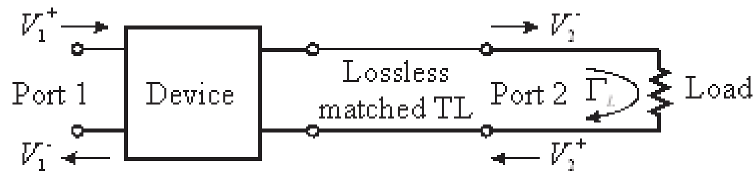
A 2-port device, characterized by $[S_{\text{dev}}]$, is connected by a lossless matched transmission line (TL) to load with a reflection coefficient Γ_L as shown. a) Find the S-parameters of the TL $[S_{\text{TL}}]$ if its electrical length is 50°. b) Create a signal flow graph (SFG) for this problem. c) Find the reflection coefficient $\Gamma_1 = \dfrac{V_1^-}{V_1^+}$ as well as the return loss at port 1.

$$[S_{\text{dev}}] = \begin{bmatrix} 0.2\angle 40^\circ & 0.8\angle -30^\circ \\ 0.8\angle -30^\circ & 0.1\angle -20^\circ \end{bmatrix}$$

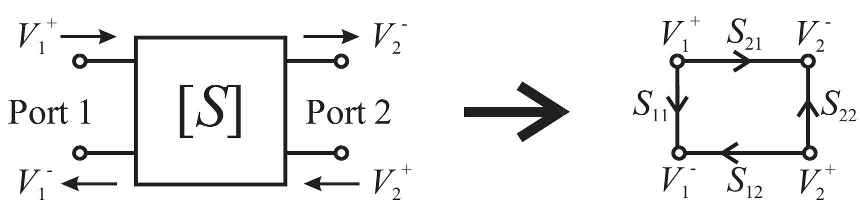


a) From earlier problem, the S-matrix for a lossless matched TL w/ electrical length $\beta\ell$ is

$$[S_{\text{TL}}] = \begin{bmatrix} 0 & e^{-j\beta\ell} \\ e^{-j\beta\ell} & 0 \end{bmatrix} = \begin{bmatrix} 0 & e^{-j50(\pi/180)} \\ e^{-j50(\pi/180)} & 0 \end{bmatrix} \Rightarrow$$

$$\boxed{[S_{\text{TL}}] = \begin{bmatrix} 0 & 0.643 - j0.766 \\ 0.643 - j0.766 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1\angle -50^\circ \\ 1\angle -50^\circ & 0 \end{bmatrix}}$$

b) Per Fig 4.14, we can get the SFG for two-port devices in terms of the S-parameters as



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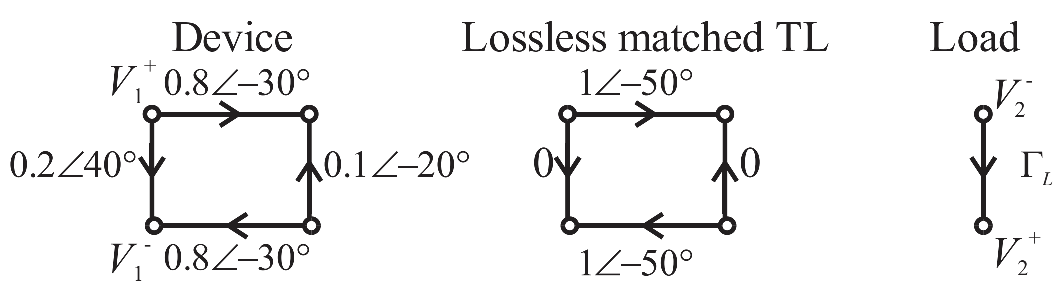
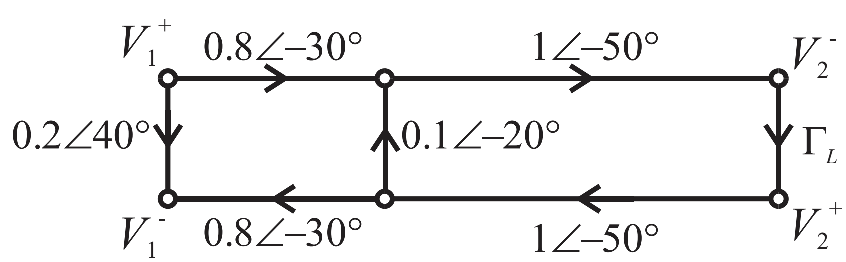
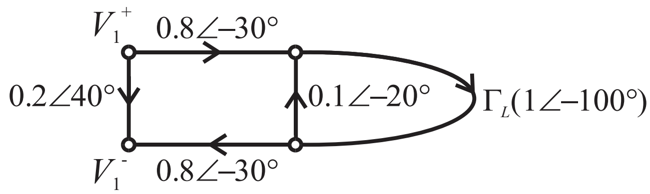
Per Fig 4.14a, we can get the SFG for a one-port device (load). Then, we get for our three devices-



