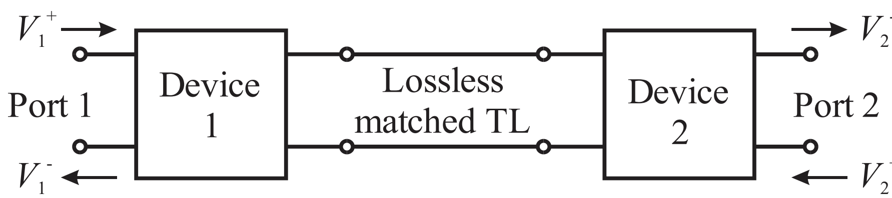
Two 2-port devices, characterized by [S_{dev1}] and [S_{dev2}], are connected by a lossless matched transmission line (TL) as shown. a) Find the S-parameters of the TL [S_{TL}] if its electrical length is 120°. b) Create a signal flow graph (SFG) for this problem. c) Assuming port 2 is matched, use the SFG to find the transmission coefficient $T_{21} = \dfrac{V_2^-}{V_1^+}$ as well as the insertion loss and phase delay from port 1 to port 2.

$$[S_{\text{dev1}}] = \begin{bmatrix} 0.16\angle 30° & 0.9\angle -135° \\ 0.9\angle -135° & 0.28\angle -25° \end{bmatrix} \quad \& \quad [S_{\text{dev2}}] = \begin{bmatrix} 0.2\angle -30° & 0.8\angle -65° \\ 0.8\angle -65° & 0.3\angle 20° \end{bmatrix}$$

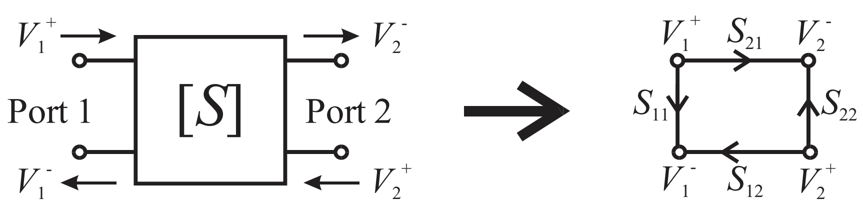


a) From earlier problem, the S-matrix for a lossless matched TL w/ electrical length $\beta\ell$ is

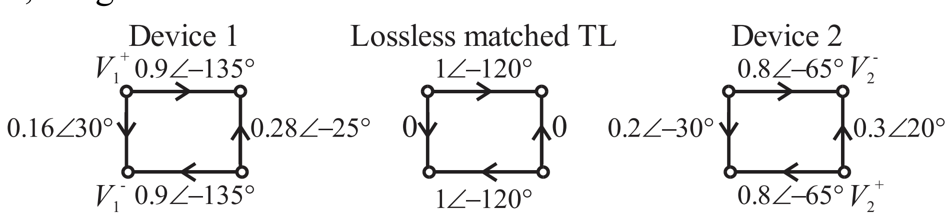
$$[S_{\text{TL}}] = \begin{bmatrix} 0 & e^{-j\beta\ell} \\ e^{-j\beta\ell} & 0 \end{bmatrix} = \begin{bmatrix} 0 & e^{-j120(\pi/180)} \\ e^{-j120(\pi/180)} & 0 \end{bmatrix} \Rightarrow$$

$$\boxed{[S_{\text{TL}}] = \begin{bmatrix} 0 & -0.5 - j0.866 \\ -0.5 - j0.866 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1\angle -120° \\ 1\angle -120° & 0 \end{bmatrix}}$$

