

A lossless transmission line (40Ω , $2.6 \times 10^8 \text{ m/s}$) has a load $Z_L = 26 + j18 \Omega$ at 1.04 GHz. Design and sketch a quarter-wave transformer (QWT), using a Smith chart, to match the load with the QWT placed as close as possible to the load. Assume that the QWT section has a phase velocity of $2.5 \times 10^8 \text{ m/s}$.

- Calculate wavelength $\lambda = v_p / f = 2.6 \times 10^8 / 1.04 \times 10^9 \Rightarrow \lambda = 0.25 \text{ m} = 25 \text{ cm}$.
- Calculate the normalized load impedance $z_L = Z_L / Z_0 = (26 + j18) / 40$
 $\Rightarrow z_L = 0.65 + j0.45 \Omega/\Omega$. Plot z_L on Smith chart.
- Use compass to draw a circle through z_L , centered on Smith chart. Use a straight edge to draw line through center of Smith chart & z_L to outer rings of Smith chart. Note $|\Gamma| = 0.5$ and SWR = 3.
- Note circle intersects the horizontal axis (real impedances) at $z_{\max} = 2 \Omega/\Omega$ (closest) and $z_{\min} = 0.5 \Omega/\Omega$. Then, $Z_{\max} = z_{\max} Z_0 = 2(40) = 80 \Omega/\Omega$.
- The distance from z_L to z_{\max} along the circle of constant $|\Gamma| = 0.5$ in the “WAVELENGTHS TOWARD GENERATOR” direction is $\ell_{\max} / \lambda = 0.25 - 0.0937$
 $\Rightarrow \boxed{\ell_{\max} = 0.1563\lambda = 3.91 \text{ cm}}$.
- Use (5.25) $Z_1 = \sqrt{Z_0 Z_L} = \sqrt{40(80)}$ to calculate the impedance of the QWT as $\Rightarrow \boxed{Z_1 = 56.57 \Omega}$.
- Calculate wavelength of QWT as $\lambda_1 = v_p' / f = 2.5 \times 10^8 / 1.04 \times 10^9 \Rightarrow \lambda_1 = 0.240385 \text{ m} = 24.04 \text{ cm}$. This yields at QWT length of $\Rightarrow \boxed{\lambda_1 / 4 = 6.01 \text{ cm}}$.