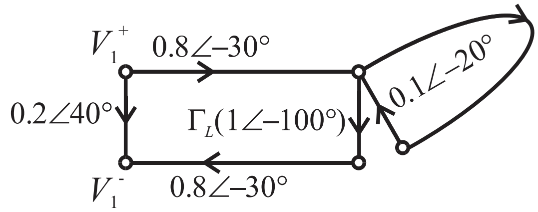
, and a SFG



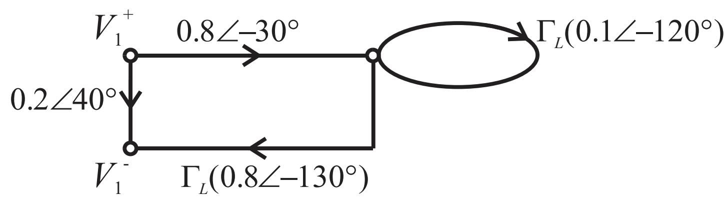
c) **<u>Step 1</u>** Use series rule on the three branches on the right to get



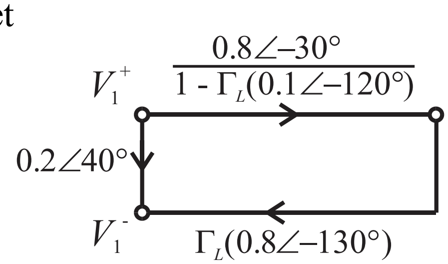
<u>Step 3</u> Split the bottom right node to create a self loop



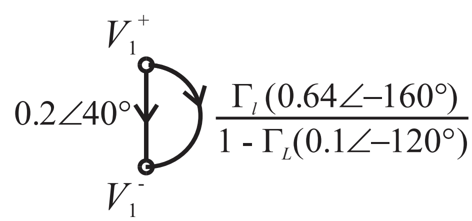
<u>Step 4</u> Use series rule on the self loop <u>and</u> the bottom two branches to get



<u>Step 5</u> Use self loop rule to get



<u>Step 6</u> Use series rule on the right two branches to get



By parallel rule we get $V_1^- = \left[0.2\angle40° + \dfrac{\Gamma_L(0.64\angle-160°)}{1-\Gamma_L(0.1\angle-120°)}\right]V_1^+ \Rightarrow$

$$\boxed{\Gamma_1 = \frac{V_1^-}{V_1^+} = 0.2\angle40° + \frac{\Gamma_L(0.64\angle-160°)}{1-\Gamma_L(0.1\angle-120°)}}$$

The return loss (2.38) is $\mathrm{RL} = -20\log|\Gamma_1|$

$$\Rightarrow \boxed{\mathrm{RL} = -20\log\left|0.2\angle40° + \frac{\Gamma_L(0.64\angle-160°)}{1-\Gamma_L(0.1\angle-120°)}\right| \text{ (dB)}}.$$