A lossless transmission line (300 Ω, 2×10^8 m/s) of length 125 cm connects a load of 180 + j240 Ω to a matched 16 V generator operating at 500 MHz. Using a Smith chart, find the unmatched input impedance and load power. Then, design and sketch a shunt double-stub tuning network with short circuit terminations. Place the stubs at λ/8 intervals and make the stubs as short as possible. Find the matched input impedance and load power.

Calculate $\lambda = v_p / f = 2 \times 10^8 / 500 \times 10^6$ ⇒ **λ = 0.4 m = 40 cm**.

Calculate $d_L = d = \lambda/8 = (0.4)/8$ ⇒ **$d_L = d = 0.05$ m = 5 cm**.

1) Calculate the normalized load impedance $z_L' = Z_L' / Z_0 = (180 + j240)/300$ and admittance $y_L' = Z_0 / Z_L' = 300/(180 + j240)$. Plot **$z_L' = 0.6 + j0.8$ Ω/Ω** and **$y_L' = 0.6 - j0.8$ S/S** on the Smith chart.

2) Calculate TL length $\ell/\lambda = 125/40 = 3.125 \Rightarrow 0.125$. Move 0.125λ on an arc/circle of constant radius |Γ| in the "WAVELENGTHS TOWARD GENERATOR" direction (CW) from z_L' to $z_{\text{in,NM}} = r_{\max} = 3$ Ω/Ω. Calculate the unmatched input impedance as $Z_{\text{in,NM}} = z_{\text{in,NM}} Z_0 = 3(300)$ ⇒ **$Z_{\text{in,NM}} = 900$ Ω**.

3) Calculate the unmatched phasor input current and voltage as

$$\bar{I}_{\text{in,NM}} = \frac{\bar{V}_g}{Z_g + Z_{\text{in,NM}}} = \frac{16\angle 0°}{300 + 900} \Rightarrow \bar{I}_{\text{in,NM}} = 0.0133\bar{3}\angle 0° \text{ A} \quad \text{and}$$

$$\bar{V}_{\text{in,NM}} = \bar{V}_g \frac{Z_{\text{in,NM}}}{Z_g + Z_{\text{in,NM}}} = (16\angle 0°)\frac{900}{300 + 900} \Rightarrow \bar{V}_{\text{in,NM}} = 12\angle 0° \text{ V}.$$

For a lossless TL, the input power and load power are the same.

$P_{L,\text{NM}} = P_{\text{in,NM}} = 0.5\,\text{Re}\{V_{\text{in,NM}}\, I_{\text{in,NM}}^*\} = 0.5\,\text{Re}\{12(0.0133)\}$ ⇒ **$P_{L,\text{NM}} = 0.08$ W = 80 mW.**

4) Draw the rotated 'match' circle $g = 1$ at $d/\lambda = \lambda/8$ in the "WAVELENGTHS TOWARD LOAD" direction, i.e., rotate 90° CCW. Note that **$g_0 = 2$ S/S.**

5) Move the selected distance d_L/λ on an arc/circle of constant radius |Γ| in the "WAVELENGTHS TOWARD GENERATOR" direction (CW) from y_L' to y_L. Plot **y_L = 0.33 S/S** on the Smith chart. Note that **$g_L < g_0$** (good).

6) From y_L, move along circle of constant $g_L = 0.33$ S/S to where it intersects the rotated g = 1 circle. There are two match points, **$y_{1a} = 0.33 + j0.255$ S/S** or $y_{1b} = 0.33 + j1.75$ S/S. To minimize the length of the first stub select **y_{1a}**. The normalized stub 1 susceptance b_{1a} = 0.255 - 0 ⇒ **$b_{1a} = 0.255$ S/S**.

