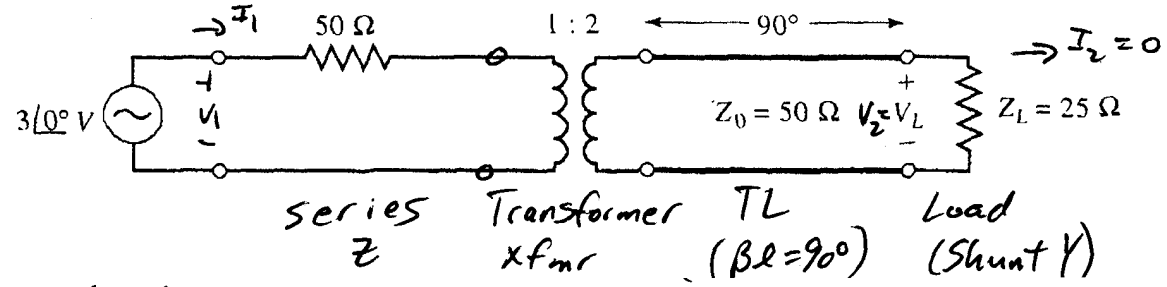
4.24 Use *ABCD* matrices to find the voltage V_L across the load resistor in the circuit shown below.



From Table 4.1

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{50\Omega} = \begin{bmatrix} 1 & 50\Omega \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{xfmr} = \begin{bmatrix} N & 0 \\ 0 & \frac{1}{N} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{TL} = \begin{bmatrix} \cos\beta\ell & jZ_0\sin\beta\ell \\ jY_0\sin\beta\ell & \cos\beta\ell \end{bmatrix} = \begin{bmatrix} \cos 90° & j50\sin 90° \\ j\frac{1}{50}\sin 90° & \cos 90° \end{bmatrix}$$

$$= \begin{bmatrix} 0 & j50\Omega \\ j0.025\ \mathrm{S} & 0 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{Load} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{25} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.04\,\mathrm{S} & 1 \end{bmatrix}$$

Per (4.71),

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{TOT} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & j50 \\ j0.02 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0.04 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 100 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & j50 \\ j0.02 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0.04 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} j2 & j25 \\ j0.04 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0.04 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} j3 & j25\Omega \\ j0.04\,\mathrm{S} & 0 \end{bmatrix}$$

Now, per (4.69)

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 = V_L \\ I_2 = 0 \end{bmatrix}$$

$$\begin{bmatrix} 3\angle 0^\circ \\ I_1 \end{bmatrix} = \begin{bmatrix} j3 & j25\Omega \\ j0.045 & 0 \end{bmatrix} \begin{bmatrix} V_L \\ 0 \end{bmatrix}$$

$$3\angle 0^\circ\ V = j3\ V_L + 0 \Rightarrow \underline{V_L = 1\angle -90^\circ\ V}$$

Bonus: $I_1 = j0.04\ V_L + 0$

$= j0.04\,(1\angle -90^\circ) \Rightarrow I_1 = 0.04\angle 0^\circ\ A$