4.4 A two-port network is driven at both ports such that the port voltages and currents have the following values ($Z_0 = 50\ \Omega$):

$$V_1 = 10\angle 90°, \quad I_1 = 0.2\angle 90°,$$
$$V_2 = 8\angle 0°, \quad I_2 = 0.16\angle -90°.$$

Determine the input impedance seen at each port, and find the incident and reflected voltages at each port.

Port 1 $\quad Z_{in,1} = \frac{V_1}{I_1} = \frac{10\angle 90°}{0.2\angle 90°} \Rightarrow Z_{in,1} = 50\,\Omega$

Port 2 $\quad Z_{in,2} = \frac{V_2}{I_2} = \frac{8\angle 0°}{0.16\angle -90°} \Rightarrow Z_{in,2} = 50\angle 90° = j50\,\Omega$

Per (2.14a) & (2.14b), $\quad V(0) = V_0^+ + V_0^-$

$$I(0) = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$ } solve

$$V_0^+ = \tfrac{1}{2}\left[V_0 + Z_0 I_0\right]$$
$$V_0^- = \tfrac{1}{2}\left[V_0 - Z_0 I_0\right]$$ where $Z_0 = 50\,\Omega$

Port 1 $\quad V_1^+ = \tfrac{1}{2}\left[10\angle 90° + 50(0.2\angle 90°)\right] \Rightarrow V_1^+ = 10\angle 90°\ V$

$$V_1^- = \tfrac{1}{2}\left[10\angle 90° - 50(0.2\angle 90°)\right] \Rightarrow V_1^- = 0$$

Port 2 $\quad V_2^+ = \tfrac{1}{2}\left[8\angle 0° + 50(0.16\angle -90°)\right] \Rightarrow V_2^+ = 5.657\angle -45°\ V$

$$V_2^- = \tfrac{1}{2}\left[8\angle 0° - 50(0.16\angle -90°)\right] \Rightarrow V_2^- = 5.657\angle 45°\ V$$