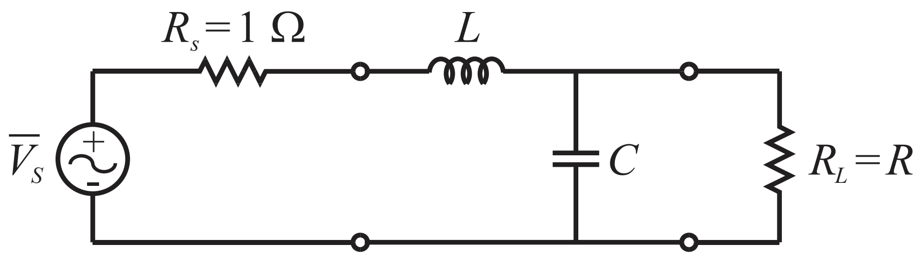
8.6 Solve the design equations in Section 8.3 for the elements of an $N = 2$ equal-ripple filter if the ripple specification is 1.0 dB.

- Per (8.54), $P_{LR} = 1 + k^2 T_N^2\left(\dfrac{\omega}{\omega_c}\right)$ for a Chebyshev low-pass filter.
- Chebyshev polynomials are equal to 1 when their argument is 1, i.e., $T_N^2(1) = 1$ at $\omega = \omega_c$.
- Therefore, $P_{LR}(\omega_c) = 1 + k^2 T_N^2\left(\dfrac{\omega_c}{\omega_c}\right) = 1 + k^2$.
- Here, we desire $P_{LR}(\omega_c) = 1 + k^2 = 1\text{ dB } = 10^{1/10} = 1.2589254 \;\Rightarrow\; \underline{k^2 = 0.2589254}$.
- Solving we get $k = \pm 0.50884714 \;\Rightarrow\; \underline{k_{pos} = 0.50884714}$ & $\underline{k_{neg} = -0.50884714}$.
- For our prototype second order low-pass filter, we let $R_s = 1$ and assume $\omega_c = 1$ rad/s.



- Now (8.54) becomes (8.61) $P_{LR} = 1 + k^2 T_N^2(\omega)$.
- For $N = 2$, per (5.56b) $T_2(x) = 2x^2 - 1$, so $T_2^2(\omega) = \left(2\omega^2 - 1\right)^2$.
- Equating the P_{LR} equation (8.60) with (8.61) yields (8.62)

$$
\begin{aligned}
P_{LR} &= 1 + k^2\left(2\omega^2 - 1\right)^2 = 1 + k^2\left(4\omega^4 - 4\omega^2 + 1\right) \\
&= 1 + \frac{1}{4R}\left[(1-R)^2 + (R^2C^2 + L^2 - 2LCR^2)\omega^2 + L^2C^2R^2\omega^4\right]
\end{aligned}
$$

- Letting $\omega = 0$, we get $1 + k^2 = 1 + \dfrac{1}{4R}\left[(1-R)^2\right] \;\Rightarrow\; k^2 = \dfrac{(1-R)^2}{4R}$.
- After some algebra & using the quadratic formula, we get (8.63) $R = 1 + 2k^2 \pm 2k\sqrt{1+k^2}$.
- For this case, $R = 1 + 2(0.258925) \pm 2(\pm 0.508847)\sqrt{1.258925}$ which has two possible solutions $\Rightarrow$ $\underline{\boldsymbol{R_{big} = 2.65972256}}$ & $\underline{\boldsymbol{R_{small} = 0.375979}}$.
- Equating the ω^4 coefficients of (8.62) gives $4k^2 = \dfrac{L^2C^2R^2}{4R} \;\Rightarrow\; L^2 = \dfrac{4^2k^2R}{C^2R^2} = \dfrac{16k^2}{C^2R}$.

