For a 50 Ω system, design a lumped-element, Butterworth high-pass filter with a cut-off frequency of 2.4 GHz with an attenuation of at least 15 dB at 1.2 GHz using the architecture of Fig. 8.25a. a) Determine the filter order N and the low-pass filter prototype element values. b) Draw a labeled sketch of the scaled and transformed filter with component values. c) Draw a labeled sketch of the filter in phasor form with $V_s = 1\angle 0°$ V. d) Plot the amplitude response $|V_L|$ in decibels with horizontal dashed lines at 20log(0.5) & 20log(0.5/√2) and a vertical dashed line at 2.4 GHz for $0 \le f \le 5$ GHz & $-25 \text{ dB} \le |V_L| \le 0$.

a) Since ω is at $0.5\omega_c$ for the HPF, calculate normalized frequency for LPF prototype at $2\omega_c$ $|\omega/\omega_c| - 1 = |2\omega_c/\omega_c| - 1 = 1$. From Figure 8.26, we see that an LP prototype filter of order ***N* = 3** is needed to meet the 15 dB attenuation specification.

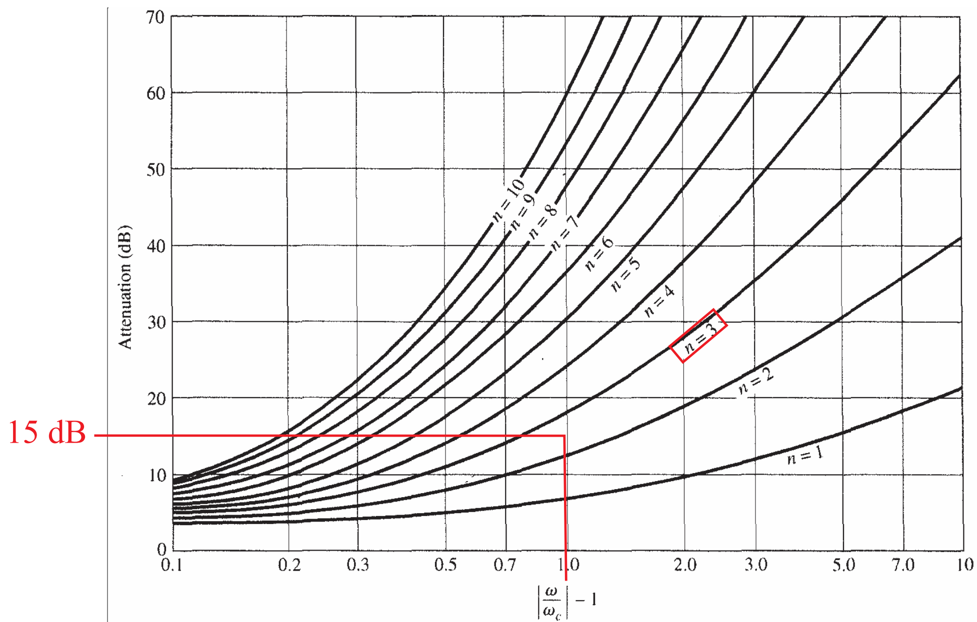


FIGURE 8.26 Attenuation versus normalized frequency for maximally flat filter prototypes.
Adapted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

From Table 8.3, we get the immittances:

$g_0 = g_4 = 1$ (resistors), $g_1 = g_3 = 1.0000$ (capacitors), and $g_2 = 2.0000$ (inductor).

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

b) For filter architecture of Fig 8.25a, use the immittances & equations (8.64cd), & (8.70ab) to get necessary scaled & transformed shunt inductances and series capacitances:

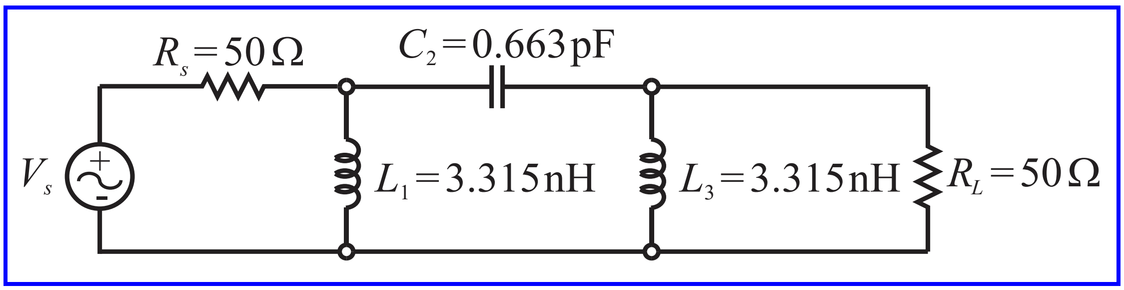
Per (8.64c), $R_S' = R_0 = 50$ $\Rightarrow$ **$R_s' = 50\ \Omega$.**

