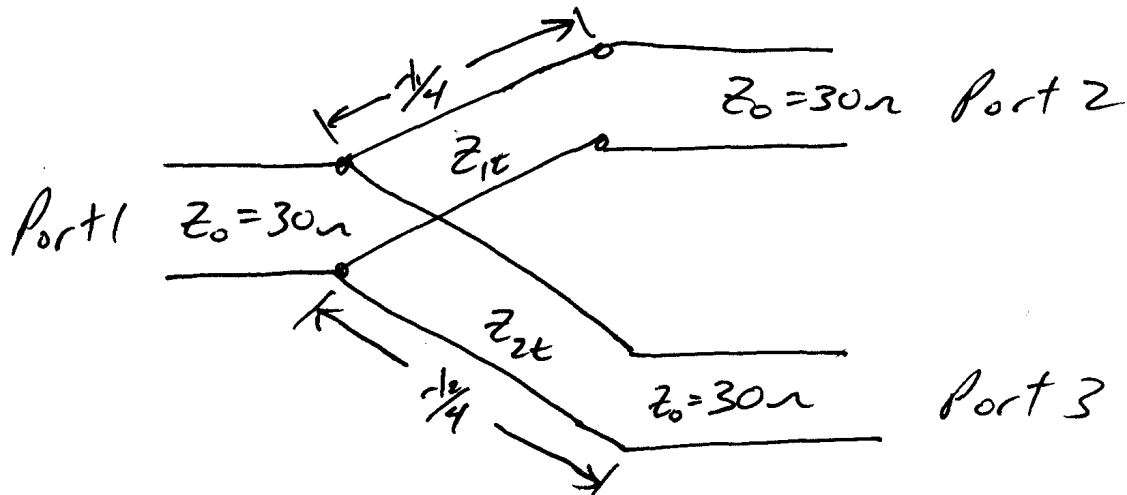


7.6 Design a lossless T-junction divider with a $30\ \Omega$ source impedance to give a 3:1 power split. Design quarter-wave matching transformers to convert the impedances of the output lines to $30\ \Omega$. Determine the magnitude of the scattering parameters for this circuit, using a $30\ \Omega$ characteristic impedance.

- Draw labeled sketch of design. EE 481: Design only. EE 581: Design and [S]-parameter magnitudes.



Per class notes: $Z_1 = \frac{Z_0}{P_2/P_{in}} + Z_2 = \frac{Z_0}{P_3/P_{in}}$

From problem description, $P_2/P_{in} = 3/4 + \frac{P_3}{P_{in}} = 1/4$.

Therefore, $Z_1 = \frac{30}{3/4} = \underline{40\ \Omega}$

$Z_2 = \frac{30}{1/4} = \underline{120\ \Omega}$

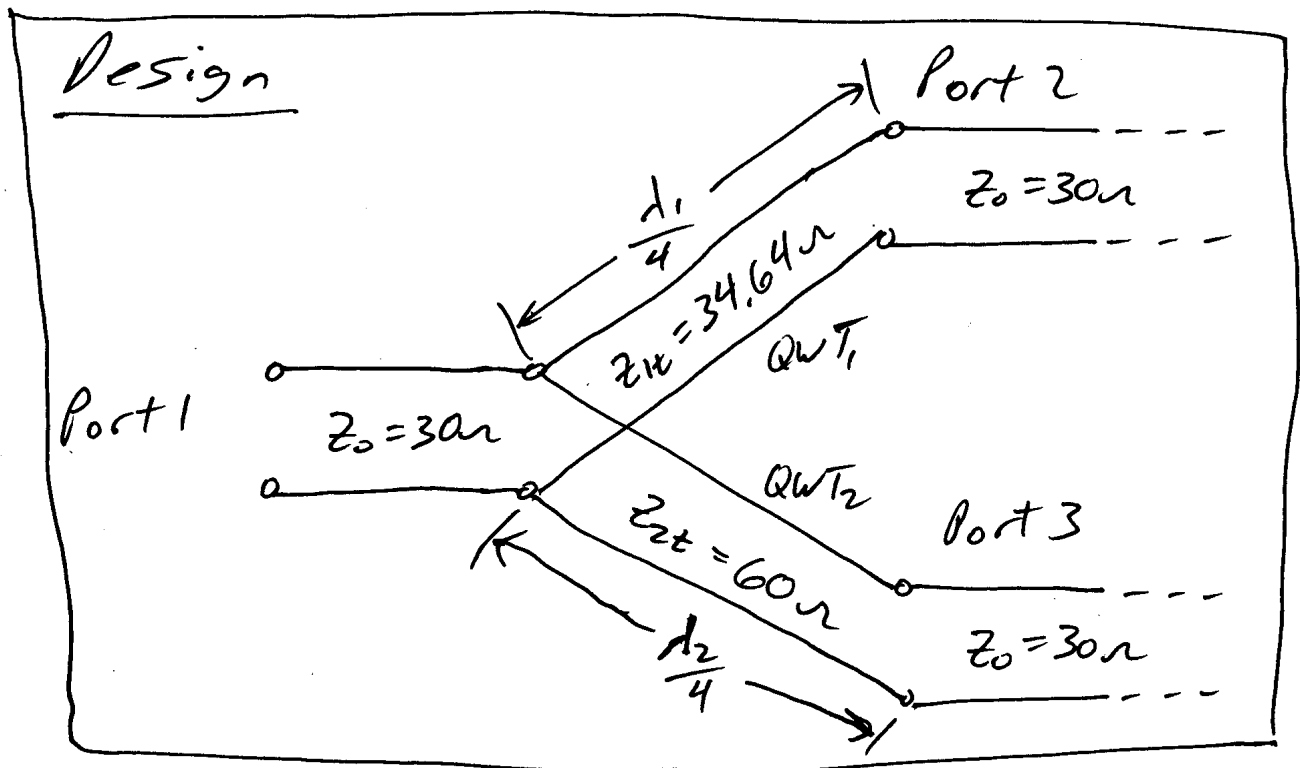
$\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{40} + \frac{1}{120} = \frac{1}{30}$ OK!