- Solving for the inductance, $L = \dfrac{\pm 4k}{C\sqrt{R}}$ which implies $\underline{L_{kneg} = \dfrac{-4k_{neg}}{C\sqrt{R}}}$ or $\underline{L_{kpos} = \dfrac{4k_{pos}}{C\sqrt{R}}}$ are the realizable solutions where $L > 0$.
- Equating the ω^2 coefficients of (8.62) gives

$$-4k^2 = \frac{1}{4R}(R^2C^2 + L^2 - 2LCR^2)$$

$$-16k^2R = R^2C^2 + \frac{16k^2}{C^2R} - 2LCR^2$$

$$16k^2RC^2 + R^2C^4 + \frac{16k^2}{R} - 2LR^2C^3 = 0$$

- If L_{kneg} is substituted in, we get a polynomial in terms of C.

$$16k^2RC^2 + R^2C^4 + \frac{16k^2}{R} - 2\left(\frac{-4k_{neg}}{C\sqrt{R}}\right)R^2C^3 = 0$$

$$\underline{R^2C^4 + \left(16k^2R + \frac{8k_{neg}R^2}{\sqrt{R}}\right)C^2 + \frac{16k^2}{R} = 0}.$$

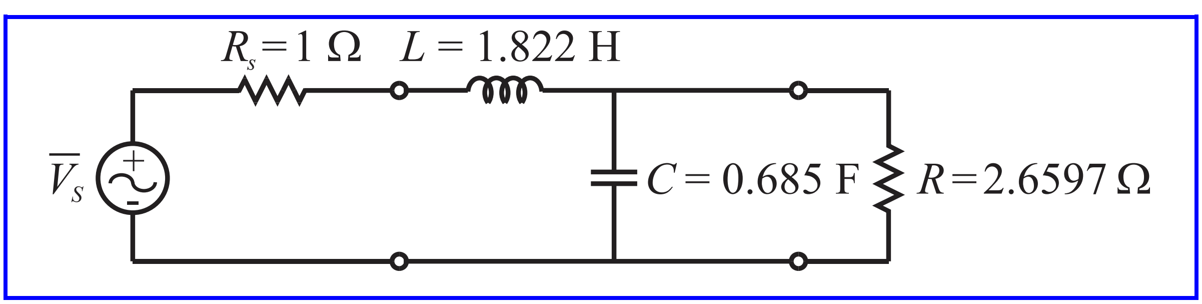
- If L_{kpos} is substituted in, we get another polynomial in terms of C.

$$16k^2RC^2 + R^2C^4 + \frac{16k^2}{R} - 2\left(\frac{4k_{pos}}{C\sqrt{R}}\right)R^2C^3 = 0$$

$$\underline{R^2C^4 + \left(16k^2R - \frac{8k_{pos}R^2}{\sqrt{R}}\right)C^2 + \frac{16k^2}{R} = 0}.$$

- MathCAD was used to solve these polynomials for C to determine which combination(s) of R_{big}, R_{small}, k_{pos}, and k_{neg} lead to realizable values of L & C.

- As shown, only the R_{big} & k_{new} or R_{big} & k_{pos} combinations work and yield the same solutions. The 2nd-order prototype Chebyshev low-pass filter w/ a ripple of 1 dB is shown below. In terms of immittances: $\boxed{\mathbf{g_0 = 1, g_1 = 1.822, g_2 = 0.685, \text{and } g_3 = 2.6597}}$.



<u>Case 1</u> k_{neg} and R_{big}- ***C* & *L* are real & positive!**

$$\text{vcase1} := \begin{pmatrix} \text{Rbig}^2 \\ 0 \\ 16\cdot\text{ksqrd}\cdot\text{Rbig} + \dfrac{8\cdot\text{kneg}\cdot\text{Rbig}^2}{\sqrt{\text{Rbig}}} \\ 0 \\ \dfrac{16\cdot\text{ksqrd}}{\text{Rbig}} \end{pmatrix} \qquad \text{vcase1} = \begin{pmatrix} 7.0741 \\ 0 \\ -6.6389 \\ 0 \\ 1.5576 \end{pmatrix}$$

$$f(C) := \text{Rbig}^2\cdot C^4 + \left(16\cdot\text{ksqrd}\cdot\text{Rbig} + \frac{8\cdot\text{kneg}\cdot\text{Rbig}^2}{\sqrt{\text{Rbig}}}\right)\cdot C^2 + \frac{16\cdot\text{ksqrd}}{\text{Rbig}}$$

$$C := 0.5 \qquad C1 := \text{root}(f(C), C) \qquad L1 := \frac{-4\cdot\text{kneg}}{C1\cdot\sqrt{\text{Rbig}}} \qquad R1 := \text{Rbig}$$

$$\boxed{Rs := 1} \qquad \boxed{C1 = 0.684997} \qquad \boxed{L1 = 1.821967} \qquad \boxed{R1 = 2.659723}$$

