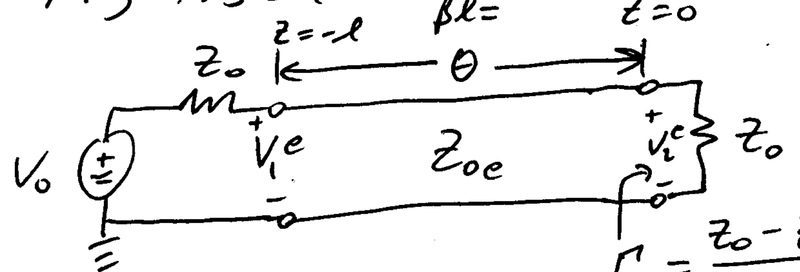
7.24 Derive Equations (7.83) and (7.84).

From Fig 7.32a



$$\Gamma_e = \frac{Z_0 - Z_{0e}}{Z_0 + Z_{0e}}$$

Per (2.69), $V(z) = V_{0e}^+ \left[e^{-j\beta z} + \Gamma_e e^{j\beta z} \right]$.

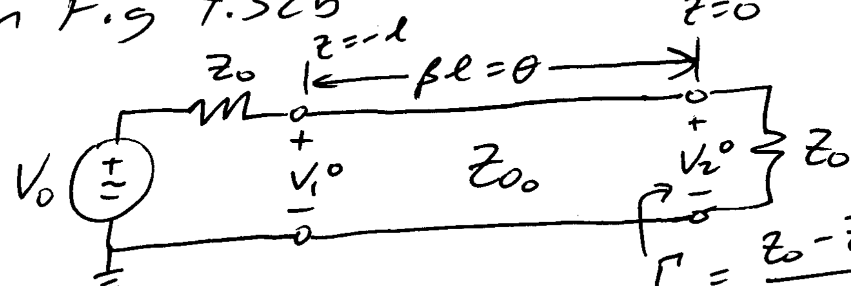
$$V_2^e = V(0) = V_{0e}^+ \left[1 + \Gamma_e \right]$$

$$V_1^e = V(-l) = V_{0e}^+ \left[e^{+j\beta l} + \Gamma_e e^{-j\beta l} \right] = V_{0e}^+ \left[e^{+j\theta} + \Gamma_e e^{-j\theta} \right]$$

$\hookrightarrow V_{0e}^+ = \dfrac{V_1^e}{e^{+j\theta} + \Gamma_e e^{-j\theta}}$ which can be put into the V_2^e expression

$$V_2^e = \frac{V_1^e (1 + \Gamma_e)}{e^{+j\theta} + \Gamma_e e^{-j\theta}} = \frac{V_1^e \left(1 + \frac{Z_0 - Z_{0e}}{Z_0 + Z_{0e}}\right)}{e^{+j\theta} + \frac{Z_0 - Z_{0e}}{Z_0 + Z_{0e}} e^{-j\theta}} \quad (A)$$

From Fig 7.32b



$$\Gamma_o = \frac{Z_0 - Z_{0o}}{Z_0 + Z_{0o}}$$

$$V_2^o = V(0) = V_{0o}^+ (1 + \Gamma_o)$$

$$V_1^o = V(-l) = V_{0o}^+ \left(e^{+j\theta} + \Gamma_o e^{-j\theta} \right)$$

$\hookrightarrow V_{0o}^+ = \dfrac{V_1^o}{e^{+j\theta} + \Gamma_o e^{-j\theta}}$

$$V_2^o = \frac{V_1^o (1 + \Gamma_o)}{e^{+j\theta} + \Gamma_o e^{+j\theta}} = \frac{V_1^o \left(1 + \frac{Z_0 - Z_{0o}}{Z_0 + Z_{0o}}\right)}{e^{+j\theta} + \frac{Z_0 - Z_{0o}}{Z_0 + Z_{0o}} e^{-j\theta}} \quad (B)$$

From earlier, $V_4 = V_4^e + V_4^o = V_2^e - V_2^o$. Use A + B

$$V_4 = \frac{V_1^e\left(1+\frac{Z_0 - Z_{oe}}{Z_0 + Z_{oe}}\right)}{e^{+j\theta} + \frac{Z_0 - Z_{oe}}{Z_0 + Z_{oe}} e^{-j\theta}} - \frac{V_1^o\left(1+\frac{Z_0 - Z_{oo}}{Z_0 + Z_{oo}}\right)}{e^{+j\theta} + \frac{Z_0 - Z_{oo}}{Z_0 + Z_{oo}} e^{-j\theta}}$$

