4.4 A two-port network is driven at both ports such that the port voltages and current have the following values ($Z_0 = 75\ \Omega$):

$$V_1 = 15\angle 90°\text{ V},\ \ I_1 = 0.2\angle 90°\text{ A},$$
$$V_2 = 12\angle 0°\text{ V},\ \ I_2 = 0.16\angle\text{-}90°\text{ A}$$

Determine the input impedance seen at each port, and find the incident and reflected voltages at each port.

Port 1

Using Ohm's Law, the input impedance $Z_{\text{in},1} = \dfrac{V_1}{I_1} = \dfrac{15\angle 90°}{0.2\angle 90°}$ $\Rightarrow$ $\boxed{Z_{\text{in},1} = 75\ \Omega}$.

Port 2

Using Ohm's Law, the input impedance $Z_{\text{in},2} = \dfrac{V_2}{I_2} = \dfrac{12\angle 0°}{0.16\angle -90°}$ $\Rightarrow$ $\boxed{Z_{\text{in},2} = j75\ \Omega}$.

Per (2.14a) & (2.14b), $V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$ & $I(z) = \dfrac{V_0^+}{Z_0} e^{-j\beta z} - \dfrac{V_0^-}{Z_0} e^{j\beta z}$ respectively.

At $z = 0$, these yield $V(0) = V_0^+ + V_0^-$ & $I(0) = \dfrac{V_0^+}{Z_0} - \dfrac{V_0^-}{Z_0}$. Solving these two equations for the incident and reflected voltage waves yields

$$V_0^+ = 0.5[V(0) + I(0)Z_0] \text{ and } V_0^- = 0.5[V(0) - I(0)Z_0].$$

Port 1

Let $V_1 = V(0)$ and $I_1 = I(0)$ and solve for the incident and reflected voltage waves

$$V_1^+ = 0.5[(15\angle 90°) + (0.2\angle 90°)75] \quad \Rightarrow \quad \boxed{V_1^+ = 15\angle 90°\text{ V}}$$

$$\text{and } V_1^- = 0.5[(15\angle 90°) - (0.2\angle 90°)75] \quad \Rightarrow \quad \boxed{V_1^- = 0}.$$

Port 2

Let $V_2 = V(0)$ and $I_2 = I(0)$ and solve for the incident and reflected voltage waves

$$V_2^+ = 0.5[(12\angle 0°) + (0.16\angle -90°)75] \quad \Rightarrow \quad \boxed{V_2^+ = 8.4853\angle\text{-}45°\text{ V}}$$

$$\text{and } V_2^- = 0.5[(12\angle 90°) - (0.16\angle -90°)75] \quad \Rightarrow \quad \boxed{V_2^- = 8.4853\angle 45°\text{ V}}.$$