For [*S*] of problem 4.18, find [*S'*] when ports 3 & 4 are separately connected with lossless matched TLs of electrical length 45°.

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4.18 A four-port network has the scattering matrix shown as follows. If ports 3 and 4 are connected with a lossless matched transmission line with an electrical length of 45°, find the resulting insertion loss and phase delay between ports 1 and 2.

$$[S] = \begin{bmatrix} 0.2\angle 50^\circ & 0 & 0 & 0.4\angle -45^\circ \\ 0 & 0.6\angle 45^\circ & 0.7\angle -45^\circ & 0 \\ 0 & 0.7\angle -45^\circ & 0.6\angle 45^\circ & 0 \\ 0.4\angle -45^\circ & 0 & 0 & 0.5\angle 45^\circ \end{bmatrix}$$

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Use (4.56), with $\theta_1 = \theta_2 = 0$ and $\theta_3 = \theta_4 = 45^\circ$.

$$\begin{bmatrix} e^{0} & 0 & 0 & 0 \\ 0 & e^{0} & 0 & 0 \\ 0 & 0 & e^{-j45^\circ} & 0 \\ 0 & 0 & 0 & e^{-j45^\circ} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1\angle -45^\circ & 0 \\ 0 & 0 & 0 & 1\angle -45^\circ \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1\angle -45^\circ & 0 \\ 0 & 0 & 0 & 1\angle -45^\circ \end{bmatrix} \begin{bmatrix} 0.2\angle 50^\circ & 0 & 0 & 0.4\angle -45^\circ \\ 0 & 0.6\angle 45^\circ & 0.7\angle -45^\circ & 0 \\ 0 & 0.7\angle -45^\circ & 0.6\angle 45^\circ & 0 \\ 0.4\angle -45^\circ & 0 & 0 & 0.5\angle 45^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 0.2\angle 50^\circ & 0 & 0 & 0.4\angle -45^\circ \\ 0 & 0.6\angle 45^\circ & 0.7\angle -45^\circ & 0 \\ 0 & 0.7\angle -90^\circ & 0.6\angle 0^\circ & 0 \\ 0.4\angle -90^\circ & 0 & 0 & 0.5\angle 0^\circ \end{bmatrix}$$

$$[S'] = \begin{bmatrix} 0.2\angle 50^\circ & 0 & 0 & 0.4\angle -45^\circ \\ 0 & 0.6\angle 45^\circ & 0.7\angle -45^\circ & 0 \\ 0 & 0.7\angle -90^\circ & 0.6\angle 0^\circ & 0 \\ 0.4\angle -90^\circ & 0 & 0 & 0.5\angle 0^\circ \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1\angle -45^\circ & 0 \\ 0 & 0 & 0 & 1\angle -45^\circ \end{bmatrix}$$

$$[S'] = \begin{bmatrix} 0.2\angle 50^\circ & 0 & 0 & 0.4\angle -90^\circ \\ 0 & 0.6\angle 45^\circ & 0.7\angle -90^\circ & 0 \\ 0 & 0.7\angle -90^\circ & 0.6\angle -45^\circ & 0 \\ 0.4\angle -90^\circ & 0 & 0 & 0.5\angle -45^\circ \end{bmatrix}$$