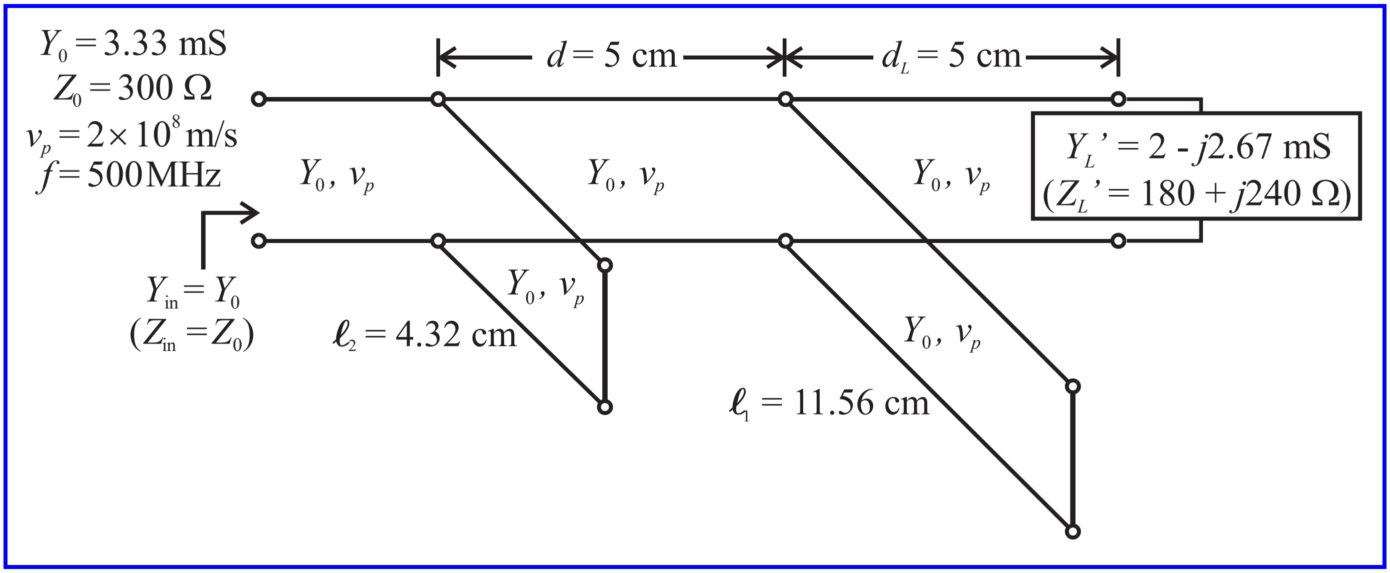
- For the desired $j0.255$ S/S, the length of a short-circuit terminated ($y_{SC} \rightarrow \infty$) stub 1 is $\ell_1 = 0.25\lambda + 0.039\lambda = 0.289(40\text{ cm}) \Rightarrow$ **$\ell_1 \approx 11.56$ cm**.

7) Next, we will move the selected distance $d/\lambda = \lambda/8 = 0.125\lambda$ on an arc/circle of constant radius $|\Gamma|$ in the "WAVELENGTHS TOWARD GENERATOR" direction (CW) from y_{1a} to y_{1at}. This places us at $\boldsymbol{y_{1at} = 1 - jb_2 = 1 + j1.24}$ **S/S** on the $g = 1$ 'match' circle.

8) Complete the match by adding shunt stub 2 with a normalized susceptance of $\boldsymbol{jb_2 = -j1.24}$ **S/S**. This yields $y_2 = y_{1at} + y_{stub2} = (1 + j1.24) - j1.24 \Rightarrow \boldsymbol{y_2 = 1}$ **S/S**.

   - For the desired $-j1.24$ S/S, the length of a short-circuit terminated ($y_{SC} \rightarrow \infty$) stub 2 is $\ell_2 = 0.358\lambda - 0.25\lambda = 0.108(40\text{ cm}) \Rightarrow$ **$\ell_2 \approx 4.32$ cm**.

Final Design



9) By definition the matched input impedance is $\Rightarrow$ **$Z_{in,M} = Z_0 = 300\ \Omega$**.