Per (8.70b), $L_1' = L_3' = \dfrac{R_0}{\omega_c C_k} = \dfrac{R_0}{\omega_c g_1} = \dfrac{50}{(2\pi)2.4\times10^9(1)}$ $\Rightarrow$ **$L_1' = L_3' = 3.315$ nH.**

Per (8.70a), $C_2' = \dfrac{1}{R_0\,\omega_c L_k} = \dfrac{1}{R_0\,\omega_c g_2} = \dfrac{1}{50(2\pi)2.4\times10^9(2)}$ **$C_2' = 0.663$ pF.**

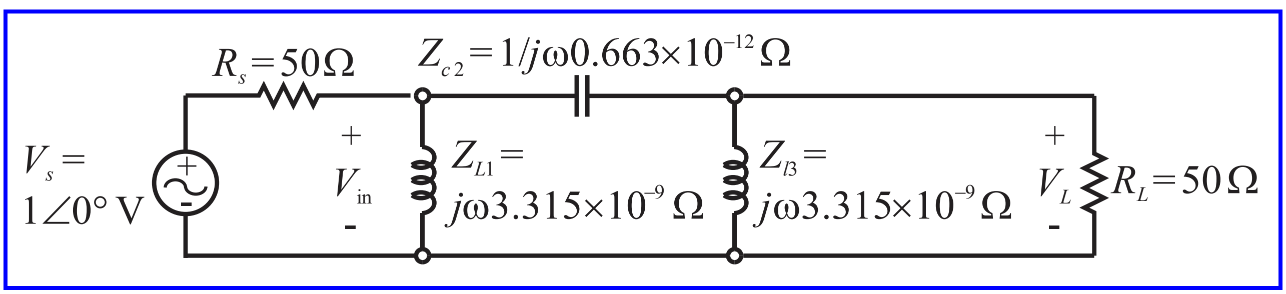
Per (8.64d), $R_L' = R_0 R_L = R_0 g_4 = 50(1)$ $\Rightarrow$ **$R_L' = 50\ \Omega$.**

- The resulting **Butterworth highpass** filter circuit with component values is:



c) From circuits, use $Z_R = R$, $Z_L = j\omega L$, and $Z_C = 1/j\omega C$.

The resulting **Butterworth highpass** filter circuit in phasor form is:



d) Using MathCAD

System impedance Z0 := 50 Ω n := 1 .. 500 $f_n := n\cdot 10^7$

Butterworth lumped element design N = 3 $fc := 2.4\cdot10^9$ Hz

Table 8.3 LPF prototype g0 := 1 g1 := 1 g2 := 2 g3 := g1 g4 := 1

RS := g0·Z0 $L1 := \dfrac{Z0}{2\cdot\pi\cdot fc\cdot g1}$ $C2 := \dfrac{1}{Z0\cdot g2\cdot 2\pi\cdot fc}$ L3 := L1 RL := Z0·g4

VS := 1 V RS = 50 Ω $L1 = 3.31573\times10^{-9}$ H

$C2 = 6.63146\times10^{-13}$ F $L3 = 3.31573\times10^{-9}$ H RL = 50 Ω

Parallel impedance of RL & C3 $Z1(f) := \left[\frac{1}{RL} + \frac{1}{(j \cdot 2 \cdot \pi \cdot f \cdot L3)}\right]^{-1}$

Series impedance of L2 & Z1 $Z2(f) := \frac{1}{(j \cdot 2 \cdot \pi \cdot f \cdot C2)} + Z1(f)$

Input impedance of LP filter w/ load $Zin(f) := \left[\frac{1}{Z2(f)} + \frac{1}{(j \cdot 2 \cdot \pi \cdot f \cdot L1)}\right]^{-1}$

Voltage divisions to get Vin & Vld $Vin(f) := VS \cdot \frac{Zin(f)}{RS + Zin(f)}$ $Vld(f) := Vin(f) \cdot \frac{Z1(f)}{Z2(f)}$

$VIN_n := Vin(f_n)$ $VL_n := Vld(f_n)$ $VL\_dB_n := 20 \cdot \log(|VL_n|)$

$vert24 := \begin{pmatrix} 0 \\ -25 \end{pmatrix}$ $vertf := \begin{pmatrix} 2.4 \cdot 10^9 \\ 2.4 \cdot 10^9 \end{pmatrix}$