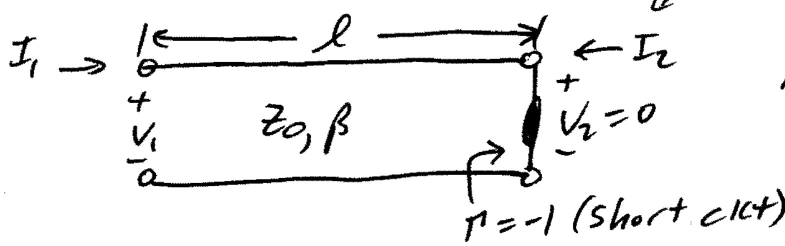
4.9 Find the **admittance** parameters of a section of transmission line with length ℓ, characteristic impedance Z_0, and propagation constant β.

Per (4.29), $Y_{ij} = \left.\dfrac{I_i}{V_j}\right|_{V_k=0 \text{ for } k \neq j}$

$\boxed{Y_{11}}$ Here, $Y_{11} = \left.\dfrac{I_1}{V_1}\right|_{V_2=0}$

Note direction



Find I_1 in terms of V_1

From (2.36a) + (2.36b), $V(z) = V_0^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$

$$I(z) = \frac{V_0^+}{Z_0}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right)$$

Euler's

So, $V_1 = V(-\ell) = V_0^+\left(e^{j\beta\ell} + (-1)e^{-j\beta\ell}\right) = V_0^+(2j\sin\beta\ell)$

$$I_1 = I(-\ell) = \frac{V_0^+}{Z_0}\left(e^{j\beta\ell} - (-1)e^{-j\beta\ell}\right) = \frac{V_0^+}{Z_0}(2\cos\beta\ell)$$

$$Y_{11} = \frac{\frac{V_0^+}{Z_0}(2\cos\beta\ell)}{V_0^+(2j\sin\beta\ell)} = \underline{\frac{1}{jZ_0\tan(\beta\ell)}} = Y_{22} \text{ (Symmetry)}$$

OR $\underline{\dfrac{-j\cot(\beta\ell)}{Z_0}}$

$\boxed{Y_{21}}$ Here, $Y_{21} = \left.\dfrac{I_2}{V_1}\right|_{V_2=0}$

So, $V_1 = V(-\ell) = V_0^+(2j\sin\beta\ell)$

$$I_2 = -I(0) = \frac{-V_0^+}{Z_0}(1+1) = \frac{-2V_0^+}{Z_0}$$

OR $+\dfrac{j\csc\beta\ell}{Z_0}$

$$Y_{21} = \frac{I_2}{V_1} = \frac{-2V_0^+/Z_0}{V_0^+(2j\sin\beta\ell)} = \underline{\frac{-1}{jZ_0\sin\beta\ell}} = Y_{12}$$

(Symmetry)

$$\boxed{[Y] = \begin{bmatrix} \dfrac{1}{jZ_0\tan(\beta\ell)} & \dfrac{-1}{jZ_0\sin(\beta\ell)} \\ \dfrac{-1}{jZ_0\sin(\beta\ell)} & \dfrac{1}{jZ_0\tan(\beta\ell)} \end{bmatrix}}$$