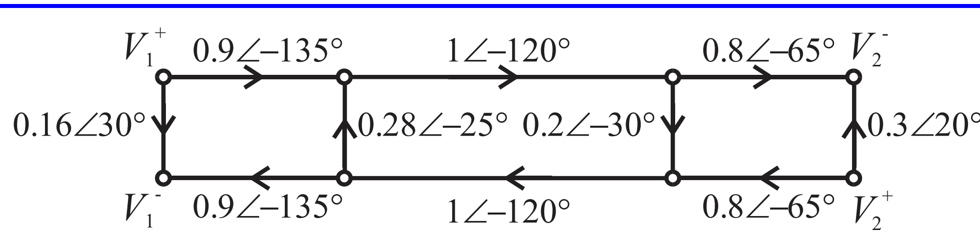
b) Per Fig 4.14, we can get the SFG for two-port devices in terms of the S-parameters as



Then, we get for our three devices-

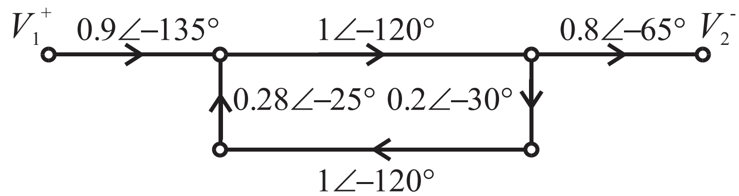


, and a SFG

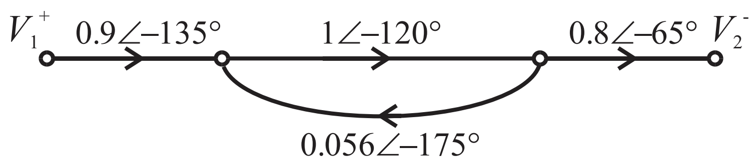


c)

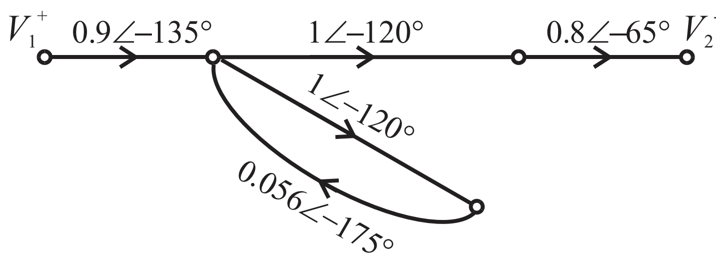
Step 1 'port 2 is matched' $\Rightarrow V_2^+ = 0$. For $T_{21} = \dfrac{V_2^-}{V_1^+}$, the node V_1^- is a dead end and can be omitted. This simplifies the SFG to



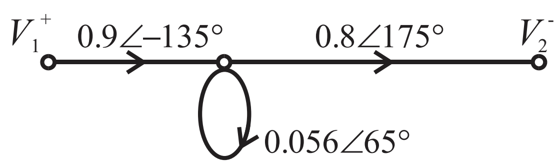
Step 2 Use series rule on the bottom feedback loop to get



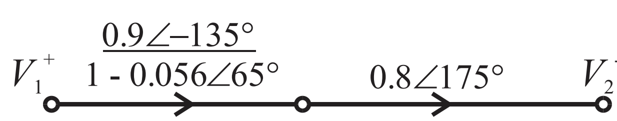
Step 3 Split the second from the right node to create a self loop



Step 4 Use series rule on the self loop <u>and</u> the rightmost two branches to get



Step 5 Use self loop rule to get



Step 6 Use series rule to get

$$V_2^- = \frac{0.9\angle -135°}{1-0.056\angle 65°}(0.8\angle 175°)\, V_1^+ = (0.736458633\angle 42.9757572°)\, V_1^+$$

$$\boxed{T_{21} = \frac{V_2^-}{V_1^+} = 0.73646\angle 42.976°}$$

The insertion loss is (2.52) IL = -20 log $|T_{21}|$ = -20 log 0.73646 $\Rightarrow$ **IL = 2.657 dB**.

From $\angle T_{21}$, the phase delay is -42.976° + 360° $\Rightarrow$ **phase delay$_{21}$ = 317.025°**.