A directional coupler has the scattering matrix given below when ports are terminated with matched loads. Find the return loss, coupling factor, directivity, insertion loss, and isolation of the coupler in decibels.

$$[S] = \begin{bmatrix} 0.11\angle 30° & 0.92\angle 90° & 0.3\angle 165° & 0.008\angle 90° \\ 0.92\angle 90° & 0.11\angle 30° & 0.008\angle 90° & 0.3\angle 165° \\ 0.3\angle 165° & 0.008\angle 90° & 0.11\angle 30° & 0.92\angle 90° \\ 0.008\angle 90° & 0.3\angle 165° & 0.92\angle 90° & 0.11\angle 30° \end{bmatrix}$$

Per (2.38), return loss RL = -20 log $|\Gamma|$ = -20 log $|S_{11}|$ = -20 log (0.11)

$\Rightarrow$ **RL = 19.172 dB**

Per (7.20a), coupling factor $C = 10\log(P_1/P_3)$ = -20 log $|S_{31}|$ = -20 log (0.3)

$\Rightarrow$ ***C* = 10.458 dB**

Per (7.20b), directivity $D = 10\log(P_3/P_4) = 20\log|S_{31}|/|S_{41}| = 20\log(0.3/0.008)$

$\Rightarrow$ ***D* = 31.481 dB**

Per (7.20d), insertion loss IL= L = -20 log $|S_{12}|$ = -20 log (0.92) $\Rightarrow$ **IL = 0.724 dB**

Per (7.20c)/(7.21), isolation I = -20 log $|S_{14}|$ = -20 log (0.008) = $D + C$ = 31.481 + 10.458

$\Rightarrow$ ***I* = 41.938 dB**