcal{J}}\cdot d\bar{s}$	$\bar{\nabla}\times\bar{\mathcal{H}} = \sigma\bar{\mathcal{E}} + \varepsilon\frac{\partial\bar{\mathcal{E}}}{\partial t} + \bar{\mathcal{J}}$
Gauss' Law	$\oint_s \bar{\mathcal{E}}\cdot d\bar{s} = \frac{1}{\varepsilon}\int_V \rho_v \, dV$	$\bar{\nabla}\cdot\bar{\mathcal{E}} = \frac{\rho_v}{\varepsilon}$
	$\oint_s \bar{\mathcal{H}}\cdot d\bar{s} = 0$	$\bar{\nabla}\cdot\bar{\mathcal{H}} = 0$

Time-harmonic/sinusoidal steady-state time-varying fields:

	Integral Form	Differential Form
Faraday's Law	$\oint_c \hat{\bar{E}} \bullet d\bar{l} = -j\omega \int_s \hat{\bar{B}} \bullet d\bar{s} - \int_s \hat{\bar{M}} \bullet d\bar{s}$	$\bar{\nabla} \times \hat{\bar{E}} = -j\omega \hat{\bar{B}} - \hat{\bar{M}}$
Ampere's Law	$\oint_c \hat{\bar{H}} \bullet d\bar{l} = \int_s \hat{\bar{J}} \bullet d\bar{s} + j\omega \int_s \hat{\bar{D}} \bullet d\bar{s}$	$\bar{\nabla} \times \hat{\bar{H}} = \hat{\bar{J}} + j\omega \hat{\bar{D}}$
Gauss' Law	$\oint_s \hat{\bar{D}} \bullet d\bar{s} = \int_V \hat{\rho}_v \, dV$	$\bar{\nabla} \bullet \hat{\bar{D}} = \hat{\rho}_v$
	$\oint_s \hat{\bar{B}} \bullet d\bar{s} = 0$	$\bar{\nabla} \bullet \hat{\bar{B}} = 0$

Time-harmonic/sinusoidal steady-state time-varying fields & simple media:

	Integral Form	Differential Form
Faraday's Law	$\oint_c \hat{\bar{E}} \bullet d\bar{l} = -j\omega\mu \int_s \hat{\bar{H}} \bullet d\bar{s} - \int_s \hat{\bar{M}} \bullet d\bar{s}$	$\bar{\nabla} \times \hat{\bar{E}} = -j\omega\mu \hat{\bar{H}} - \hat{\bar{M}}$
Ampere's Law	$\oint_c \hat{\bar{H}} \bullet d\bar{l} = (\sigma + j\omega\varepsilon) \int_s \hat{\bar{E}} \bullet d\bar{s} + \int_s \hat{\bar{J}} \bullet d\bar{s}$	$\bar{\nabla} \times \hat{\bar{H}} = (\sigma + j\omega\varepsilon) \hat{\bar{E}} + \hat{\bar{J}}$
Gauss' Law	$\oint_s \hat{\bar{E}} \bullet d\bar{s} = \frac{1}{\varepsilon} \int_V \hat{\rho}_v \, dV$	$\bar{\nabla} \bullet \hat{\bar{E}} = \frac{\hat{\rho}_v}{\varepsilon}$
	$\oint_s \hat{\bar{H}} \bullet d\bar{s} = 0$	$\bar{\nabla} \bullet \hat{\bar{H}} = 0$

Other important relationships:

	Integral Form	Differential Form
Eqn of Continuity / Conservation of Charge	$\oint_s \bar{\mathcal{J}} \bullet d\bar{s} = -\frac{d}{dt}\int_V \rho_v dV$	$\bar{\nabla} \bullet \bar{\mathcal{J}} = -\frac{\partial \rho_v}{\partial t}$

Lorentz Force Eqn. $\bar{\mathcal{F}} = q(\bar{\mathcal{E}} + \bar{u} \times \bar{\mathcal{B}})$

Constitutive Relations $\bar{\mathcal{D}} = \varepsilon \bar{\mathcal{E}} = \varepsilon_r \varepsilon_0 \bar{\mathcal{E}}$ $\quad$ $\bar{\mathcal{B}} = \mu \bar{\mathcal{H}} = \mu_r \mu_0 \bar{\mathcal{H}}$

