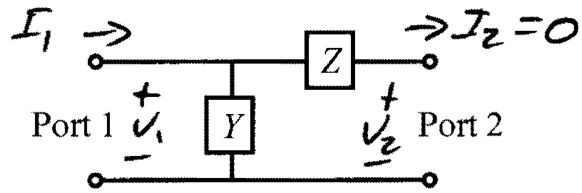
For the circuit shown below: a) Find the *ABCD* matrix by direct calculation using the definition of the *ABCD* matrix. b) Next, compute the *ABCD* matrices for each element separately using Table 4.1. c) Lastly, multiply the individual *ABCD* matrices and compare the result with the first answer.

a)

Per Ex. 4.6, $A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$

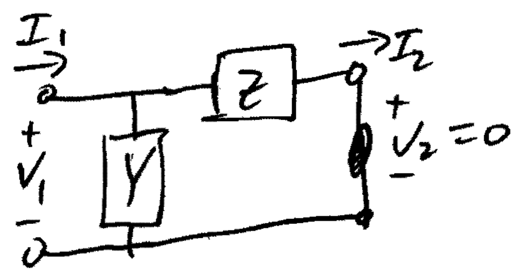


By KVL, $-V_1 + 0(Z) + V_2 = 0 \Rightarrow V_1 = V_2 \Rightarrow \underline{\underline{A = 1}}$

Ohm's Law $I_1 = V_1 Y = V_2 Y \Rightarrow \underline{\underline{C = Y}}$

Per Ex. 4.6, $B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$

$D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$



Ohm's Law $I_2 = V_1/Z \Rightarrow Z = V_1/I_2 \Rightarrow \underline{\underline{B = Z}}$

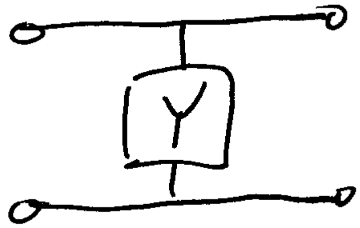
KCL $I_1 = YV_1 + I_2 = Y(I_2 Z) + I_2$

$= I_2(1 + YZ) \Rightarrow \frac{I_1}{I_2} = 1 + YZ \Rightarrow \underline{\underline{D = 1 + YZ}}$

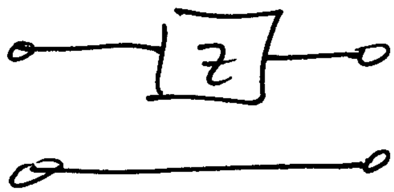
$$\boxed{[ABCD] = \begin{bmatrix} 1 & Z \\ Y & 1+YZ \end{bmatrix}}$$

b)

From Table 4.1



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_Y = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_Z = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

c) Per (4.71),

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{TOT} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_Y \begin{bmatrix} A & B \\ C & D \end{bmatrix}_Z$$

$$= \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ Y & 1+YZ \end{bmatrix}}$$

Same answer!