Treating Z_1 & Z_2 as the input impedances of QWTs terminated in $Z_0 = 30\ \Omega$ loads, we can use (2.63) to find

$Z_{1t} = \sqrt{Z_0 Z_1} = \sqrt{30(40)}$

$Z_{1t} = \underline{\underline{34.641\ \Omega}}$

$$Z_{2t} = \sqrt{Z_0 Z_2} = \sqrt{30(120)} \Rightarrow \underline{\underline{Z_{2t} = 60\Omega}}$$



With ports 2 & 3 terminated in matched loads,

$$S_{11} = \Gamma_{11} = \frac{Z_{in,1} - Z_0}{Z_{in,1} + Z_0} = \frac{30 - 30}{30 + 30} \Rightarrow \underline{\underline{|S_{11}| = 0}}$$

With ports 1 & 3 terminated in $Z_0 = 30\Omega$, the load at the end of the QWT_1 is

$$Z_0 \parallel Z_2 = 30 \parallel 120 = 24\Omega.$$

Per (2.62), the input impedance at port 2 is

$$Z_{in,2} = \frac{Z_{1t}^2}{24} = \frac{34.641^2}{24} = 50\Omega$$

$$\text{Therefore, } S_{22} = \frac{Z_{in,2} - Z_0}{Z_{in,2} + Z_0} = \frac{50 - 30}{50 + 30} = 0.25$$

$$\Rightarrow \underline{\underline{|S_{22}| = 0.25}}$$

With ports 1 & 2 terminated in $z_0 = 30\Omega$, the load at the end of QWT₂ is

$$z_0 \parallel z_1 = 30 \parallel 40 = 17.143\Omega$$

$$\text{Per (2.62), } z_{in,3} = \frac{z_{2t}^2}{17.143} = \frac{60^2}{17.143} = 210\Omega$$

$$\text{Therefore, } S_{33} = \frac{z_{in,3} - z_0}{z_{in,3} + z_0} = \frac{210 - 30}{210 + 30} = 0.75$$

$$\underline{|S_{33}| = 0.75}$$

For a 3:1 power split to ports 2 & 3, we must have $|S_{21}| = |S_{12}| = \sqrt{3/4} = \underline{0.866}$

$$\text{and } |S_{31}| = |S_{13}| = \sqrt{1/4} = \underline{0.5}$$

In order to satisfy (4.51) $[S]^t [S]^* = [I]$,

$$\begin{bmatrix} S_{11} & S_{21} & S_{31} \\ S_{12} & S_{22} & S_{32} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{21}^* & S_{22}^* & S_{23}^* \\ S_{31}^* & S_{32}^* & S_{33}^* \end{bmatrix} = [I]$$

Therefore (2nd row x 2nd column), we get

$$|S_{12}|^2 + |S_{22}|^2 + |S_{32}|^2 = 1$$

$$0.866^2 + 0.25^2 + |S_{32}|^2 = 1$$

$$\hookrightarrow \underline{|S_{32}| = |S_{23}| = 0.433}$$