7.3 A directional coupler has the scattering matrix given below. Find the return loss, coupling factor, directivity, and insertion loss. Assume that the ports are terminated in matched loads.

$$[S] = \begin{bmatrix} 0.1\angle 40^\circ & 0.944\angle 90^\circ & 0.178\angle 180^\circ & 0.0056\angle 90^\circ \\ 0.944\angle 90^\circ & 0.1\angle 40^\circ & 0.0056\angle 90^\circ & 0.178\angle 180^\circ \\ 0.178\angle 180^\circ & 0.0056\angle 90^\circ & 0.1\angle 40^\circ & 0.944\angle 90^\circ \\ 0.0056\angle 90^\circ & 0.178\angle 180^\circ & 0.944\angle 90^\circ & 0.1\angle 40^\circ \end{bmatrix}$$

➢ Also, find the isolation of the coupler (dB).

Per (2.38), the return loss is

$$RL = -20\log|\Gamma| = -20\log_{10}|S_{11}| = -20\log_{10}|0.1\angle 40^\circ|$$

$$\Rightarrow \underline{\underline{RL = 20\,dB}}$$

Per (7.20a), the coupling factor is

$$C = 10\log\frac{P_1}{P_3} = -20\log\beta = -20\log_{10}|S_{31}| = -20\log|0.178\angle 180^\circ|$$

$$\Rightarrow \underline{\underline{C = 14.9916\,dB}}$$

Per (7.20b), the directivity is

$$D = 10\log\frac{P_3}{P_4} = 20\log\frac{\beta}{|S_{14}|} = 20\log\frac{|S_{31}|}{|S_{41}|}$$

$$= 20\log\frac{|0.178\angle 180^\circ|}{|0.0056\angle 90^\circ|} = 20\log\frac{0.178}{0.0056}$$

$$\Rightarrow \underline{\underline{D = 30.0446\,dB}}$$

Per (7.20d), the insertion loss is

$$IL = L = 10\log P_1/P_2 = -20\log|S_{12}|$$

$$= -20\log|0.944\angle 90^\circ| \Rightarrow \underline{\underline{L = 0.5006\,dB}}$$

Isolation (7.20c)

$$I = 10\log P_1/P_4 = -20\log|S_{14}| = -20\log 0.0056 \Rightarrow \underline{\underline{I = 45.04\,dB}}$$