8.6 Solve the design equations in Section 8.3 for the elements of an $N = 2$ equal-ripple filter if the ripple specification is 1.0 dB.

- 8.6 with ripple specification changed to 1.5 dB. a) Find the parameter k^2 as well as values k_{neg} & k_{pos}. b) Find the equations for the P_{LR} in terms of ω and the circuit component values. c) Find the two possible values for R, i.e., R_{small} & R_{big}. d) Find equations for realizable values of the inductance $L_{k\text{neg}}$ and $L_{k\text{pos}}$ in terms of R, C, and k_{neg} or k_{pos}. e) Equate ω^2 coefficients and then use the $L_{k\text{neg}}$ & $L_{k\text{pos}}$ equations to generate two polynomials in terms of C. f) Find R_s, L, C, & R and draw prototype low-pass filter circuit with component values. [Hints: Try all four permutations of k_{neg}, k_{pos}, R_{small}, and R_{big} in the polynomials to find the solution with realizable components. Consider using quadratic formula or numerical polynomial root solver.]

a) Per (8.54), $P_{LR} = 1 + k^2 T_N^2\left(\frac{\omega}{\omega_c}\right)$ for a Chebyshev low-pass filter.

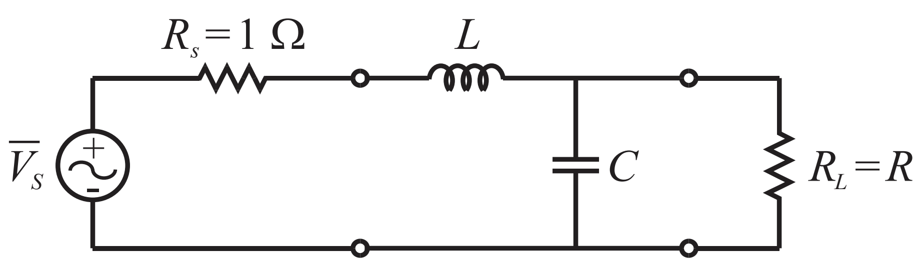
Chebyshev polynomials are equal to 1 when their argument is 1, i.e., $T_N^2(1) = 1$ at $\omega = \omega_c$.

Therefore, $P_{LR}(\omega_c) = 1 + k^2 T_N^2\left(\frac{\omega_c}{\omega_c}\right) = 1 + k^2$. Here, we desire

$$P_{LR}(\omega_c) = 1 + k^2 = 1.5 \text{ dB} = 10^{1.5/10} = 1.412537545 \quad \Rightarrow \quad \mathbf{k^2 = 0.412537545}.$$

Solving we get $k = \pm 0.642290857$ $\Rightarrow$ $\mathbf{k_{pos} = 0.642290857}$ & $\mathbf{k_{neg} = -0.642290857}$.

b) For our prototype second order low-pass filter, we let $R_s = 1$ and assume $\omega_c = 1$ rad/s.



Now (8.54) becomes (8.61) $P_{LR} = 1 + k^2 T_N^2(\omega)$. For $N = 2$, per (5.56b) $T_2(x) = 2x^2 - 1$, so $T_2^2(\omega) = (2\omega^2 - 1)^2$. Equating the P_{LR} equation (8.60) with (8.61) yields (8.62)

$$P_{LR} = 1 + k^2(2\omega^2 - 1)^2 = 1 + k^2(4\omega^4 - 4\omega^2 + 1)$$
$$= 1 + \frac{1}{4R}\left[(1-R)^2 + (R^2C^2 + L^2 - 2LCR^2)\omega^2 + L^2C^2R^2\omega^4\right]$$

c) Letting $\omega = 0$, we get $1 + k^2 = 1 + \frac{1}{4R}\left[(1-R)^2\right] \quad \Rightarrow \quad k^2 = \frac{(1-R)^2}{4R}$.

After some algebra & using the quadratic formula, we get (8.63) $R = 1 + 2k^2 \pm 2k\sqrt{1 + k^2}$.