$$= \frac{V_1^e(Z_0 + Z_{oe} + Z_0 - Z_{oe})}{(Z_0 + Z_{oe})e^{+j\theta} + (Z_0 - Z_{oe})e^{-j\theta}} - \frac{V_1^o(Z_0 + Z_{oo} + Z_0 - Z_{oo})}{(Z_0 + Z_{oo})e^{+j\theta} + (Z_0 - Z_{oo})e^{-j\theta}}$$

$$= \frac{V_1^e(2Z_0)}{Z_0(e^{+j\theta} + e^{-j\theta}) + Z_{oe}(e^{+j\theta} - e^{-j\theta})} - \frac{V_1^o(2Z_0)}{Z_0(e^{+j\theta} + e^{-j\theta}) + Z_{oo}(e^{+j\theta} - e^{-j\theta})}$$

$$= \frac{V_1^e(2Z_0)}{Z_0 2\cos\theta + Z_{oe}(+j2\sin\theta)} - \frac{V_1^o(2Z_0)}{Z_0 2\cos\theta + Z_{oo}(+j2\sin\theta)}$$

$$= \frac{V_1^e Z_0}{Z_0\cos\theta + jZ_{oe}\sin\theta} - \frac{V_1^o Z_0}{Z_0\cos\theta + jZ_{oo}\sin\theta}$$

From (7.74a) & after (7.79), $V_1^o = V_0 \frac{Z_{in}^o}{Z_{in}^o + Z_0} = V_0 \frac{Z_0 + jZ_{oo}\tan\theta}{2Z_0 + j(Z_{oe} + Z_{oo})\tan\theta}$ C

From (7.74b) & after (7.79), $V_1^e = V_0 \frac{Z_{in}^e}{Z_{in}^e + Z_0} = V_0 \frac{Z_0 + jZ_{oe}\tan\theta}{2Z_0 + j(Z_{oe} + Z_{oo})\tan\theta}$ D

$$V_4 = \frac{V_0 Z_0 \frac{Z_0 + jZ_{oe}\tan\theta}{2Z_0 + j(Z_{oe}+Z_{oo})\tan\theta}}{Z_0\cos\theta + jZ_{oe}\sin\theta} - \frac{V_0 Z_0 \frac{Z_0 + jZ_{oo}\tan\theta}{2Z_0 + j(Z_{oe}+Z_{oo})\tan\theta}}{Z_0\cos\theta + jZ_{oo}\sin\theta}$$

$$= \frac{V_0 Z_0}{2Z_0 + j(Z_{oe}+Z_{oo})\tan\theta}\left[\frac{Z_0 + jZ_{oe}\tan\theta}{Z_0\cos\theta + jZ_{oe}\sin\theta} - \frac{Z_0 + jZ_{oo}\tan\theta}{Z_0\cos\theta + jZ_{oo}\sin\theta}\right]$$

$$V_4 = \frac{V_0 Z_0 \frac{1}{\cos\theta}}{2Z_0 + j(Z_{0e}+Z_{0o})\tan\theta}\left[\underbrace{\frac{Z_0 + jZ_{0e}\tan\theta}{Z_0 + jZ_{0e}\tan\theta}}_{\to 1} - \underbrace{\frac{Z_0 + jZ_{0o}\tan\theta}{Z_0 + jZ_{0o}\tan\theta}}_{\to 1}\right]$$

$$= \frac{V_0 Z_0 \frac{1}{\cos\theta}}{2Z_0 + j(Z_{0e}+Z_{0o})\tan\theta}\left(\underbrace{1 - 1}_{\to 0}\right)$$

$\underline{\underline{V_4 = 0}}$ $\therefore$ Eq'n (7.83) is confirmed!

Next, $V_2 = V_2^e + V_2^o$. Follow steps to bottom of page 2 to get:

$$V_2 = \frac{V_0 Z_0 \frac{1}{\cos\theta}}{2Z_0 + j(Z_{0e}+Z_{0o})\tan\theta}(1+1)$$

$$= \frac{V_0 Z_0 \frac{1}{\cos\theta}\, 2\left(\frac{1}{Z_{0e}+Z_{0o}}\right)}{\frac{2Z_0}{Z_{0e}+Z_{0o}} + j\tan\theta}$$

Earlier in notes/text (p.354), it was shown that $\frac{2Z_0}{Z_{0e}+Z_{0o}} = \sqrt{1-C^2}$ where $C = \frac{Z_{0e}-Z_{0o}}{Z_{0e}+Z_{0o}}$

So,

$$\underline{\underline{V_2 = \frac{\frac{V_0}{\cos\theta}\sqrt{1-C^2}}{\sqrt{1-C^2} + j\tan\theta} = \frac{V_0\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta}}} \quad \therefore$$

Eq'n (7.84) is confirmed!