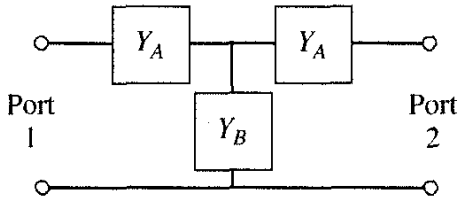
4.7 Derive the [Z] and [Y] matrices for the two-port networks shown in the figure below.

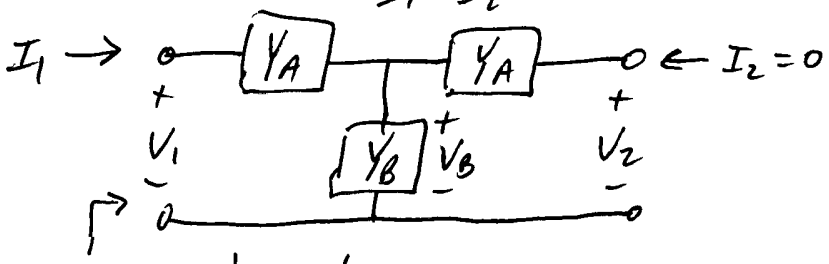


(b)

Note that the Tee-network is symmetric.

Per (4.28), $z_{ij} = \left.\frac{V_i}{I_j}\right|_{I_{k\neq j}=0}$

z_{11} Here $z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$ and the circuit is



Find I_1 in terms of V_1.

$z_{eq} = \frac{1}{Y_A} + \frac{1}{Y_B}$

$I_1 = V_1 / z_{eq} = V_1 / \left(\frac{1}{Y_A} + \frac{1}{Y_B}\right)$

$z_{11} = \frac{V_1}{I_1} = \frac{V_1}{V_1/\left(\frac{1}{Y_A}+\frac{1}{Y_B}\right)} = \frac{1}{Y_A} + \frac{1}{Y_B} \Rightarrow z_{11} = \frac{Y_A + Y_B}{Y_A Y_B} = z_{22}$

z_{21} Here $z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$ w/ same circuit.

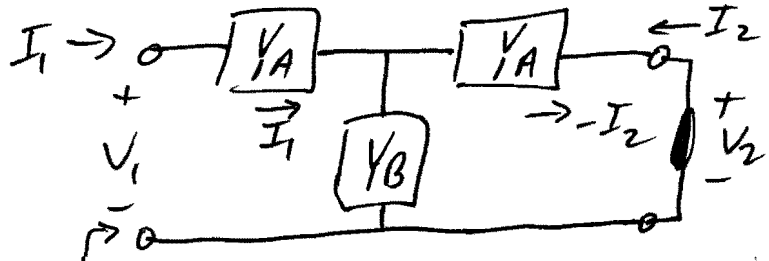
By Ohm's Law, $V_B = I_1 / Y_B$. By KVL, $-V_B + 0 + V_2 = 0$
$\hookrightarrow V_2 = V_B$

$V_B = V_2 = I_1/Y_B \Rightarrow z_{21} = \frac{I_1/Y_B}{I_1} \Rightarrow z_{21} = \frac{1}{Y_B} = z_{12}$

$$[Z] = \begin{bmatrix} \frac{Y_A + Y_B}{Y_A Y_B} & \frac{1}{Y_B} \\ \frac{1}{Y_B} & \frac{Y_A + Y_B}{Y_A Y_B} \end{bmatrix}$$

Per (4.29), $Y_{ij} = \left.\frac{I_i}{V_j}\right|_{V_k=0\ k\neq j}$.

Y_{11} Here $Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$ and the circuit is



Find V_1 in terms of I_1

$$Z_{eq} = \frac{1}{Y_A} + \frac{1}{Y_A+Y_B} = \frac{(Y_A+Y_B)+Y_A}{Y_A(Y_A+Y_B)} = \frac{2Y_A+Y_B}{Y_A(Y_A+Y_B)}$$

$$V_1 = I_1 Z_{eq} = I_1 \frac{2Y_A+Y_B}{Y_A(Y_A+Y_B)}$$

$$Y_{11} = \frac{I_1}{I_1 \frac{2Y_A+Y_B}{Y_A(Y_A+Y_B)}} \Rightarrow \underline{Y_{11} = \frac{Y_A(Y_A+Y_B)}{2Y_A+Y_B} = Y_{22}}$$

Y_{21} Here $Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$ w/ same circuit.

By current division, $-I_2 = I_1 \frac{Y_A}{Y_A+Y_B}$. So,

$$I_2 = -I_1 \frac{Y_A}{Y_A+Y_B} = \frac{-V_1}{Z_{eq}}\left(\frac{Y_A}{Y_A+Y_B}\right) = -V_1 \frac{Y_A(Y_A+Y_B)}{2Y_A+Y_B}\left(\frac{Y_A}{Y_A+Y_B}\right)$$

$$Y_{21} = \frac{-V_1 Y_A^2/(2Y_A+Y_B)}{V_1} \Rightarrow \underline{Y_{21} = \frac{-Y_A^2}{2Y_A+Y_B} = Y_{12}}$$

$$[Y] = \begin{bmatrix} \frac{Y_A(Y_A+Y_B)}{2Y_A+Y_B} & \frac{-Y_A^2}{2Y_A+Y_B} \\ \frac{-Y_A^2}{2Y_A+Y_B} & \frac{Y_A(Y_A+Y_B)}{2Y_A+Y_B} \end{bmatrix}$$