For a 50 Ω system, design a 4th-order, lumped-element, linear phase bandpass filter with a center frequency of 2.4 GHz and a bandwidth of 20% using the architecture of Fig. 8.25a. a) Determine the low-pass filter prototype element values. b) Draw a labeled sketch of the scaled and transformed filter with component values. c) Draw a labeled sketch of the filter in phasor form with $V_s = 1\angle 0°$ V. d) Plot the amplitude response $|V_L|$ in decibels with horizontal dashed lines at 20log(0.5) & 20log(0.5/√2) and vertical dashed lines at 2.4 ± 10% GHz for $1 \le f \le 4$ GHz and $-25 \le |V_L| \le -5$ dB. Plot $\angle V_L$ (deg) for $1 \le f \le 4$ GHz with vertical dashed lines at 2.4 ± 10% GHz.

a) From Table 8.5, we get the immittances:

$g_0 = g_5 = 1$ (resistors), $g_1 = 1.0598$ (capacitor), $g_2 = 0.5116$ (inductor), $g_3 = 0.3181$ (capacitor), and $g_4 = 0.1104$ (inductor).

TABLE 8.5 Element Values for Maximally Flat Time Delay Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.5774	0.4226	1.0000								
3	1.2550	0.5528	0.1922	1.0000							
4	1.0598	0.5116	0.3181	0.1104	1.0000						
5	0.9303	0.4577	0.3312	0.2090	0.0718	1.0000					
6	0.8377	0.4116	0.3158	0.2364	0.1480	0.0505	1.0000				
7	0.7677	0.3744	0.2944	0.2378	0.1778	0.1104	0.0375	1.0000			
8	0.7125	0.3446	0.2735	0.2297	0.1867	0.1387	0.0855	0.0289	1.0000		
9	0.6678	0.3203	0.2547	0.2184	0.1859	0.1506	0.1111	0.0682	0.0230	1.0000	
10	0.6305	0.3002	0.2384	0.2066	0.1808	0.1539	0.1240	0.0911	0.0557	0.0187	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

b) Given $\Delta = 0.2$ and $\omega_0 = 2\pi(2.4\times 10^9)$ rad/s. Per Table 8.6 and using (8.64a) & (8.64b) for impedance scaling, each shunt capacitor in the prototype becomes a shunt L & C:

$C_1 = g_1 = 1.0598$ becomes: $$L_1' = \frac{\Delta R_0}{\omega_0 g_1} = \frac{0.2(50)}{(2\pi)2.4\times 10^9(1.0598)} \Rightarrow \mathbf{L_1' = 0.626\ nH.}$$

and $$C_1' = \frac{g_1}{\omega_0 \Delta R_0} = \frac{1.0598}{(2\pi)2.4\times 10^9(0.2)50} \Rightarrow \mathbf{C_1' = 7.028\ pF.}$$

$C_3 = g_3 = 0.3181$ becomes: $$L_3' = \frac{\Delta R_0}{\omega_0 g_3} = \frac{0.2(50)}{(2\pi)2.4\times 10^9(0.3181)} \Rightarrow \mathbf{L_3' = 2.085\ nH.}$$

and $$C_3' = \frac{g_3}{\omega_0 \Delta R_0} = \frac{0.3181}{(2\pi)2.4\times 10^9(0.2)50} \Rightarrow \mathbf{C_3' = 2.109\ pF.}$$

- Per Table 8.6 and using (8.64a) & (8.64b) for impedance scaling, each series inductor in the prototype becomes a series *L* & *C*:

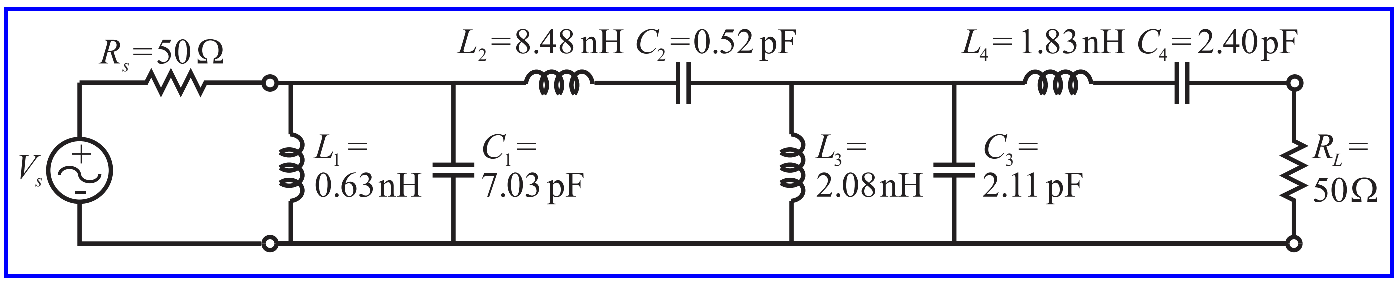
$L_2 = g_2 = 0.5116$ becomes: $$L_2' = \frac{g_2 R_0}{\omega_0 \Delta} = \frac{0.5116(50)}{(2\pi)2.4\times10^9(0.2)} \Rightarrow \underline{\mathbf{L_2' = 8.482\ nH}}.$$

