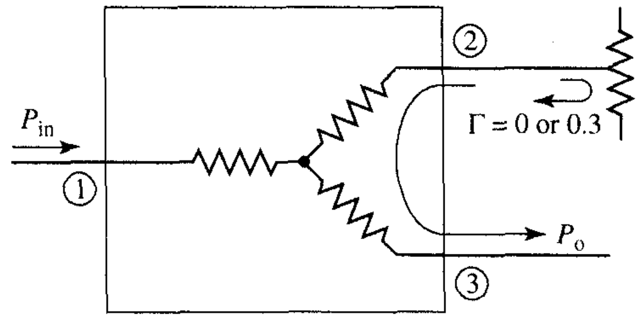
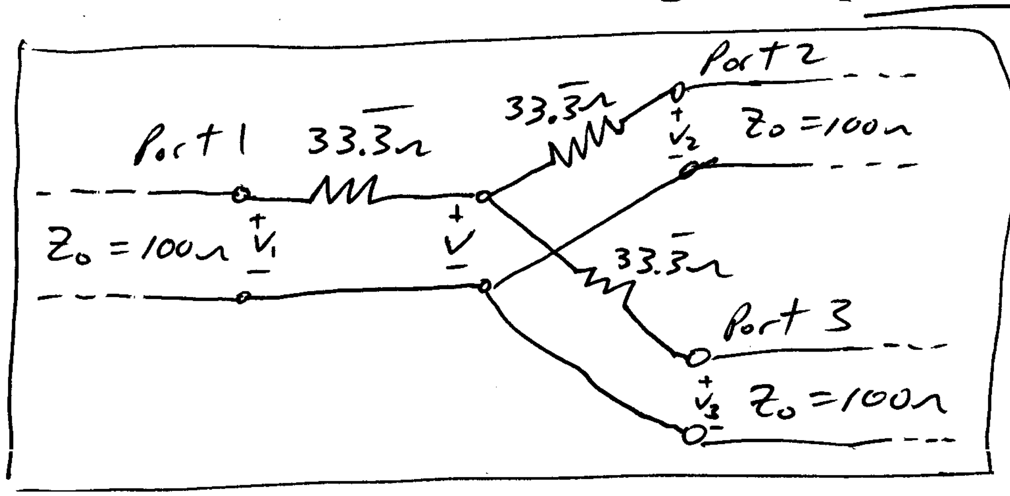
7.8 Design a three-port resistive divider for an equal power split and a 100 Ω system impedance. If port 3 is matched, calculate the change in output power at port 3 (in dB) when port 2 is connected first to a matched load, and then to a load having a mismatch of $\Gamma = 0.3$. See the figure below.



- Also, draw labeled sketch of design.

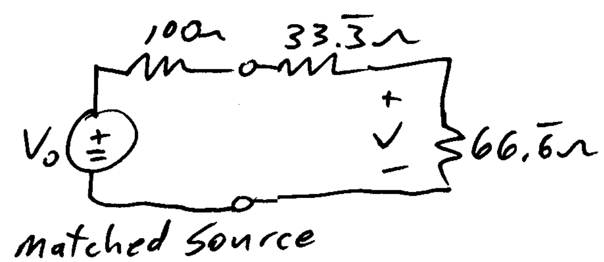
Per Fig. 7.7, the required resistance for an equal split resistive divider is

$$R = Z_0/3 = \frac{100}{3} \Rightarrow R = 33.\bar{3}\,\Omega$$



To calculate the change in power, we can use S-param. or cKt theory.

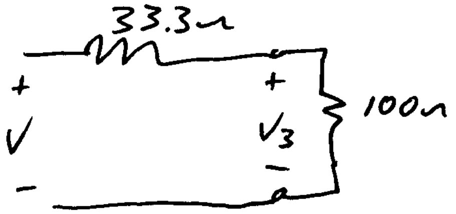
Ports 2 & 3 matched Input V_0 will 'see' a load of $33.\bar{3}\,\Omega$ in series with the parallel combination of $(33.\bar{3}+100)//(33.\bar{3}+100) = 66.\bar{6}\,\Omega$



At the center junction,

$$V = \frac{V_0\, 66.\bar{6}}{100 + 33.\bar{3} + 66.\bar{6}} = 0.\overline{33}\, V_0$$

Looking down to port 3, the voltage V 'sees' $33.\bar{3}\,\Omega$ in series w/ $100\,\Omega$. The voltage into port 3.



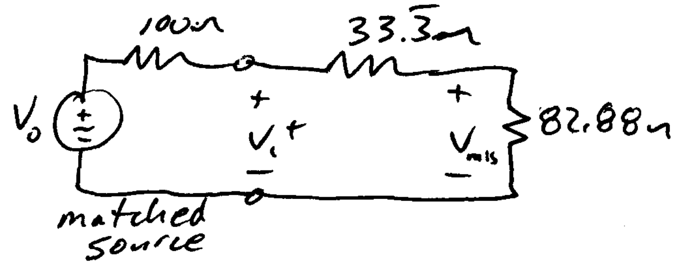
$$V_3 = V\frac{100}{100+33.\bar{3}} = 0.\bar{3}V_1^+\frac{100}{133.\bar{3}} = 0.25V_0$$

$$P_{3,mat} = \frac{1}{2}\frac{|V_3|^2}{Z_0} = \frac{0.25^2|V_0|^2}{200} = \frac{|V_0|^2}{3200}$$

Port 2 w/ $\Gamma = 0.3$ + Port 3 matched

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.3 = \frac{Z_L - 100}{Z_L + 100} \Rightarrow Z_L = \frac{130}{0.7} = 185.7143\,\Omega$$

Now, input V_0 will see a load of $33.\bar{3}\,\Omega$ in series with the parallel combination $(33.\bar{3}+185.71)\,//\,(33.\bar{3}+100) = 82.883\,\Omega$



$$V_{mis} = \frac{V_0\;82.883}{100+33.\bar{3}+82.883} = 0.38\bar{3}\,V_0$$

Looking down to port 3, the voltage V_{mis} 'sees' $33.\bar{3}\,\Omega$ in series w/ $100\,\Omega$. The voltage into port 3 is

$$V_3 = V_{mis}\frac{100}{100+33.\bar{3}} = 0.38\bar{3}\,V_0\frac{100}{133.\bar{3}} = 0.2875\,V_0$$

Now, $P_{3,mis} = \frac{1}{2}\frac{|V_3^+|^2}{Z_0} = \frac{0.2875^2|V_1^+|^2}{200} = \frac{|V_1^+|^2}{2419.66}$

$$\frac{P_{3,mis}}{P_{3,mat}}(\text{dB}) = 10\log_{10}\left[\frac{\frac{|V_1^+|^2}{2419.66}}{\frac{|V_1^+|^2}{3200}}\right] = 10\log_{10}\frac{3200}{2419.66} = 1.214\,\text{dB}$$