Ohm's Law $\bar{\mathcal{J}}_c = \sigma \bar{\mathcal{E}}$

Boundary Conditions		Electric	Magnetic
	Tangential-	$\bar{\mathcal{E}}_{1t} = \bar{\mathcal{E}}_{2t}$ or $\hat{a}_{\mathrm{n12}} \times (\bar{\mathcal{E}}_2 - \bar{\mathcal{E}}_1) = 0$	$\hat{a}_{\mathrm{n12}} \times (\bar{\mathcal{H}}_2 - \bar{\mathcal{H}}_1) = \bar{\mathcal{J}}_s$
	Normal-	$\hat{a}_{\mathrm{n12}} \bullet (\bar{\mathcal{D}}_2 - \bar{\mathcal{D}}_1) = \rho_s$	$\bar{\mathcal{B}}_{1n} = \bar{\mathcal{B}}_{2n}$ or $\hat{a}_{\mathrm{n12}} \bullet (\bar{\mathcal{B}}_2 - \bar{\mathcal{B}}_1) = 0$

where surface normal $\hat{a}_{\mathrm{n12}}$ points from region 1 into region 2, and $\mathcal{B}_{1n}$ & $\mathcal{D}_{1n}$ point away from boundry while $\mathcal{B}_{2n}$ & $\mathcal{D}_{2n}$ point toward from boundry

Permittivity of free space, $\varepsilon_0 = 8.8541878 \times 10^{-12}$ F/m

Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ H/m

Poynting Vector $\bar{\mathcal{S}} = \bar{\mathcal{E}} \times \bar{\mathcal{H}}$

Poynting Theorem

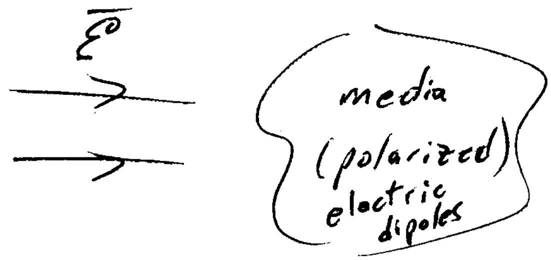
Differential Form- $-\bar{\nabla} \bullet \bar{\mathcal{S}} = \bar{\mathcal{E}} \bullet \bar{\mathcal{J}} + \bar{\mathcal{E}} \bullet \frac{\partial \bar{\mathcal{D}}}{\partial t} + \bar{\mathcal{H}} \bullet \frac{\partial \bar{\mathcal{B}}}{\partial t}$

Integral Form- $-\oint_s \bar{\mathcal{S}} \bullet d\bar{s} = \int_V \bar{\mathcal{E}} \bullet \bar{\mathcal{J}}\, dV + \int_V \left(\bar{\mathcal{E}} \bullet \frac{\partial \bar{\mathcal{D}}}{\partial t} \right) dV + \int_V \left(\bar{\mathcal{H}} \bullet \frac{\partial \bar{\mathcal{B}}}{\partial t} \right) dV$

1.3 Fields in Media & Boundary Conditions

For microwave engineering, we will often be concerned w/ how fields interact with materials/media as well as how fields act at the interface/boundary between different materials.

Electric Fields



$$\bar{\mathcal{D}} = \epsilon_0 \bar{\mathcal{E}} + \bar{\mathcal{P}} \quad \text{or} \quad \bar{D} = \epsilon_0 \bar{E} + \bar{P}$$ Polarization vector (C/m^2)

In linear media, $\bar{P} = \epsilon_0 \chi_e \bar{E} = \bar{P}_e$ ← Text
where χ_e is the electric susceptibility. ← complex

This leads to

$$\bar{D} = \epsilon_0 \bar{E} + \bar{P}_e = \epsilon_0 \bar{E} + \epsilon_0 \chi_e \bar{E} = (1+\chi_e)\epsilon_0 \bar{E} = \underline{\epsilon \bar{E} = \bar{D}}$$

$$\underline{\bar{D} = \epsilon_r \epsilon_0 \bar{E}} \quad \text{(lossless)}$$

As an added 'feature', the permittivity ϵ can be complex

$$\epsilon = \epsilon' - j\epsilon'' = \epsilon_0(1+\chi_e) = \epsilon_0 \epsilon_r$$

where the imaginary part accounts for losses in electric dipole moments due to damping

1.3 cont.

