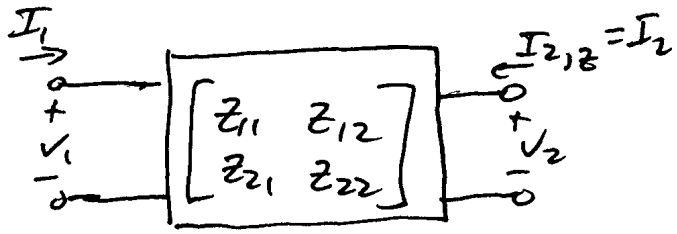
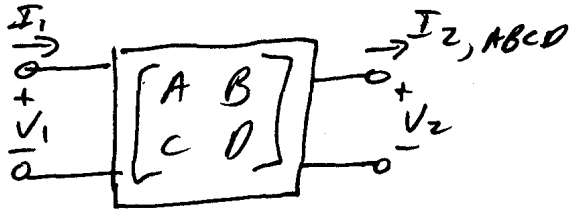
4.22 Derive expressions that give the impedance parameters in terms of the *ABCD* parameters.





$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_{2,ABCD} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Per (4.28)

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} \quad Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} \quad Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

Per Text (pp. 190-191), $A = \frac{V_1}{V_2}\big|_{I_2=0}$ & $C = \frac{I_1}{V_2}\big|_{I_2=0}$

$$\frac{A}{C} = \frac{V_1/V_2}{I_1/V_2}\bigg|_{I_2=0} = \frac{V_1}{I_1}\bigg|_{I_2=0} \Rightarrow \underline{\underline{Z_{11} = A/C}}$$

$$\frac{1}{C} = \frac{1}{\frac{I_1}{V_2}\big|_{I_2=0}} = \frac{V_2}{I_1}\bigg|_{I_2=0} \Rightarrow \underline{\underline{Z_{21} = \frac{1}{C}}}$$

Multiply out [ABCD] matrices & set $I_1 = 0$

$$V_1 = AV_2 - BI_2 = A\frac{D}{C}I_2 - BI_2 \Rightarrow \frac{V_1}{I_2}\bigg|_{I_1=0} = \frac{AD}{C} - B = \underline{\underline{\frac{AD-BC}{C} = Z_{12}}}$$

$$I_1 = 0 = CV_2 - DI_2 \Rightarrow \frac{V_2}{I_2}\bigg|_{I_1=0} = \underline{\underline{D/C = Z_{22}}}$$

$\hookrightarrow CV_2 = DI_2$

$\hookrightarrow V_2 = \frac{D}{C}I_2$ (Sub above)

$$\boxed{[Z] = \begin{bmatrix} A/C & \frac{AD-BC}{C} \\ \frac{1}{C} & D/C \end{bmatrix}}$$