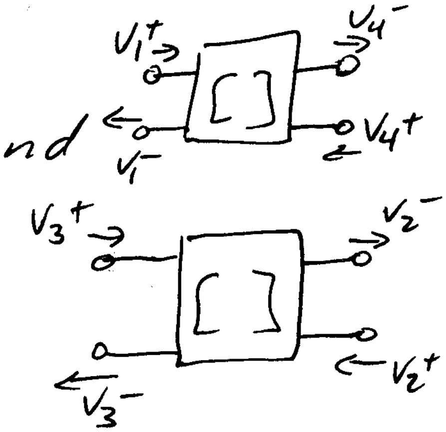
4.18 A four-port network has the scattering matrix shown as follows. If ports 3 and 4 are connected with a lossless matched transmission line with an electrical length of 45°, find the resulting insertion loss and phase delay between ports 1 and 2.

$$[S] = \begin{bmatrix} 0.2\angle 50^\circ & 0 & 0 & 0.4\angle -45^\circ \\ 0 & 0.6\angle 45^\circ & 0.7\angle -45^\circ & 0 \\ 0 & 0.7\angle -45^\circ & 0.6\angle 45^\circ & 0 \\ 0.4\angle -45^\circ & 0 & 0 & 0.5\angle 45^\circ \end{bmatrix}$$

From the 'interesting' arrangement of the [S]-matrix, we can create two separate 2x2 systems.

$$\begin{bmatrix} V_1^- \\ V_4^- \end{bmatrix} = \begin{bmatrix} 0.2\angle 50^\circ & 0.4\angle -45^\circ \\ 0.4\angle -45^\circ & 0.5\angle 45^\circ \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_4^+ \end{bmatrix}$$

and



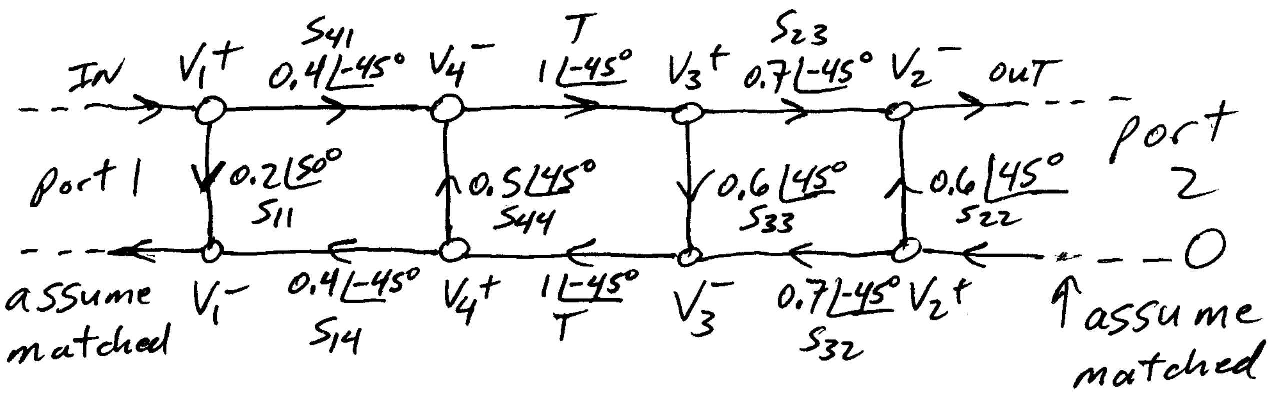
$$\begin{bmatrix} V_2^- \\ V_3^- \end{bmatrix} = \begin{bmatrix} 0.6\angle 45^\circ & 0.7\angle -45^\circ \\ 0.7\angle -45^\circ & 0.6\angle 45^\circ \end{bmatrix} \begin{bmatrix} V_2^+ \\ V_3^+ \end{bmatrix}$$

Given that ports 3 & 4 are connected w/ a lossless matched TL w/ electrical length $45^\circ = \beta \ell$

