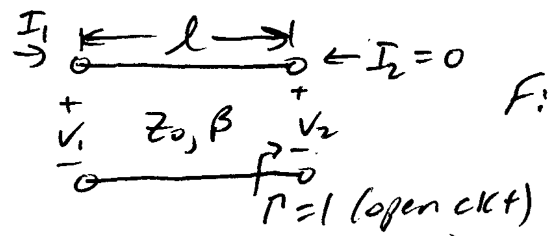
4.9 Find the impedance parameters of a section of transmission line with length ℓ, characteristic impedance Z_0, and propagation constant β.

Per (4.28), $Z_{ij} = \left.\dfrac{V_i}{I_j}\right|_{I_k=0 \text{ for } k \neq j}$

Z_{11}



Find I_1 in terms of V_1

From (2.36a) & (2.36b),

$$V(z) = V_0^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$$

$$I(z) = \frac{V_0^+}{Z_0}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right)$$

$$V_1 = V(-\ell) = V_0^+\left(e^{+j\beta\ell} + (1)e^{-j\beta\ell}\right) = 2V_0^+\cos\beta\ell$$

$$I_1 = I(-\ell) = \frac{V_0^+}{Z_0}\left(e^{+j\beta\ell} - (1)e^{-j\beta\ell}\right) = j\frac{2V_0^+}{Z_0}\sin\beta\ell$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{2V_0^+\cos(\beta\ell)}{j\frac{2V_0^+}{Z_0}\sin(\beta\ell)} = \underline{-jZ_0\cot(\beta\ell)} = Z_{22} \text{ (symmetry)}$$

Z_{21} Same circuit.

$$V_2 = V(0) = V_0^+\left(e^0 + 1e^0\right) = 2V_0^+$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{2V_0^+}{j\frac{2V_0^+}{Z_0}\sin(\beta\ell)} = \frac{-jZ_0}{\sin(\beta\ell)} = \underline{-jZ_0\csc(\beta\ell)}$$

$$= Z_{12} \text{ (symmetry)}$$

$$\boxed{[Z] = \begin{bmatrix} -jZ_0\cot(\beta\ell) & -jZ_0\csc(\beta\ell) \\ -jZ_0\csc(\beta\ell) & -jZ_0\cot(\beta\ell) \end{bmatrix}}$$