and $$C_2' = \frac{\Delta}{\omega_0 g_2 R_0} = \frac{0.2}{(2\pi)2.4\times10^9(0.5116)50} \Rightarrow \underline{\mathbf{C_2' = 0.518\ pF}}.$$

$L_4 = g_4 = 0.1104$ becomes: $$L_4' = \frac{g_4 R_0}{\omega_0 \Delta} = \frac{0.1104\,(50)}{(2\pi)2.4\times10^9(0.2)} \Rightarrow \underline{\mathbf{L_4' = 1.830\ nH}}.$$

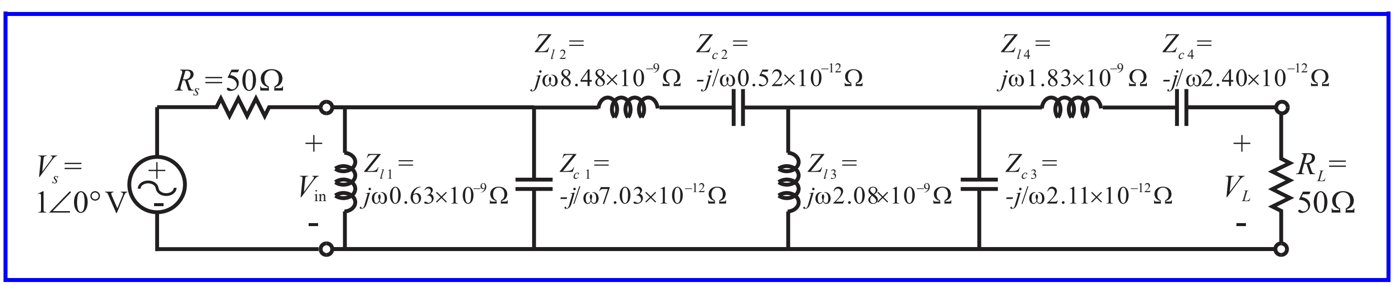
and $$C_4' = \frac{\Delta}{\omega_0 g_4 R_0} = \frac{0.2}{(2\pi)2.4\times10^9(0.1104)50} \Rightarrow \underline{\mathbf{C_4' = 2.403\ pF}}.$$

- Per (8.64c), the source resistance $R_S = g_0$ becomes: $R_S' = R_0\, g_0 = (50)1 \Rightarrow \underline{\mathbf{R_S' = 50\ \Omega}}.$
- Per (8.64d), the load resistance $R_L = g_5$ becomes: $R_L' = R_0\, g_5 = 50(1) \Rightarrow \underline{\mathbf{R_L' = 50\ \Omega}}.$
- The resulting **linear phase bandpass** filter circuit with component values is:



c) From circuits, use $Z_R = R$, $Z_L = j\omega L$, and $Z_C = -j/\omega C$.

The resulting **linear phase bandpass** filter circuit in phasor form is:



d) Using MathCAD

System impedance $Z0 := 50 \quad \Omega$ $\qquad n := 100..400 \qquad f_n := n \cdot 10^7$

Linear phase lumped element design N = 4 $\qquad fc := 2.4 \cdot 10^9 \ \text{Hz} \qquad \Delta := 0.2$

Table 8.3 LPF prototype $g0 := 1 \qquad g1 := 1.0598 \qquad g2 := 0.5116 \qquad g3 := 0.3181$

