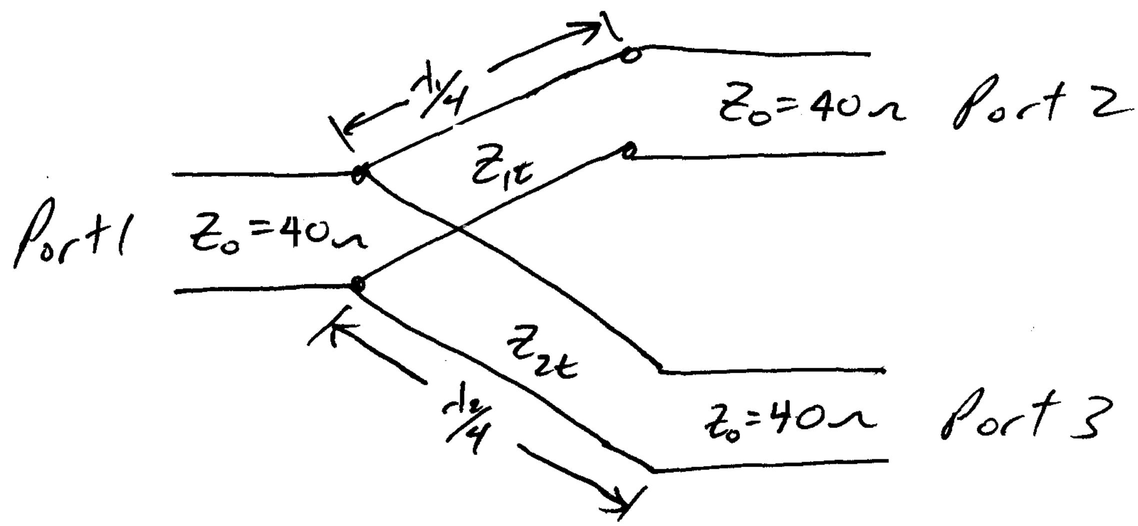
7.6 Design a lossless T-junction divider with a **40** Ω source impedance to give a 3:1 power split. Design quarter-wave matching transformers to convert the impedances of the output lines to **40** Ω. Determine the magnitude of the scattering parameters for this circuit, using a **40** Ω characteristic impedance.

- Change to 40 Ω impedances. Also, draw labeled sketch of design. **EE 481:** Design only. **EE 581:** Design and [*S*]-parameter magnitudes.



Per class notes: $Z_1 = \dfrac{Z_0}{P_2/P_{in}}$ + $Z_2 = \dfrac{Z_0}{P_3/P_{in}}$

From problem description, $P_2/P_{in} = 3/4$ + $\dfrac{P_3}{P_{in}} = 1/4$.

Therefore, $Z_1 = \dfrac{40}{3/4} = \underline{53.\bar{3}\,\Omega}$

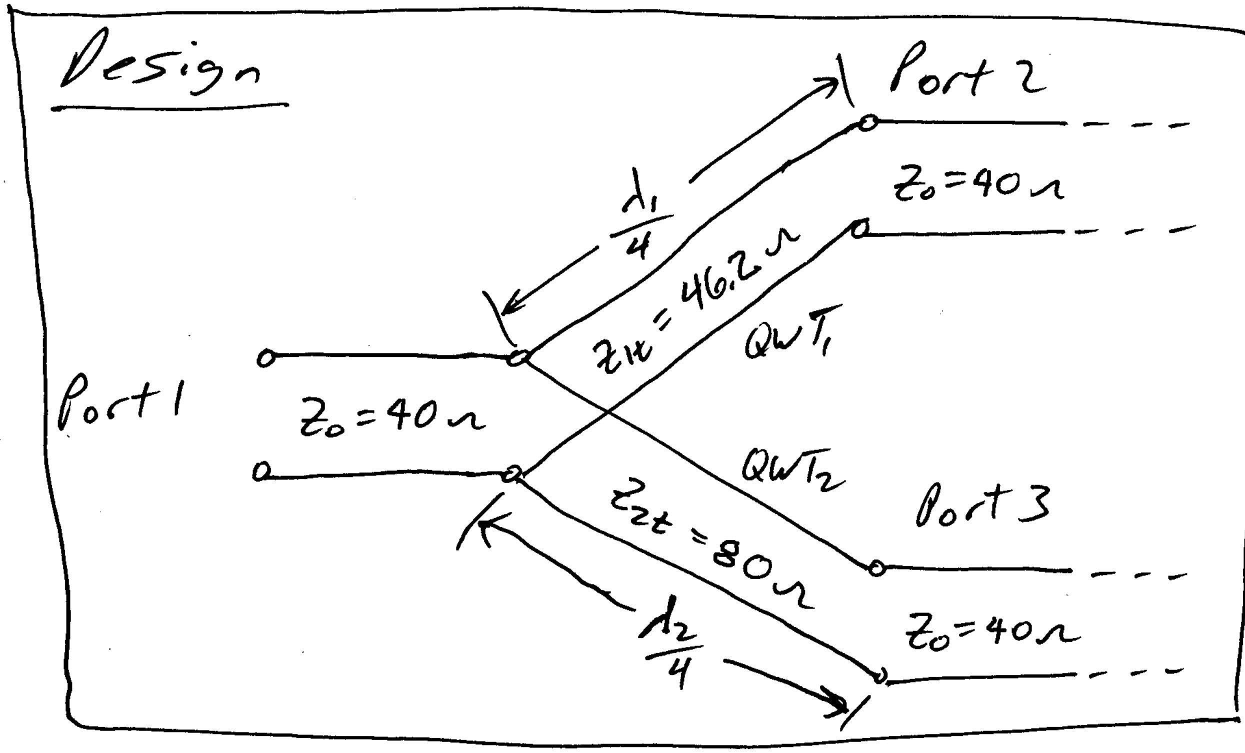
$$Z_2 = \frac{40}{1/4} = \underline{160\,\Omega}$$

