A four-port network has the scattering matrix shown as follows. If ports 3 and 4 are connected with a lossless matched transmission line with an electrical length of 45°, find the resulting insertion loss and phase delay between ports 1 and 2.

$$[S] = \begin{bmatrix} 0.2\angle 50^{\circ} & 0 & 0 & 0.4\angle -45^{\circ} \\ 0 & 0.6\angle 45^{\circ} & 0.7\angle -45^{\circ} & 0 \\ 0 & 0.7\angle -45^{\circ} & 0.6\angle 45^{\circ} & 0 \\ 0.4\angle -45^{\circ} & 0 & 0 & 0.5\angle 45^{\circ} \end{bmatrix}$$

From the interesting arrangement of the [5]-matrix we can create two separate 2x2 systems.

$$\begin{bmatrix} V_{1}^{-} \\ V_{4}^{-} \end{bmatrix} = \begin{bmatrix} 0.2150^{\circ} & 0.4145^{\circ} \\ 0.4145^{\circ} & 0.5145^{\circ} \end{bmatrix} \begin{bmatrix} V_{1}^{+} \\ V_{4}^{+} \end{bmatrix} \text{ and } \begin{bmatrix} V_{1}^{+} \\ V_{4}^{-} \end{bmatrix}$$

$$\begin{bmatrix} V_{4}^{-} \end{bmatrix} \begin{bmatrix} 0.76.15 & 0.545 \end{bmatrix} \begin{bmatrix} V_{4}^{+} \end{bmatrix} = \begin{bmatrix} 0.66.145^{\circ} & 0.72-45^{\circ} \\ V_{3}^{-} \end{bmatrix} \begin{bmatrix} V_{2}^{+} \\ 0.76.45^{\circ} & 0.666.145^{\circ} \end{bmatrix} \begin{bmatrix} V_{2}^{+} \\ V_{2}^{+} \end{bmatrix}$$

Given that ports 344 are connected w/ a lossless matched TL welectrical length 450 = Bl V4+ = V3- (11-450) = V2-e3450 = V3-(T)

$$V_3^{\dagger} = V_4 - e^{-545^{\circ}} = V_4 - (16-45^{\circ}) = V_4 - (7)$$

Using the above and Fig 4.14, the SFG below can be created.

IN VI+ 0.41-45° V4 11-45° V3+ 0.71-45° V2 OUT $V_0.2150^{\circ}$ 0.5145° $V_0.6145^{\circ}$ 0.6145° $V_0.6145^{\circ}$ $V_0.6145$ Step! Given that ports 142 are matched, we can eliminate nodes Vi (deadend, no impact on forward path) and Vzt (nothing coming in port 2)

IN - > 0 $V_1^{+} > 541$ $V_4^{-} T V_3^{+} > 523 V_2^{-}$ $V_2^{-} > 00T$ $V_3^{+} = V_3^{-} > 00T$ $V_4^{+} = V_3^{-} > 00T$

Step 2 Eliminate nodes Vy and V3 using series rule.

Vit Syl Vy T V3 523 Vz

See Sunt

Step 3 Split node V3+

Vit S41 V4- T V3+ S23 V2
S33 544 T

Step4 Apply series rule to eliminate 13th 415th

V+0 541 V41 523T V21

Szz S44T2

Step 5 Apply <u>Self-loof</u> rule

<u>S41</u>
V+0 1-533 S44T2 V4' S25T
OV2'

Step 6 Apply series rule to get

$$V_{2} = \frac{S_{23} S_{41} T}{1 - S_{33} S_{44} T^{2}} V_{1}^{+}$$

$$T_{21} = \frac{V_{2}^{-}}{V_{1}^{+}} = \frac{S_{22} S_{41} T}{1 - S_{33} S_{44} T^{2}}$$

$$= \frac{(0.7 + 45^{\circ})(0.4 + 45^{\circ})(1 + 45^{\circ})}{1 - (0.6 + 45^{\circ})(0.5 + 45^{\circ})(1 + 45^{\circ})^{2}} = \frac{0.28 (-135^{\circ})}{1 - 0.3}$$

$$T_{21} = 0.4 (-135^{\circ})$$

$$\text{Phase delay}_{1 \leftrightarrow 2} = 135^{\circ}$$

$$\text{Per } (2.52), \quad IL = -20 \log |T|$$

$$IL_{21} = -20 \log (0.4)$$

$$IL_{21} = 7.9588 dB$$