$g4 := 0.1104 \qquad g5 := 1 \qquad RS := g0 \cdot Z0 \qquad RL := Z0 \cdot g5$

parallel L & C **series L & C**

$$L1 := \frac{\Delta \cdot Z0}{2 \cdot \pi \cdot fc \cdot g1} \qquad C1 := \frac{g1}{Z0 \cdot \Delta \cdot 2\pi \cdot fc} \qquad L2 := \frac{g2 \cdot Z0}{2 \cdot \pi \cdot fc \cdot \Delta} \qquad C2 := \frac{\Delta}{Z0 \cdot g2 \cdot 2\pi \cdot fc}$$

$$L3 := \frac{\Delta \cdot Z0}{2 \cdot \pi \cdot fc \cdot g3} \qquad C3 := \frac{g3}{Z0 \cdot \Delta \cdot 2\pi \cdot fc} \qquad L4 := \frac{g4 \cdot Z0}{2 \cdot \pi \cdot fc \cdot \Delta} \qquad C4 := \frac{\Delta}{Z0 \cdot g4 \cdot 2\pi \cdot fc}$$

$VS := 1 \ \text{V} \qquad RS = 50 \ \Omega \qquad RL = 50 \ \Omega$

$L1 = 6.257 \times 10^{-10} \ \text{H} \qquad C1 = 7.028 \times 10^{-12} \ \text{F}$

$L2 = 8.482 \times 10^{-9} \ \text{H} \qquad C2 = 5.185 \times 10^{-13} \ \text{F}$

$L3 = 2.085 \times 10^{-9} \ \text{H} \qquad C3 = 2.109 \times 10^{-12} \ \text{F}$

$L4 = 1.83 \times 10^{-9} \ \text{H} \qquad C4 = 2.403 \times 10^{-12} \ \text{F}$

Series impedance of RL, C4, & L4 $\qquad Z1(f) := RL + \frac{1}{(j \cdot 2 \cdot \pi \cdot f \cdot C4)} + j \cdot 2 \cdot (\pi \cdot f \cdot L4)$

Parallel impedance of Z1, L3, & C3 $\qquad Z2(f) := \left[\frac{1}{Z1(f)} + \frac{1}{(j \cdot 2 \cdot \pi \cdot f \cdot L3)} + j \cdot 2 \cdot \pi \cdot f \cdot C3\right]^{-1}$

Series impedance of L2, C2, & Z2 $\qquad Z3(f) := j \cdot 2 \cdot \pi \cdot f \cdot L2 + \frac{1}{(j \cdot 2 \cdot \pi \cdot f \cdot C2)} + Z2(f)$

Input impedance of BPF filter w/ load $\qquad Zin(f) := \left[\frac{1}{Z3(f)} + \frac{1}{(j \cdot 2 \cdot \pi \cdot f \cdot L1)} + j \cdot 2 \cdot \pi \cdot f \cdot C1\right]^{-1}$

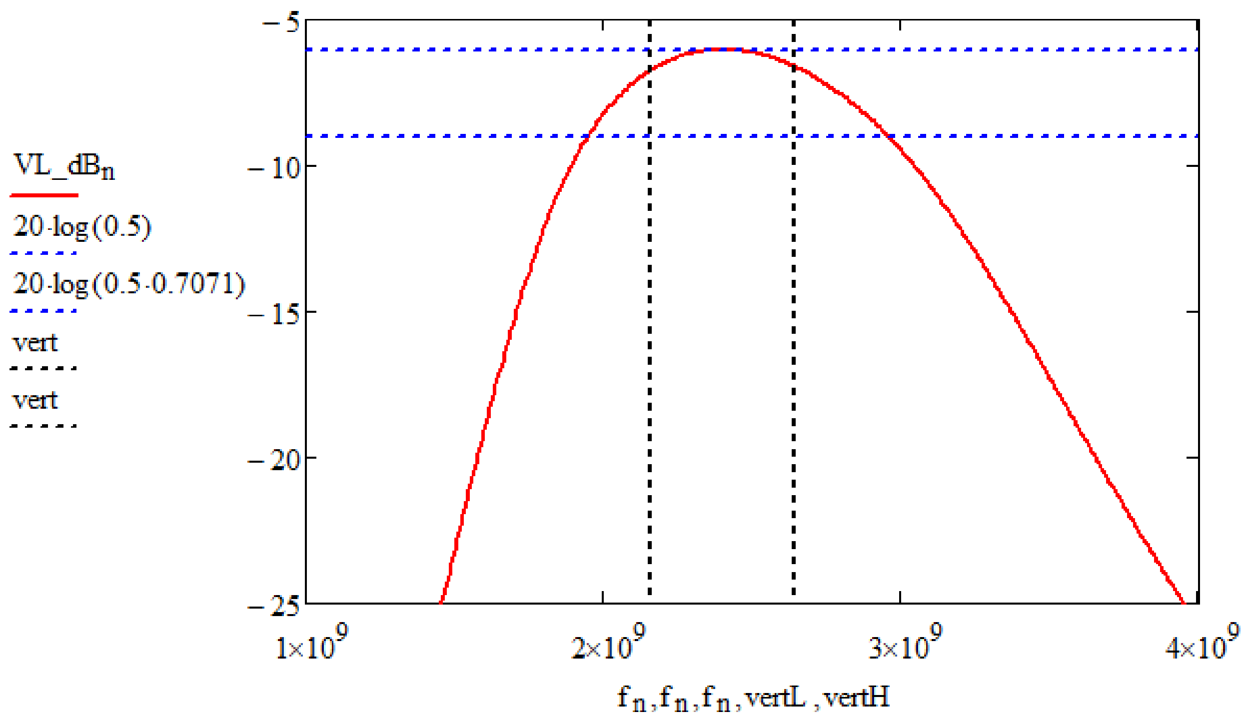
Voltage divisions to get Vin & VRL $\qquad Vin(f) := VS \cdot \frac{Zin(f)}{RS + Zin(f)}$