$$\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{53.\bar{3}} + \frac{1}{160} = \frac{1}{40} \quad \underline{\text{ok!}}$$

Treating Z_1 + Z_2 as the input impedances of QWTs terminated in $Z_0 = 40\Omega$ loads, we can use (2.63) to find

$$Z_{1t} = \sqrt{Z_0 Z_1} = \sqrt{40(53.\bar{3})} \Rightarrow \underline{\underline{Z_{1t} = 46.188\,\Omega}}$$

$$Z_{2t} = \sqrt{Z_0 Z_2} = \sqrt{40(160)} \Rightarrow \underline{\underline{Z_{2t} = 80\,\Omega}}$$



With ports 2 & 3 terminated in matched loads,

$$S_{11} = \Gamma_{11} = \frac{Z_{in,1} - Z_0}{Z_{in,1} + Z_0} = \frac{40-40}{40+40} \Rightarrow \underline{\underline{|S_{11}| = 0}}$$

With ports 1 & 3 terminated in $Z_0 = 40\,\Omega$, the load at the end of the QWT_1 is

$$Z_0 \| Z_2 = 40 \| 160 = 32\,\Omega.$$

Per (2.62), the input impedance at port 2 is

$$Z_{in,2} = \frac{Z_{1t}^2}{R_L} = \frac{46.188^2}{32} = 66.\bar{6}\,\Omega$$

Therefore, $S_{22} = \dfrac{Z_{in,2} - Z_0}{Z_{in,2} - Z_0} = \dfrac{66.\bar{6} - 40}{66.\bar{6} + 40} = 0.25$

$$\Rightarrow \underline{\underline{|S_{22}| = 0.25}}$$

With ports 1 & 2 terminated in $Z_0 = 40\Omega$, the load at the end of QWT_2 is

$$Z_0 \parallel Z_1 = 40 \parallel 53.\bar{3} = 22.857\ \Omega$$

Per (2.62), $Z_{in,3} = \dfrac{Z_{2t}^2}{R_L} = \dfrac{80^2}{22.86} = 280\Omega$

Therefore, $S_{33} = \dfrac{Z_{in,3} - Z_0}{Z_{in,3} + Z_0} = \dfrac{280 - 40}{280 + 40} = 0.75$

$$\underline{\underline{|S_{33}| = 0.75}}$$

For a 3:1 power split to ports 2 & 3, we must have $|S_{21}| = |S_{12}| = \sqrt{3/4} = \underline{\underline{0.866}}$

and $|S_{31}| = |S_{13}| = \sqrt{1/4} = \underline{\underline{0.5}}$

In order to satisfy (4.51) $[S]^t [S]^* = [U]$,

$$\begin{bmatrix} S_{11} & S_{21} & S_{31} \\ S_{12} & S_{22} & S_{32} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{21}^* & S_{22}^* & S_{23}^* \\ S_{31}^* & S_{32}^* & S_{33}^* \end{bmatrix} = \begin{bmatrix} U \end{bmatrix}$$

Therefore (2nd row x 2nd column), we get

$$|S_{12}|^2 + |S_{22}|^2 + |S_{32}|^2 = 1$$

$$0.866^2 + 0.25^2 + |S_{32}|^2 = 1$$

$$\hookrightarrow \underline{\underline{|S_{32}| = |S_{23}| = 0.433}}$$