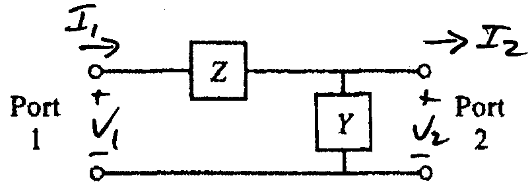
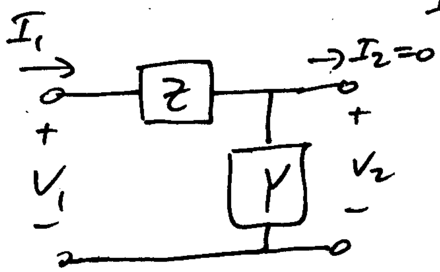
**4.23** Find the *ABCD* matrix for the circuit shown below by direct calculation using the definition of the *ABCD* matrix, and compare with the *ABCD* matrix of the appropriate cascade of canonical circuits from Table 4.1.




Direct Calculation

Per text (pp. 190-191), $A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ & $C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} \text{ \& } D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

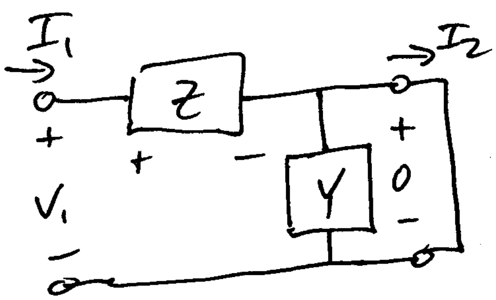



By voltage division

$$V_2 = V_1 \frac{1/Y}{Z+\frac{1}{Y}} = V_1\left(\frac{1}{YZ+1}\right)$$

$$\underline{\underline{A = \frac{V_1}{V_2} = YZ+1 = 1+YZ}}$$

By Ohm's Law $V_2 = I_1/Y \Rightarrow \underline{\underline{C = Y}}$



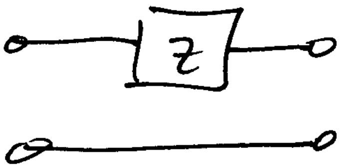

$$I_2 = I_1 \Rightarrow \underline{\underline{D = \frac{I_1}{I_2} = 1}}$$

By KVL, $V_1 = I_1 Z + 0$

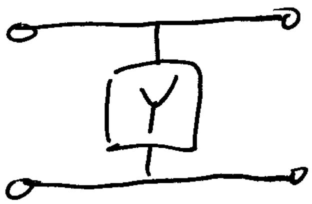
$$\hookrightarrow \underline{\underline{B = \frac{V_1}{I_1} = Z}}$$

$$\boxed{\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+YZ & Z \\ Y & 1 \end{bmatrix}}$$

From Table 4.1



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_Z = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_Y = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

Per (4.71),

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{TOT} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_Z \begin{bmatrix} A & B \\ C & D \end{bmatrix}_Y$$

$$= \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+YZ & Z \\ Y & 1 \end{bmatrix}}$$

Same answer!