For this case, $R = 1 + 2(0.412537545) \pm 2(\pm 0.642290857)\sqrt{1.412537545}$ which has two possible solutions $\Rightarrow$ **R_{big} = 3.351803** & **R_{small} = 0.298347**.

d) Equating the ω^4 coefficients of (8.62) gives $4k^2 = \dfrac{L^2C^2R^2}{4R} \Rightarrow L^2 = \dfrac{4^2k^2R}{C^2R^2} = \dfrac{16k^2}{C^2R}$.

Solving for the inductance, $L = \dfrac{\pm 4k}{C\sqrt{R}}$ which implies $\underline{L_{kneg} = \dfrac{-4k_{neg}}{C\sqrt{R}}}$ or $\underline{L_{kpos} = \dfrac{4k_{pos}}{C\sqrt{R}}}$ are the realizable solutions where $L > 0$.

e) Equating the ω^2 coefficients of (8.62) gives

$$-4k^2 = \frac{1}{4R}(R^2C^2 + L^2 - 2LCR^2)$$

$$-16k^2R = R^2C^2 + \frac{16k^2}{C^2R} - 2LCR^2$$

$$16k^2RC^2 + R^2C^4 + \frac{16k^2}{R} - 2LR^2C^3 = 0$$

If L_{kneg} is substituted in, we get a polynomial in terms of C.

$$16k^2RC^2 + R^2C^4 + \frac{16k^2}{R} - 2\left(\frac{-4k_{neg}}{C\sqrt{R}}\right)R^2C^3 = 0$$

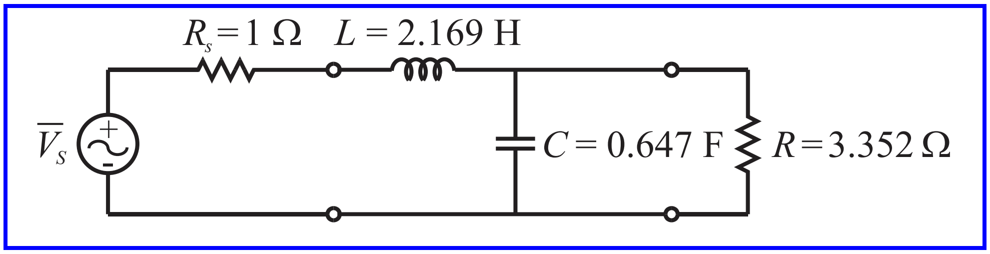
$$\boxed{R^2C^4 + \left(16k^2R + \frac{8k_{neg}R^2}{\sqrt{R}}\right)C^2 + \frac{16k^2}{R} = 0.}$$

If L_{kpos} is substituted in, we get another polynomial in terms of C.

$$16k^2RC^2 + R^2C^4 + \frac{16k^2}{R} - 2\left(\frac{4k_{pos}}{C\sqrt{R}}\right)R^2C^3 = 0$$

$$\boxed{R^2C^4 + \left(16k^2R - \frac{8k_{pos}R^2}{\sqrt{R}}\right)C^2 + \frac{16k^2}{R} = 0.}$$

f) MathCAD was used to solve these polynomials for C to determine which combination(s) of R_{big}, R_{small}, k_{pos}, and k_{neg} lead to realizable values of L & C. As shown, only the R_{big} & k_{new} or R_{big} & k_{pos} combinations work and yield the same solutions. The 2[nd]-order prototype Chebyshev low-pass filter w/ a ripple of 1.5 dB is shown below. In terms of immittances: **g_0 =1, g_1 = 2.1688, g_2 = 0.6470, and g_3 = 3.3518**.



Case 1 k_{neg} and R_{big}- ***C & L* are real & positive!**

$$vcase1 := \begin{pmatrix} Rbig^2 \\ 0 \\ 16 \cdot ksqrd \cdot Rbig + \frac{8 \cdot kneg \cdot Rbig^2}{\sqrt{Rbig}} \\ 0 \\ \frac{16 \cdot ksqrd}{Rbig} \end{pmatrix} \qquad vcase1 = \begin{pmatrix} 11.2346 \\ 0 \\ -9.4072 \\ 0 \\ 1.9693 \end{pmatrix}$$

$$f(C) := Rbig^2 \cdot C^4 + \left(16 \cdot ksqrd \cdot Rbig + \frac{8 \cdot kneg \cdot Rbig^2}{\sqrt{Rbig}}\right) \cdot C^2 + \frac{16 \cdot ksqrd}{Rbig}$$