Another loss mechanism in materials is conduction loss, related to conduction current

$$\bar{J} = \sigma \bar{E}$$

Putting these into Ampere's Law

$$\nabla \times \bar{H} = j\omega \bar{D} + \bar{J} = j\omega\epsilon \bar{E} + \bar{J}$$

$$= j\omega\epsilon' \bar{E} + (\omega\epsilon'' + \sigma)\bar{E}$$

$$= j\omega\left(\epsilon' - j\epsilon'' - j\frac{\sigma}{\omega}\right)\bar{E}$$

A common way of characterizing losses is the loss tangent $\equiv \tan\delta = \dfrac{\omega\epsilon'' + \sigma}{\omega\epsilon'}$ # given in PCB datasheets

To confuse things, some texts/references lump the conduction losses (σ) and dipole moment damping losses ($\omega\epsilon''$) together, saying

$$\tan\delta = \frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon}$$

← 'effective' σ
← real

and

$$\epsilon_c = \epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right) = \epsilon(1 - j\tan\delta) = \epsilon' - j\epsilon''$$

So that $\epsilon' = \epsilon = \epsilon_r\epsilon_0$ and $\epsilon'' = \sigma/\omega$

☆ We will let $\epsilon' = \epsilon_r\epsilon_0$

& $\epsilon'' = \epsilon'\tan\delta = \epsilon_r\epsilon_0\tan\delta$

So that $\epsilon = \epsilon' - j\epsilon'' = \epsilon'(1 - j\tan\delta) = \epsilon_r\epsilon_0(1 - j\tan\delta)$

1.3 cont

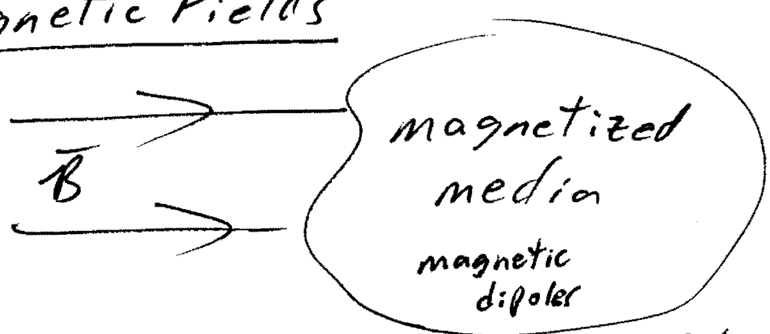
Other complications, some materials are NOT isotropic, i.e., $\bar{P}$ & $\bar{E}$ are not exactly in the same direction, which makes it appear that ϵ changes w/ direction in the material.

We cover this by introducing tensor permittivities for anisotropic materials

$$\bar{D} = [\epsilon]\bar{E} = \bar{\bar{\epsilon}}\,\bar{E} \quad \leftarrow \text{Two different notations}$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

Magnetic Fields



$$\bar{B} = \mu_0(\bar{H} + \bar{M}) = \mu_0(\bar{H} + \bar{P}_m)$$

← EE381/382 ↙ This text

$\bar{P}_m = \chi_m \bar{H}$ in linear media where χ_m is the magnetic susceptibility. ← complex

$$\bar{B} = \mu_0(1+\chi_m)\bar{H} = \mu_0\mu_r\bar{H} \quad \text{(lossless)}$$

$$\bar{B} = \mu\bar{H}$$

1.3 cont.

As with the electric field case, there are losses associated with damping of the magnetic dipoles. We account for this by making μ complex $\mu = \mu_0(1+\chi_m) = \mu' - j\mu''$

However, since there are no magnetic charge carriers, we stop there/here.

