

For $[S]$ of problem 4.18, find $[S']$ when ports 1 & 3 are connected using lossless matched adapters of electrical length 10° while ports 2 & 4 are connected using lossless matched adapters of electrical length 15° .

$$\text{From 4.18, } [S] = \begin{bmatrix} 0.2\angle 50^\circ & 0 & 0 & 0.4\angle -45^\circ \\ 0 & 0.6\angle 45^\circ & 0.7\angle -45^\circ & 0 \\ 0 & 0.7\angle -45^\circ & 0.6\angle 45^\circ & 0 \\ 0.4\angle -45^\circ & 0 & 0 & 0.5\angle 45^\circ \end{bmatrix}.$$

$$\text{Use (4.56), } [S'] = \begin{bmatrix} e^{-j\theta_1} & 0 & 0 & 0 \\ 0 & e^{-j\theta_2} & 0 & 0 \\ 0 & 0 & e^{-j\theta_3} & 0 \\ 0 & 0 & 0 & e^{-j\theta_4} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & 0 & 0 & 0 \\ 0 & e^{-j\theta_2} & 0 & 0 \\ 0 & 0 & e^{-j\theta_3} & 0 \\ 0 & 0 & 0 & e^{-j\theta_4} \end{bmatrix} \text{ where we are}$$

given that $\theta_1 = \theta_3 = 10^\circ$ and $\theta_2 = \theta_4 = 15^\circ$. Going to all polar format, multiply the first two matrices to get

$$\begin{bmatrix} 1\angle -10^\circ & 0 & 0 & 0 \\ 0 & 1\angle -15^\circ & 0 & 0 \\ 0 & 0 & 1\angle -10^\circ & 0 \\ 0 & 0 & 0 & 1\angle -15^\circ \end{bmatrix} \begin{bmatrix} 0.2\angle 50^\circ & 0 & 0 & 0.4\angle -45^\circ \\ 0 & 0.6\angle 45^\circ & 0.7\angle -45^\circ & 0 \\ 0 & 0.7\angle -45^\circ & 0.6\angle 45^\circ & 0 \\ 0.4\angle -45^\circ & 0 & 0 & 0.5\angle 45^\circ \end{bmatrix} \\ = \begin{bmatrix} 0.2\angle 40^\circ & 0 & 0 & 0.4\angle -55^\circ \\ 0 & 0.6\angle 30^\circ & 0.7\angle -60^\circ & 0 \\ 0 & 0.7\angle -55^\circ & 0.6\angle 35^\circ & 0 \\ 0.4\angle -60^\circ & 0 & 0 & 0.5\angle 30^\circ \end{bmatrix}.$$

Then, matrix multiply this result with the last matrix in polar format-

$$[S'] = \begin{bmatrix} 0.2\angle 40^\circ & 0 & 0 & 0.4\angle -55^\circ \\ 0 & 0.6\angle 30^\circ & 0.7\angle -60^\circ & 0 \\ 0 & 0.7\angle -55^\circ & 0.6\angle 35^\circ & 0 \\ 0.4\angle -60^\circ & 0 & 0 & 0.5\angle 30^\circ \end{bmatrix} \begin{bmatrix} 1\angle -10^\circ & 0 & 0 & 0 \\ 0 & 1\angle -15^\circ & 0 & 0 \\ 0 & 0 & 1\angle -10^\circ & 0 \\ 0 & 0 & 0 & 1\angle -15^\circ \end{bmatrix}$$

$$[S'] = \begin{bmatrix} 0.2\angle 30^\circ & 0 & 0 & 0.4\angle -70^\circ \\ 0 & 0.6\angle 15^\circ & 0.7\angle -70^\circ & 0 \\ 0 & 0.7\angle -70^\circ & 0.6\angle 25^\circ & 0 \\ 0.4\angle -70^\circ & 0 & 0 & 0.5\angle 15^\circ \end{bmatrix}$$