$C := 0.5$ $\quad C1 := root(f(C), C)$ $\quad L1 := \frac{-4 \cdot kneg}{C1 \cdot \sqrt{Rbig}}$ $\quad R1 := Rbig$

$Rs := 1$ $\quad C1 = 0.647038$ $\quad L1 = 2.168815$ $\quad R1 = 3.351803$

Case 2 k_{neg} and R_{small}- **NO, *C & L*** are complex

$$vcase2 := \begin{pmatrix} Rsmall^2 \\ 0 \\ 16 \cdot ksqrd \cdot Rsmall + \frac{8 \cdot kneg \cdot Rsmall^2}{\sqrt{Rsmall}} \\ 0 \\ \frac{16 \cdot ksqrd}{Rsmall} \end{pmatrix} \qquad vcase2 = \begin{pmatrix} 0.089 \\ 0 \\ 1.1319 \\ 0 \\ 22.1239 \end{pmatrix}$$

$$f(C) := Rsmall^2 \cdot C^4 + \left(16 \cdot ksqrd \cdot Rsmall + \frac{8 \cdot kneg \cdot Rsmall^2}{\sqrt{Rsmall}}\right) \cdot C^2 + \frac{16 \cdot ksqrd}{Rsmall}$$

$C := 0.5$ $\quad C2 := root(f(C), C)$ $\quad L2 := \frac{-4 \cdot kneg}{C2 \cdot \sqrt{Rsmall}}$ $\quad R2 := Rsmall$

$Rs = 1$ $\quad C2 = 2.169 - 3.326i$ $\quad L2 = 0.647 + 0.992i$ $\quad R2 = 0.298347$

Case 3 k_{pos} and R_{big}- ***C* & *L* are real & positive and same values as Case 1!**

$$vcase3 := \begin{pmatrix} Rbig^2 \\ 0 \\ 16 \cdot ksqrd \cdot Rbig - \dfrac{8 \cdot kpos \cdot Rbig^2}{\sqrt{Rbig}} \\ 0 \\ \dfrac{16 \cdot ksqrd}{Rbig} \end{pmatrix} \qquad vcase3 = \begin{pmatrix} 11.2346 \\ 0 \\ -9.4072 \\ 0 \\ 1.9693 \end{pmatrix}$$

$$f(C) := Rbig^2 \cdot C^4 + \left(16 \cdot ksqrd \cdot Rbig - \frac{8 \cdot kpos \cdot Rbig^2}{\sqrt{Rbig}}\right) \cdot C^2 + \frac{16 \cdot ksqrd}{Rbig}$$

$C := 0.5 \qquad C3 := root(f(C), C) \qquad L3 := \dfrac{4 \cdot kpos}{C3 \cdot \sqrt{Rbig}} \qquad R3 := Rbig$

$Rs := 1 \qquad C3 = 0.647038 \qquad L3 = 2.168815 \qquad R3 = 3.351803$

Case 4 k_{pos} and R_{small}- **NO**, ***C*** & ***L*** are complex

$$vcase4 := \begin{pmatrix} Rsmall^2 \\ 0 \\ 16 \cdot ksqrd \cdot Rsmall - \dfrac{8 \cdot kpos \cdot Rsmall^2}{\sqrt{Rsmall}} \\ 0 \\ \dfrac{16 \cdot ksqrd}{Rsmall} \end{pmatrix} \qquad vcase4 = \begin{pmatrix} 0.089 \\ 0 \\ 1.1319 \\ 0 \\ 22.1239 \end{pmatrix}$$

$$f(C) := Rsmall^2 \cdot C^4 + \left(16 \cdot ksqrd \cdot Rsmall - \frac{8 \cdot kpos \cdot Rsmall^2}{\sqrt{Rsmall}}\right) \cdot C^2 + \frac{16 \cdot ksqrd}{Rsmall}$$

$C := 0.5 \qquad C4 := root(f(C), C) \qquad L4 := \dfrac{4 \cdot kpos}{C4 \cdot \sqrt{Rsmall}} \qquad R4 := Rsmall$

$Rs = 1 \qquad C4 = 2.169 - 3.326i \qquad L4 = 0.647 + 0.992i \qquad R4 = 0.298347$