Again, some materials are NOT isotropic, i.e., $\bar{B}$ and $\bar{P}_m$ point in slightly different directions, leading to tensor permeabilities for anisotropic materials

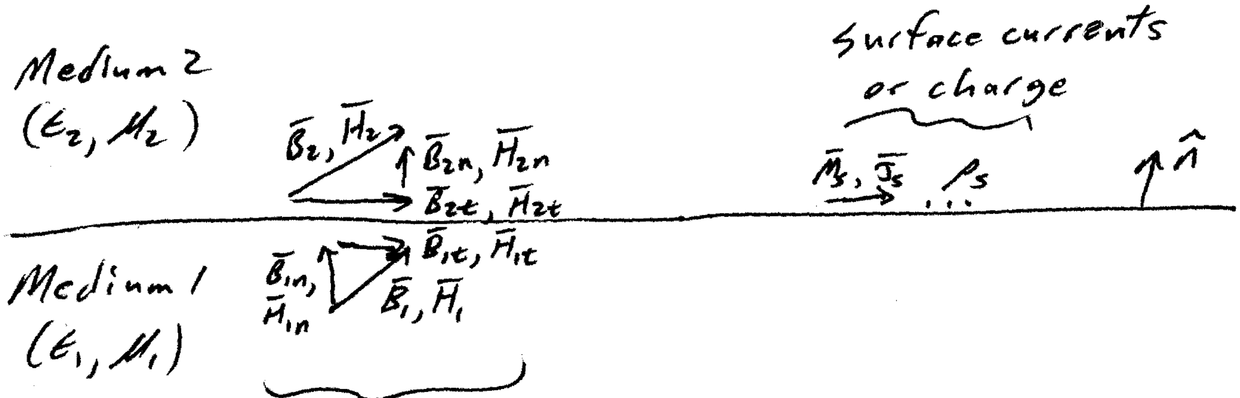
$$\bar{B} = \bar{\bar{\mu}}\,\bar{H} = [\mu]\,\bar{H}$$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \mu_{xx} & \mu_{yx} & \mu_{zx} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}$$

Magnetic materials are notorious for being lossy, anisotropic, & non-linear. unless you stay w/in well defined limits for field strengths, directions, frequencies...

1.3 cont.

Boundary Conditions



we can decompose fields into normal & tangential components

Using Maxwell's Equations, we find

Normal:

$$D_{2n} - D_{1n} = \rho_s \quad \text{or} \quad \hat{n} \cdot (\bar{D}_2 - \bar{D}_1) = \rho_s$$

Note: when $\rho_s = 0$, $D_{1n} = D_{2n}$ or $\bar{D}_{1n} = \bar{D}_{2n}$

Gauss' Law

Tangential:

$$E_{1t} - E_{2t} = -M_s$$ ← fictious magnetic surface current

OR

$$(\bar{E}_2 - \bar{E}_1) \times \hat{n} = \bar{M}_s$$

Note: when $\bar{M}_s = 0$, $E_{1t} = E_{2t}$ or $\bar{E}_{1t} = \bar{E}_{2t}$

Faraday's Law

Most often $\rho_s = 0$ and $\bar{M}_s = 0$ in practical situations.

1.3 cont.

Normal

$$\boxed{\begin{array}{l} B_{1n} = B_{2n} \text{ or } \hat{n}\cdot\bar{B}_1 = \hat{n}\cdot\bar{B}_2 \\ \text{No magnetic surface charges} \end{array}}$$

Law of conservation of magnetic flux

Tangential

$$\boxed{\begin{array}{l} \hat{n} \times (\bar{H}_2 - \bar{H}_1) = \bar{J}_s \\ \text{Note: when } \bar{J}_s = 0,\ \bar{H}_{1t} = \bar{H}_{2t} \end{array}}$$

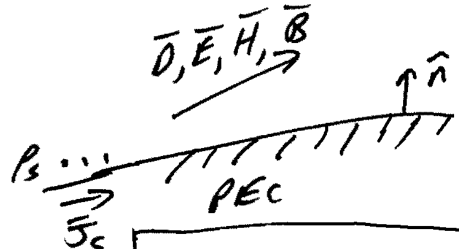
Ampere's Law

Most often $\bar{J}_s = 0$ for practical situations.

