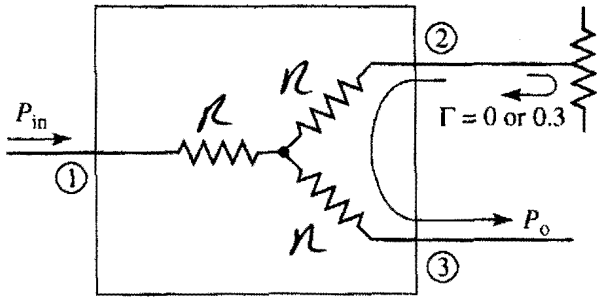
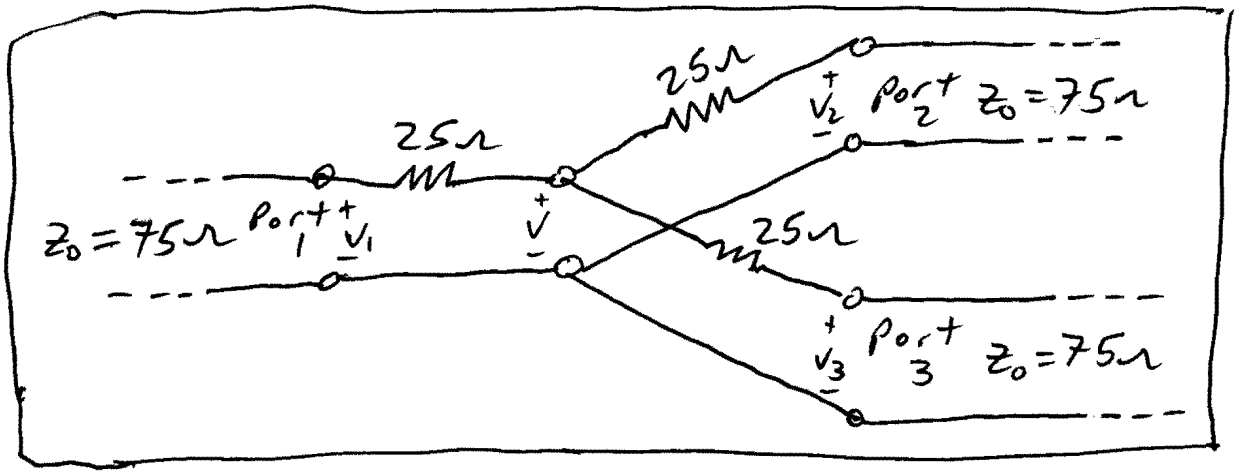
7.8 Design a three-port resistive divider for an equal power split and a 75 Ω system impedance. If port 3 is matched, calculate the change in output power at port 3 (in dB) when port 2 is connected first to a matched load, and then to a load having a mismatch of $\Gamma = 0.3$. See the figure below.



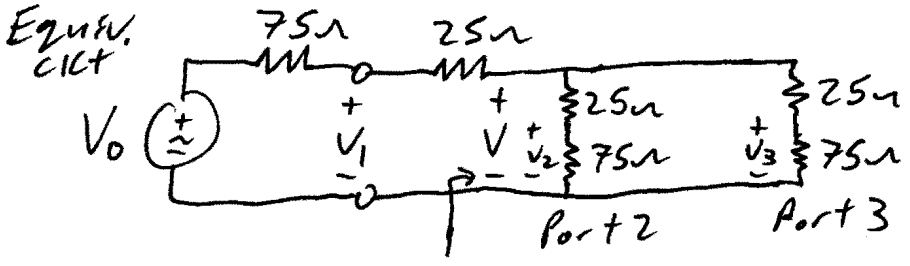
- Change to 75 Ω system impedance. Also, draw labeled sketch of design.

Per Figure 7.7, for an equal-split 3-port resistive power divider

$$R = Z_0/3 = \frac{75}{3} \Rightarrow \underline{R = 25\,\Omega}$$



First, find the power to Port 3 when both Ports 2 & 3 are terminated in matched loads (75Ω) and Port 1 is driven by matched source.



$$Z_{in} = (25+75)\,//\,(25+75) = 50\,\Omega$$

By voltage division, $V = V_0 \frac{50}{75+25+50} = 0.\overline{3}\, V_0$

By voltage division, $V_3 = V \frac{75}{25+75} = 0.\overline{3}\, V_0 (3/4) = 0.25\, V_0$

The power to Port 3 w/ Port 2 also matched is

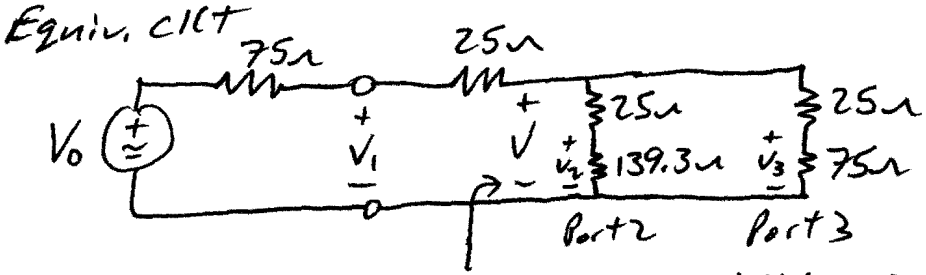
$$P_{3,match} = \tfrac{1}{2} \frac{|V_3|^2}{Z_0} = \frac{0.25^2 |V_0|^2}{2(75)} \Rightarrow \underline{P_{3,mat} = 4.1\overline{6} \times 10^{-4} |V_0|^2}$$

Next, find the power to Port 3 when it is matched, Port 2 has $\Gamma = 0.3$, and Port 1 is driven by a matched source.

Per (2.35), $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow 0.3 = \frac{Z_{L,2} - 75}{Z_{L,2} + 75}$

$$\hookrightarrow Z_{L,2} = \frac{75(1+0.3)}{1-0.3} = 139.2857\,\Omega$$

Equiv. ckt



$Z_{in} = (25+139.3) \,||\, (25+75) = 62.\overline{162}\,\Omega$

$$V = V_0 \frac{62.\overline{162}}{75+25+62.\overline{162}} = 0.38\overline{3}\, V_0$$

$$V_3 = V \frac{75}{25+75} = 0.38\overline{3}\, V_0 (0.75) = 0.2875\, V_0$$

$$P_{3,mis} = \tfrac{1}{2} \frac{0.2875^2 |V_0|^2}{75} = \underline{5.510416\overline{6} \times 10^{-4} |V_0|^2}$$

$$\frac{P_{3,mis}}{P_{3,mat}} = \frac{5.510416\overline{6}}{4.1\overline{6}} \Rightarrow \boxed{\frac{P_{3,mis}}{P_{3,mat}} = 1.3225 = 1.214\,dB \text{ (increase)}}$$