10) Calculate the matched phasor input current and voltage as

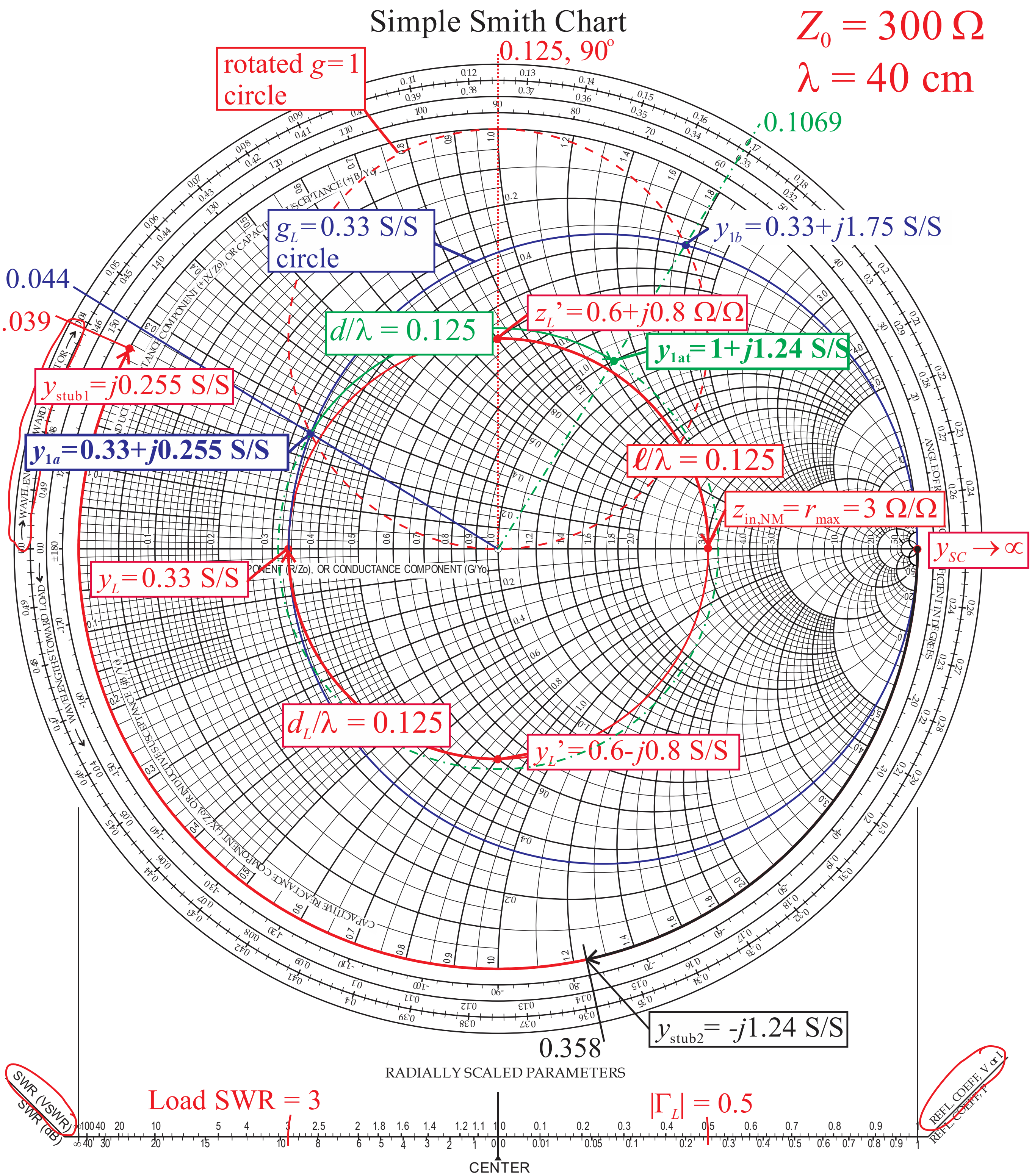
$$\bar{I}_{in,M} = \frac{\bar{V}_g}{Z_g + Z_{in,M}} = \frac{16\angle 0°}{300+300} \Rightarrow \bar{I}_{in,M} = 0.02\overline{6}\angle 0°\text{ A} \quad \text{and}$$

$$\bar{V}_{in,NM} = \bar{V}_g \frac{Z_{in,M}}{Z_g + Z_{in,M}} = (16\angle 0°)\frac{300}{300+300} \Rightarrow \bar{V}_{in,M} = 8\angle 0°\text{ V}.$$

For a lossless TL, the input power and load power are the same.

$P_{L,M} = P_{in,M} = 0.5\,\text{Re}\{V_{in,M}\, I_{in,M}{}^*\} = 0.5\,\text{Re}\{8(0.0267)\} \Rightarrow$ **$P_{L,M} = 0.1067$ W $= 106.7$ mW.**

This is 33.3% higher than the unmatched case!


Simple Smith Chart
Z0 = 300 Ω
λ = 40 cm
0.125, 90°
rotated g=1 circle
0.1069
gL=0.33 S/S circle
y1b=0.33+j1.75 S/S
0.044
0.039
zL'=0.6+j0.8 Ω/Ω
d/λ = 0.125
y1at=1+j1.24 S/S
ystub1=j0.255 S/S
y1a=0.33+j0.255 S/S
ℓ/λ = 0.125
zin,NM=rmax=3 Ω/Ω
yL=0.33 S/S
ySC →∞
dL/λ = 0.125
yL'=0.6-j0.8 S/S
ystub2= -j1.24 S/S
0.358
RADIALLY SCALED PARAMETERS
Load SWR = 3
|ΓL| = 0.5
SWR (VSWR)
SWR (dB)
REFL. COEFF. V or I
REFL. COEFF. P
CENTER