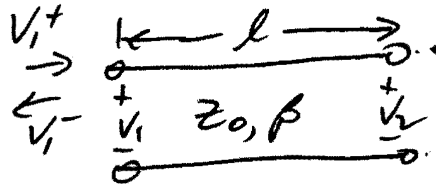
Find the *S*-parameters of a matched lossless transmission line (TL) with a characteristic impedance Z_0 and an electrical length $\beta\ell$.

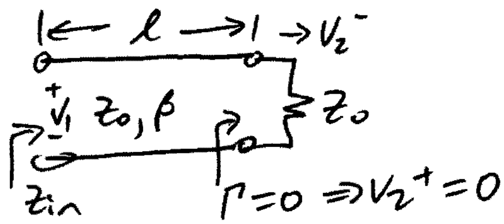
$V_1 = V_1^+ + V_1^-$



$V_2 = V_2^+ + V_2^-$

Per (4.41), $S_{ij} = \left.\dfrac{V_i^-}{V_j^+}\right|_{V_k^+=0 \text{ for } k \neq j}$

$\boxed{S_{11} \,\&\, S_{21}}$ $V_2^+ = 0 \Rightarrow Z_L = Z_0$ on port 2



$\underline{\underline{S_{11} = \dfrac{V_1^-}{V_1^+} = \Gamma_1 = 0}}$ since $Z_{in} = Z_0$

By symmetry, $\underline{\underline{S_{22} = S_{11} = 0}}$

$S_{21} = \dfrac{V_2^-}{V_1^+}$

Per (2.36a), $V(z) = V_0^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$

Here, $V_0^+ = V_2^-$ and $\Gamma = 0 \Rightarrow V(z) = V_2^- e^{-j\beta z}$

@ $z = -\ell$, $V(-\ell) = V_1 = V_2^- e^{-j\beta(-\ell)}$

$V_1^+ + \underbrace{V_1^-}_{0 \text{ (since } \Gamma_1 = 0)} = V_2^- e^{j\beta\ell} \Rightarrow \underline{\underline{S_{21} = e^{-j\beta\ell} = S_{12}}}$

$$\boxed{[S] = \begin{bmatrix} 0 & e^{-j\beta\ell} \\ e^{-j\beta\ell} & 0 \end{bmatrix}}$$