Boundary w/ PEC (perfect electric conductor)

[AKA: Electric Wall] assumes $\sigma \to \infty$

* Most good conductors can be well approximated by this PEC assumption.
* We'll also assume $\bar{M}_s = 0$



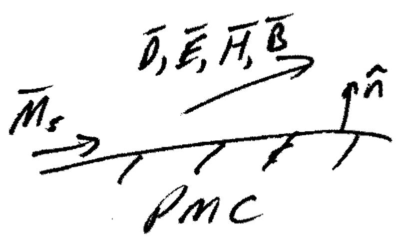
$$\boxed{\begin{array}{ll} \hat{n}\cdot\bar{D} = \rho_s & \text{or } D_n = \rho_s \\ \hat{n}\cdot\bar{B} = 0 & \text{or } B_n = 0 \\ \hat{n}\times\bar{E} = 0 & \text{or } E_t = 0 \\ \hat{n}\times\bar{H} = \bar{J}_s & \end{array}}$$

1.3 cont.

Boundary w/ PMC (perfect magnetic conductor)

[AKA: Magnetic Wall]

* This does not really exist, but is useful to approximate certain situations where the tangential magnetic field is zero.



$$\boxed{\begin{array}{l} \hat{n}\cdot\bar{D}=0 \quad \text{or} \quad D_n=0 \\ \hat{n}\cdot\bar{B}=0 \quad \text{or} \quad B_n=0 \\ \hat{n}\times\bar{E}=-\bar{M}_s \\ \hat{n}\times\bar{H}=0 \quad \text{or} \quad H_t=0 \end{array}}$$

Radiation Condition

To satisfy Conservation of Energy, we will assume that as we go to infinity away from any structures, sources, or devices that all electric + magnetic field quantities will go to zero, e.g., $\bar{E}(r\to\infty)\to 0$.

1.4 The Wave Equation ...

Starting with the phasor curl equations (Ampere's + Faraday's Law) in a source-free, linear, isotropic, + homogeneous region

$$\bar{\nabla} \times \bar{E} = -j\omega\mu \bar{H}$$

$$\bar{\nabla} \times \bar{H} = j\omega\epsilon \bar{E}$$

we can get the Helmholtz or Wave Equations

$$\nabla^2 \bar{E} + \omega^2 \mu\epsilon \bar{E} = 0$$

$$\nabla^2 \bar{H} + \omega^2 \mu\epsilon \bar{H} = 0$$

We define-

can be complex

propagation constant or wave number $\equiv K = \omega\sqrt{\mu\epsilon}$ $\left(\frac{1}{m}\right)$

Plane Waves in lossless medium

→ Assume wave propagating in the ± z-directions w/ x-oriented electric field and y-oriented magnetic field

$$\frac{\partial^2 E_x}{\partial z^2} + K^2 E_x = 0$$

$$\frac{\partial^2 H_y}{\partial z^2} + K^2 H_y = 0$$

where $K^2 = \omega^2 \mu\epsilon$, μ, + ϵ are all <u>real</u>!

1.4 cont.

The solutions are of the form

$$E_x(z) = E^+ e^{-jkz} + E^- e^{jkz}$$

$$H_y(z) = H^+ e^{-jkz} + H^- e^{jkz}$$

$$= \frac{1}{\eta}\left(E^+ e^{-jkz} - E^- e^{jkz}\right)$$

plane waves!

We'll define several parameters for our plane waves (lossless)

phase velocity $\equiv v_p = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}}$ (m/s)

wavelength $\equiv \lambda = \frac{2\pi}{k} = \frac{2\pi v_p}{\omega} = \frac{v_p}{f}$ (m)

prop. constant, wave number, &/or phase constant $\equiv k = \omega\sqrt{\mu\epsilon}$ ← real # in this case $\left(\frac{\text{rad}}{\text{m}}\right)$

intrinsic impedance (wave impedance) $\equiv \eta = \frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\epsilon}}$ (Ω)

In free space, $v_p = c = 2.9979 \times 10^8$ m/s

$$\eta_o = 376.7303\ \Omega$$

1.4 cont.