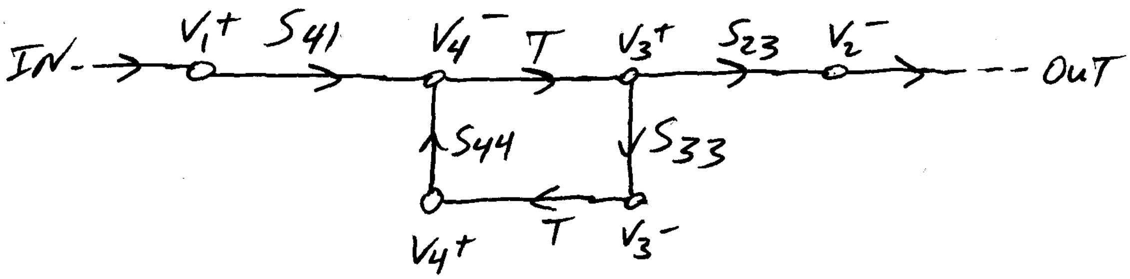
$$V_4^+ = V_3^-(1\angle -45^\circ) = V_3^- e^{-j45^\circ} = V_3^-(T)$$

$$V_3^+ = V_4^- e^{-j45^\circ} = V_4^-(1\angle -45^\circ) = V_4^-(T)$$

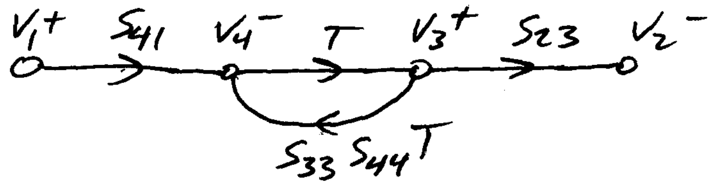
Using the above and Fig 4.14, the SFG below can be created.



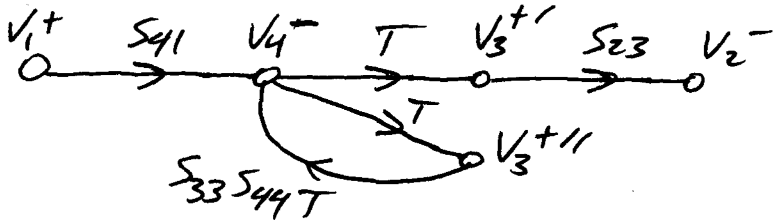
Step 1 Given that ports 1 & 2 are matched, we can eliminate nodes V_1^- (deadend, no impact on forward path) and V_2^+ (nothing coming *in* port 2)



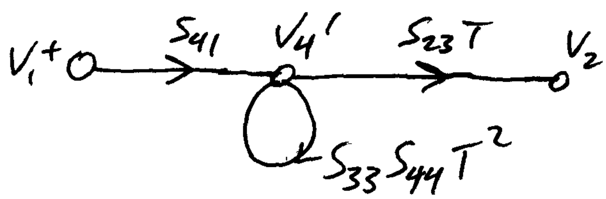
Step 2 Eliminate nodes V_4^+ and V_3^- using series rule.



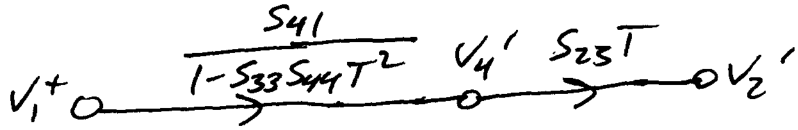
Step 3 Split node V_3^+



Step 4 Apply series rule to eliminate $V_3^{+\prime}$ & $V_3^{+\prime\prime}$.



Step 5 Apply self-loop rule



Step 6 Apply series rule to get

$$V_2 = \frac{S_{23} S_{41} T}{1 - S_{33} S_{44} T^2} V_1^+$$

$$T_{21} = \frac{V_2^-}{V_1^+} = \frac{S_{23} S_{41} T}{1 - S_{33} S_{44} T^2}$$

$$= \frac{(0.7\angle -45^\circ)(0.4\angle -45^\circ)(1\angle -45^\circ)}{1 - (0.6\angle 45^\circ)(0.5\angle 45^\circ)(1\angle -45^\circ)^2} = \frac{0.28\angle -135^\circ}{1 - 0.3}$$

$$T_{21} = 0.4\angle -135^\circ$$

$$\text{Phase delay}_{1\leftrightarrow 2} = 135^\circ$$

Per (2.52), $IL = -20 \log |T|$

$$IL_{21} = -20 \log (0.4)$$

$$IL_{21} = 7.9588 \text{ dB}$$