4.14 A four-port network has the scattering matrix shown as follows. (a) Is this network lossless? (b) Is this network reciprocal? (c) What is the return loss at port 1 when all other ports are terminated with matched loads? (d) What is the insertion loss and phase delay between ports 2 and 4 when all other ports are terminated with matched loads? (e) What is the reflection coefficient seen at port 1 if a short circuit is placed at the terminal plane of port 3 and all other ports are terminated with matched loads?

$$[S] = \begin{bmatrix} 0.178\angle 90° & 0.6\angle 45° & 0.4\angle 45° & 0 \\ 0.6\angle 45° & 0 & 0 & 0.3\angle -45° \\ 0.4\angle 45° & 0 & 0 & 0.5\angle -45° \\ 0 & 0.3\angle -45° & 0.5\angle -45° & 0 \end{bmatrix}.$$

a) Per (4.51) $[S]^t[S]^* = [U]$ for a lossless network,

$$\begin{bmatrix} 0.178\angle 90° & 0.6\angle 45° & 0.4\angle 45° & 0 \\ 0.6\angle 45° & 0 & 0 & 0.3\angle -45° \\ 0.4\angle 45° & 0 & 0 & 0.5\angle -45° \\ 0 & 0.3\angle -45° & 0.5\angle -45° & 0 \end{bmatrix} \begin{bmatrix} 0.178\angle -90° & 0.6\angle -45° & 0.4\angle -45° & 0 \\ 0.6\angle -45° & 0 & 0 & 0.3\angle 45° \\ 0.4\angle -45° & 0 & 0 & 0.5\angle 45° \\ 0 & 0.3\angle 45° & 0.5\angle 45° & 0 \end{bmatrix} \stackrel{?}{=} [U]$$

Check first element by multiplying top row + first column

$$(0.178\angle 90°)(0.178\angle -90°) + (0.6\angle 45°)(0.6\angle -45°) + (0.4\angle 45°)(0.4\angle -45°) + 0^2 \stackrel{?}{=} 1$$

$$0.178^2 + 0.6^2 + 0.4^2 + 0 \stackrel{?}{=} 1$$

$$0.551684 \neq 1$$

$\Rightarrow$ $[S]$ is NOT lossless

b) By observation, $[S] = [S]^t$ (see part a).

By (4.48), $[S]$ is reciprocal.

c) By definition (2.38), $RL = -20\log|\Gamma|$ (dB)

$$RL_1 = -20\log|S_{11}| = -20\log 0.178 \Rightarrow RL_1 = 14.9916 \text{ dB}$$

d) With ports 1 + 3 terminated in matched loads, $V_1^+ = V_3^+ = 0$. Then, from (4.40) $[V^-] = [S][V^+]$

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \\ V_4^- \end{bmatrix} = \begin{bmatrix} 0.178\angle 90^\circ & 0.6\angle 45^\circ & 0.4\angle 45^\circ & 0 \\ 0.6\angle 45^\circ & 0 & 0 & 0.3\angle -45^\circ \\ 0.4\angle 45^\circ & 0 & 0 & 0.5\angle -45^\circ \\ 0 & 0.3\angle -45^\circ & 0.5\angle -45^\circ & 0 \end{bmatrix} \begin{bmatrix} 0 \\ V_2^+ \\ 0 \\ V_4^+ \end{bmatrix}$$

$$V_2^- = 0.6\angle 45^\circ(0) + 0V_2^+ + 0(0) + 0.3\angle -45^\circ V_4^+$$

$$V_2^- = 0.3\angle -45^\circ V_4^+ \Rightarrow T_{24} = \frac{V_2^-}{V_4^+} = 0.3\angle -45^\circ$$

Per (2.52), $IL = -20\log|T|$ (dB)

$$IL_{24} = -20\log 0.3 \Rightarrow \underline{\underline{IL_{24} = 10.458\,dB}}$$

$$V_2^- = |V_2^-|\angle\theta_2^- = (0.3\angle -45^\circ)\,|V_4^+|\angle\theta_4^+$$

$$\Rightarrow \theta_2^- = \theta_4^+ - 45^\circ \Rightarrow \underline{\underline{\text{Phase delay}_{24} = 45^\circ}}$$

e) With a short at port 3, $\Gamma_3 = -1 \Rightarrow V_3^+ = -V_3^-$. With ports 2 + 4 matched, $V_2^+ = V_4^+ = 0$. Using (4.40),

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \\ V_4^- \end{bmatrix} = \begin{bmatrix} 0.178\angle 90^\circ & 0.6\angle 45^\circ & 0.4\angle 45^\circ & 0 \\ 0.6\angle 45^\circ & 0 & 0 & 0.3\angle -45^\circ \\ 0.4\angle 45^\circ & 0 & 0 & 0.5\angle -45^\circ \\ 0 & 0.3\angle -45^\circ & 0.5\angle -45^\circ & 0 \end{bmatrix} \begin{bmatrix} V_1^+ \\ 0 \\ -V_3^- \\ 0 \end{bmatrix}$$

e) cont

$$V_1^- = 0.178\angle 90^\circ\, V_1^+ + (0.6\angle 45^\circ)0 - (0.4\angle 45^\circ)V_3^- + 0(0)$$

$$V_1^- = 0.178\angle 90^\circ\, V_1^+ - 0.4\angle 45^\circ\, V_3^- \quad (A)$$

and

$$V_3^- = 0.4\angle 45^\circ\, V_1^+ + 0(0) + 0(-V_3^-) + 0.5\angle -45^\circ (0)$$

$$V_3^- = 0.4\angle\ V_1^+ \quad (B)$$

Substitute (B) into (A)

$$V_1^- = 0.178\angle 90^\circ\, V_1^+ - (0.4\angle 45^\circ)(0.4\angle 45^\circ)V_1^+$$

$$\Gamma_{11} = \frac{V_1^-}{V_1^+} = 0.178\angle 90^\circ - (0.4\angle 45^\circ)(0.4\angle 45^\circ)$$

$$\underline{\underline{\Gamma_{11} = 0.018\angle 90^\circ}}$$