Plane Waves in lossy material/medium

Now, treating ϵ as real & lumping all losses into σ, we have (ignore mag. losses)

$$\bar{\nabla} \times \bar{E} = -j\omega\mu\bar{H}$$
$$\bar{\nabla} \times \bar{H} = j\omega\epsilon\bar{E} + \sigma\bar{E}$$
$$\Downarrow$$
$$\nabla^2\bar{E} + \omega^2\mu\epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right)\bar{E} = 0$$

Define complex propagation constant $\equiv \gamma = j\omega\sqrt{\mu\epsilon}\sqrt{1 - j\frac{\sigma}{\omega\epsilon}}$
$= \alpha + j\beta$

attenuation constant $\equiv \alpha$ (Np/m)

phase constant $\equiv \beta$ (rad/m)

Solution is of form

$$E_x(z) = E^+e^{-\gamma z} + E^-e^{\gamma z}$$
$$= E^+e^{-\alpha z}e^{-j\beta z} + E^-e^{\alpha z}e^{j\beta z}$$

As an alternative, we can let $\epsilon = \epsilon' - j\epsilon''$ & $\sigma = 0$ to get: $\gamma = j\omega\sqrt{\mu\epsilon} = jk = j\omega\sqrt{\mu\epsilon'(1 - j\tan\delta)}$

1.4 cont.

lossy case parameters

$$v_p = \omega/\beta$$

$$\lambda = 2\pi/\beta$$

$$\eta = \frac{j\omega\mu}{\gamma}$$

Plane Waves in a Good Conductor

$\Rightarrow$ assume $\sigma >> \omega\epsilon$ (conduction >> displacement currents)

Here, $\gamma = \alpha + j\beta \simeq j\omega\sqrt{\mu\epsilon}\sqrt{\frac{\sigma}{j\omega\epsilon}} = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}}$

$$\alpha \cong \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

we define

$$\text{skin depth} \equiv \delta_s = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$\hookrightarrow$ distance for waves to attenuate to e^{-1} of original value

$\Rightarrow$ Table 1.1 Summary $\Leftarrow$

Sections 1.5 to 1.9 Skip

TABLE 1.1 Summary of Results for Plane Wave Propagation in Various Media

Quantity	Type of Medium: Lossless ($\epsilon'' = \sigma = 0$)	General Lossy	Good Conductor ($\epsilon'' \gg \epsilon'$ or $\sigma \gg \omega\epsilon'$)
Complex propagation constant (1/m)	$\gamma = j\omega\sqrt{\mu\epsilon}$	$\gamma = j\omega\sqrt{\mu\epsilon} = j\omega\sqrt{\mu\epsilon'}\sqrt{1 - j\dfrac{\sigma}{\omega\epsilon'}}$	$\gamma = (1+j)\sqrt{\omega\mu\sigma/2}$
Phase constant (wave number) (rad/m)	$\beta = k = \omega\sqrt{\mu\epsilon}$	$\beta = \mathrm{Im}\{\gamma\}$	$\beta = \mathrm{Im}\{\gamma\} = \sqrt{\omega\mu\sigma/2}$
Attenuation constant (Np/m)	$\alpha = 0$	$\alpha = \mathrm{Re}\{\gamma\}$	$\alpha = \mathrm{Re}\{\gamma\} = \sqrt{\omega\mu\sigma/2}$
Impedance (Ω)	$\eta = \sqrt{\mu/\epsilon} = \omega\mu/k$	$\eta = j\omega\mu/\gamma$	$\eta = (1+j)\sqrt{\omega\mu/2\sigma}$
Skin depth (m)	$\delta_s = \infty$	$\delta_s = 1/\alpha$	$\delta_s = \sqrt{2/\omega\mu\sigma}$
Wavelength (m)	$\lambda = 2\pi/\beta$	$\lambda = 2\pi/\beta$	$\lambda = 2\pi/\beta$
Phase velocity (m/s)	$v_p = \omega/\beta$	$v_p = \omega/\beta$	$v_p = \omega/\beta$

Microwave Engineering (Fourth Edition), Pozar, Wiley, 2012, ISBN 978-0-470-63155-3.