<u>Case 2</u> k_{neg} and R_{small}- **NO, *C* & *L*** are complex

$$\text{vcase2} := \begin{pmatrix} \text{Rsmall}^2 \\ 0 \\ 16\cdot\text{ksqrd}\cdot\text{Rsmall} + \dfrac{8\cdot\text{kneg}\cdot\text{Rsmall}^2}{\sqrt{\text{Rsmall}}} \\ 0 \\ \dfrac{16\cdot\text{ksqrd}}{\text{Rsmall}} \end{pmatrix} \qquad \text{vcase2} = \begin{pmatrix} 0.1414 \\ 0 \\ 0.6191 \\ 0 \\ 11.0187 \end{pmatrix}$$

$$f(C) := \text{Rsmall}^2\cdot C^4 + \left(16\cdot\text{ksqrd}\cdot\text{Rsmall} + \frac{8\cdot\text{kneg}\cdot\text{Rsmall}^2}{\sqrt{\text{Rsmall}}}\right)\cdot C^2 + \frac{16\cdot\text{ksqrd}}{\text{Rsmall}}$$

$$C := 0.5 \qquad C2 := \text{root}(f(C), C) \qquad L2 := \frac{-4\cdot\text{kneg}}{C2\cdot\sqrt{\text{Rsmall}}} \qquad R2 := \text{Rsmall}$$

$$\boxed{Rs = 1} \qquad \boxed{C2 = -1.822 - 2.347i} \qquad \boxed{L2 = -0.685 + 0.882i} \qquad \boxed{R2 = 0.375979}$$

Case 3 k_{pos} and R_{big}- ***C* & *L* are real & positive and same values as Case 1!**

$$vcase3 := \begin{pmatrix} Rbig^2 \\ 0 \\ 16 \cdot ksqrd \cdot Rbig - \dfrac{8 \cdot kpos \cdot Rbig^2}{\sqrt{Rbig}} \\ 0 \\ \dfrac{16 \cdot ksqrd}{Rbig} \end{pmatrix} \qquad vcase3 = \begin{pmatrix} 7.0741 \\ 0 \\ -6.6389 \\ 0 \\ 1.5576 \end{pmatrix}$$

$$f(C) := Rbig^2 \cdot C^4 + \left(16 \cdot ksqrd \cdot Rbig - \frac{8 \cdot kpos \cdot Rbig^2}{\sqrt{Rbig}}\right) \cdot C^2 + \frac{16 \cdot ksqrd}{Rbig}$$

$C := 0.5$ $\qquad C3 := root(f(C), C)$ $\qquad L3 := \dfrac{4 \cdot kpos}{C3 \cdot \sqrt{Rbig}}$ $\qquad R3 := Rbig$

$\boxed{Rs := 1}$ $\qquad \boxed{C3 = 0.684997}$ $\qquad \boxed{L3 = 1.821967}$ $\qquad \boxed{R3 = 2.659723}$

Case 4 k_{pos} and R_{small}- **NO**, ***C* & *L*** are complex

$$vcase4 := \begin{pmatrix} Rsmall^2 \\ 0 \\ 16 \cdot ksqrd \cdot Rsmall - \dfrac{8 \cdot kpos \cdot Rsmall^2}{\sqrt{Rsmall}} \\ 0 \\ \dfrac{16 \cdot ksqrd}{Rsmall} \end{pmatrix} \qquad vcase4 = \begin{pmatrix} 0.1414 \\ 0 \\ 0.6191 \\ 0 \\ 11.0187 \end{pmatrix}$$

$$f(C) := Rsmall^2 \cdot C^4 + \left(16 \cdot ksqrd \cdot Rsmall - \frac{8 \cdot kpos \cdot Rsmall^2}{\sqrt{Rsmall}}\right) \cdot C^2 + \frac{16 \cdot ksqrd}{Rsmall}$$

$C := 0.5$ $\qquad C4 := root(f(C), C)$ $\qquad L4 := \dfrac{4 \cdot kpos}{C4 \cdot \sqrt{Rsmall}}$ $\qquad R4 := Rsmall$

$\boxed{Rs = 1}$ $\qquad \boxed{C4 = -1.822 - 2.347i}$ $\qquad \boxed{L4 = -0.685 + 0.882i}$ $\qquad \boxed{R4 = 0.375979}$