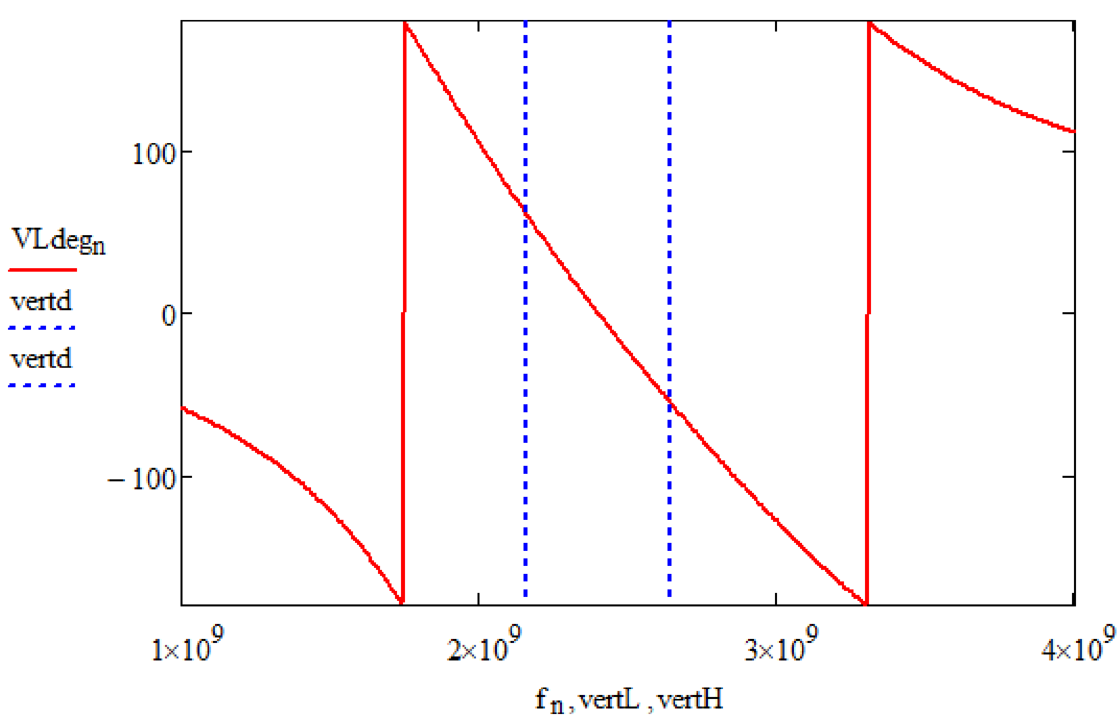
$$VZ2(f) := Vin(f) \cdot \frac{Z2(f)}{Z3(f)} \qquad VRL(f) := VZ2(f) \cdot \frac{RL}{Z1(f)}$$

d) cont.

$VIN_n := Vin(f_n) \quad VL_n := VRL(f_n) \quad VL\_dB_n := 20 \cdot \log(|VL_n|) \quad VLdeg_n := \arg(VL_n) \cdot \frac{180}{\pi}$

$vertL := \begin{pmatrix} 2.16 \cdot 10^9 \\ 2.16 \cdot 10^9 \end{pmatrix} \quad vertH := \begin{pmatrix} 2.64 \cdot 10^9 \\ 2.64 \cdot 10^9 \end{pmatrix} \quad vert := \begin{pmatrix} -5 \\ -25 \end{pmatrix} \quad vertd := \begin{pmatrix} 180 \\ -180 \end{pmatrix}$





- Note: The linear phase BPF has NOT rolled off 3 dB at $f_c = 2.4 \pm 10\%$ GHz.