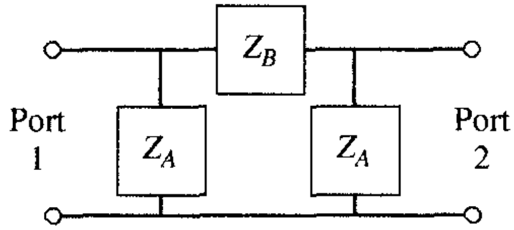
Evaluate [S] for the pi-network of problem 4.7a when $Z_A = -j50\ \Omega$, $Z_B = 100 + j40\ \Omega$, and $Z_0 = 50\ \Omega$.



(a)

Method 1

Use equations from prior problem with $Z_A = -j50\ \Omega$ and $Z_B = 100 + j40\ \Omega$.

$ZA := -j\cdot 50 \quad \Omega \qquad ZB := 100 + j\cdot 40 \quad \Omega \qquad Z0 := 50 \quad \Omega$

$$S11 := \frac{ZA^2\cdot ZB - ZB\cdot Z0^2 - 2\cdot ZA\cdot Z0^2}{ZA^2\cdot ZB + 2\cdot ZA\cdot ZB\cdot Z0 + 2\cdot ZA^2\cdot Z0 + ZB\cdot Z0^2 + 2\cdot ZA\cdot Z0^2}$$

$$S21 := \left[\frac{2\cdot\left(ZA^2\cdot ZB + ZA\cdot ZB\cdot Z0 + ZA^2\cdot Z0\right)}{ZA^2\cdot ZB + 2\cdot ZA\cdot ZB\cdot Z0 + 2\cdot ZA^2\cdot Z0 + ZB\cdot Z0^2 + 2\cdot ZA\cdot Z0^2}\right]\cdot\left(\frac{ZA\cdot Z0}{ZA\cdot ZB + ZA\cdot Z0 + ZB\cdot Z0}\right)$$

$S12 := S21 \qquad S22 := S11$

$\boxed{|S11| = 0.66851} \qquad \boxed{\arg(S11)\cdot\frac{180}{\pi} = -91.89652}$ deg

$\boxed{|S12| = 0.3326} \qquad \boxed{\arg(S12)\cdot\frac{180}{\pi} = -86.18593}$ deg

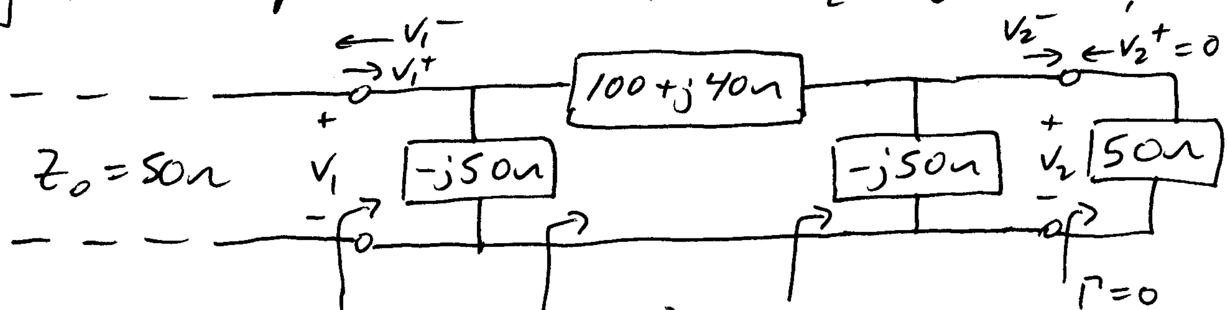
$\boxed{|S21| = 0.3326} \qquad \boxed{\arg(S21)\cdot\frac{180}{\pi} = -86.18593}$ deg

$\boxed{|S22| = 0.66851} \qquad \boxed{\arg(S22)\cdot\frac{180}{\pi} = -91.89652}$ deg

$$\boxed{[S] = \begin{bmatrix} 0.6685\angle -91.897^\circ & 0.3326\angle -86.186^\circ \\ 0.3326\angle -86.186^\circ & 0.6685\angle -91.897^\circ \end{bmatrix}}$$

Method 2 Direct calculation

$\boxed{S_{11}}$ Match port 2 with load $Z_L = Z_0 = 50\Omega$,



$Z_{eq_1} = 50 \| -j50 = 25 - j25\,\Omega$

$Z_{eq_2} = (100+j40) + (25-j25) = 125 + j15\,\Omega$

$Z_{in} = -j50 \| (125+j15) = 18.546 - j44.807\,\Omega$

$$S_{11} = \Gamma_{11} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{(18.546 - j44.807) - 50}{(18.546 - j44.807) + 50}$$

$$\underline{\underline{S_{11} = 0.668508 \angle -91.89652^\circ = S_{22} \text{ (symmetry)}}}$$

$\boxed{S_{21}}$ Same Circuit. Find $V_2^- = V_2$ (voltage across $Z_L = Z_0$) in terms of V_1^+.

By voltage division $\quad V_2 = V_2^- = V_1 \dfrac{Z_{eq_1}}{Z_{eq_2}} = V_1^+(1+\Gamma_{11}) \dfrac{Z_{eq_1}}{Z_{eq_2}}$

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0} = (1+\Gamma_{11}) \frac{Z_{eq_1}}{Z_{eq_2}}$$

$$= \left(1 + 0.668508 \angle -91.89652^\circ\right)\left(\frac{25-j25}{125+j15}\right)$$

$$\underline{\underline{S_{21} = 0.332595 \angle -86.185925^\circ = S_{12} \text{ (symmetry)}}}$$